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PERMANENCE OF A LOTKA-VOLTERRA RATIO-DEPENDENT PREDATOR-PREY MODEL WITH FEEDBACK CONTROLS AND PREY DIFFUSION

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Abstract:-

A three species multi-delay Lotka-Volterra ratio-dependent predator-prey model with feedback controls and prey diffusion is investigated. By developing some new analysis methods, some sufficient conditions are derived for the permanence of the system.

Keywords: - *Predator-Prey Model, Feedback Control, Time-Delay, Permanence*

1. INTRODUCTION

The traditional predator-prey model has received great attention from both theoretical and mathematical biologists and has been studied extensively (see [1-4] and the references cited therein). Based on growing biological and physiological evidences, some scholars generally recognized that some kind of time delays are inevitable in population interactions and tend to be destabilizing in the sense that longer delays may destroy the stability of the system. Some results are shownin [5-9] and the references cited therein.In[9], the authors considered the following three species Lotka-Volterra type competitive-mutualism system with discrete time delays

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_{11}^1(t)x_1(t-\tau) - a_{11}^2(t)x_1(t-2\tau) - a_{12}(t)x_2(t-2\tau) + a_{13}(t)x_3(t-\tau)], \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_{21}(t)x_1(t-2\tau) - a_{22}^1(t)x_2(t-\tau) - a_{22}^2(t)x_2(t-2\tau) + a_{23}(t)x_3(t-\tau)], \\ \dot{x}_3(t) = x_3(t)[r_3(t) + a_{31}(t)x_1(t-\tau) + a_{32}(t)x_2(t-\tau) - a_{33}^1(t)x_3(t) - a_{33}^2(t)x_3(t-\tau)], \end{cases} \tag{1.1}$$

Where $x_i(t)(i=1,2,3)$ denote the density of the ith species at time t, τ is a positive constant, $r_i(t)(i=1,2,3), a_{11}^l(t), a_{22}^l(t), a_{33}^l(t)(l=1,2), a_{12}(t), a_{13}(t), a_{21}(t), a_{23}(t), a_{31}(t)$ and $a_{32}(t)$ are continuous, bounded and strictly positive functions on $[0,+\infty)$. The authors established some conditions on the boundedness, permanence and global attractivity for system (1.1).

Furthermore, in the real world, species can diffuse between patches. So we should consider the effect of dispersal on the permanence and global stability of the ecosystem. Some research results can see for example [10-13].Song and Chen in [10] studied the following two-species predator-prey system with diffusion

$$\begin{cases} \dot{x}_1 = x_1[a_1(t) - b_1(t)x_1 - c(t)y] + D_1(t)(x_2 - x_1), \\ \dot{x}_2 = x_2[a_2(t) - b_2(t)x_2] + D_2(t)(x_1 - x_2), \\ \dot{y} = y[-d(t) + e(t)x_1 - q(t)y], \end{cases} \tag{1.2}$$

Where x_1 and y are population density of prey species x and predator species y in patch 1,
And x_2 is the density of prey species x in patch 2. Predator species y is confined to patch 1, while the prey species x can diffuse between two patches. $D_i(t)$ (i □1, 2) are diffusive coefficients of prey species x. It is proved that the system can be made persistent, further, if the system is a periodic system, it can have a strictly positive periodic orbit which is globally asymptotically stable under the appropriate conditions.

Moreover, predators have to search for food and have to share or compete for food, a more suitable general predator-prey theory should be based on the so-called “ratio-dependent” theory. That is to say, the per capita predator growth rate should be a function of the ratio of prey to predator abundance (e.g., see [14-17]).In [14], Wang et al. considered the following a ratio-dependent predator-prey system with two competing prey predated by one predator

$$\begin{cases} \dot{x}_1(t) = x_1(t)[a_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - \dfrac{a_{13}(t)x_3(t)}{b_{13}(t)x_3(t) + x_1(t)}], \\ \dot{x}_2(t) = x_2(t)[a_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - \dfrac{a_{23}(t)x_3(t)}{b_{23}(t)x_3(t) + x_2(t)}], \\ \dot{x}_3(t) = x_3(t)[a_3(t) + \dfrac{a_{31}(t)x_1(t)}{b_{13}(t)x_3(t) + x_1(t)} + \dfrac{a_{32}(t)x_2(t)}{b_{23}(t)x_3(t) + x_2(t)}]. \end{cases} \tag{1.3}$$

In order to study the combined effects of time-delay, dispersion and ratio-dependent on the dynamics of predator-prey systems, some models have been studied by many authors [18-24]. In 2004, Xu et al. [23] studied the following Lotka-Volterra predator-prey model with dispersion and time-delays.

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{13}(t)y_3(t)] + D_1(t)(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_{22}(t)x_2(t)] + D_2(t)(x_1(t) - x_2(t)), \\ \dot{y}(t) = y(t)[-r_3(t) + a_{31}(t)x_1(t-\tau_1) - a_{33}(t)y(t-\tau_2)], \end{cases} \tag{1.4}$$

The authors by using Ganies and Mawhin's continuation theorem of coincidence degree theory and by constructing the appropriate Lyapunov function, a set of easily verifiable sufficient conditions are derived for the existence, uniqueness And global stability of positive periodic solutions of the system (1.4).

Xu et al. in [18] studied a three-species predator–prey model both with time delay and ratiodependent,

$$\begin{cases} \dot{x}_1 = x_1(t)[a_1 - a_{11}x_1(t-\tau_{11}) - \dfrac{a_{12}x_2(t)}{m_1 + x_1(t)}], \\ \dot{x}_2 = x_2(t)[-a_2 + \dfrac{a_{21}x_1(t-\tau_{21})}{m_1 + x_1(t-\tau_{21})} - a_{22}x_2(t-\tau_{22}) - \dfrac{a_{23}x_3(t)}{m_2 + x_2(t)}], \\ \dot{x}_3 = x_3(t)[-a_3 + \dfrac{a_{32}x_2(t-\tau_{32})}{m_2 + x_2(t-\tau_{32})} - a_{33}x_3(t-\tau_{33})], \end{cases} \tag{1.5}$$

They proved that the system (1.5) is uniformly persistent under some appropriate conditions and by means of constructing suitable Lyapunov functional, sufficient conditions are derived for the global asymptotic stability of the positive equilibrium of the system.

In [21], Sun and Yuan considered the following nonautomous mixture model both with ratiodependent and diffusion

$$\begin{cases} \dot{x}_1 = x_1[a_1(t) - b_1(t)x_1 - c_1(t)z - \dfrac{d(t)y}{y + \alpha(t)x_1}] + D_1(t)(x_2 - x_1), \\ \dot{x}_2 = x_2[a_2(t) - b_2(t)x_2] + D_2(t)(x_1 - x_2), \\ \dot{z} = z[a_3(t) - b_3(t)z - c_2(t)x_1], \\ \dot{y} = y[-a_4(t) - b_4(t)y + \dfrac{e(t)x_1}{y + \alpha(t)x_1}], \end{cases} \tag{1.6}$$

It is shown that the system (1.6) is uniformly and persistently related to the dispersion rate. Furthermore, the sufficient conditions are obtained for the global asymptotic stability of a periodic solution of the system (1.6).

In [19], the authors considered a delayed two-predator-one-prey ratio-dependent model with ratio-dependent in a two-patch environment

Where $x_i(t)$ $(i = 1, 2)$ represents the prey density in the ith patch, and $x_j(t)$ $(j = 3, 4)$ represents the predator density, $\tau_1, \tau_2 > 0$ are constant delays due to gestation, and D_i $(i = 1, 2)$ is a positive constant and denotes the dispersal rate, $a_i, a_{ij}(i, j = 1, 2, 3, 4), m_{13}$ and m_{14} are positive constants. It is shown that the system (1.7) is uniformly persistent under some appropriate conditions, and sufficient conditions are obtained for the global stability of the positive equilibrium of the system.

$$\begin{cases} \dot{x}_1 = x_1(t)[a_1 - a_{11}x_1(t) - \dfrac{a_{13}x_3(t)}{m_{13}x_3(t) + x_1(t)} - \dfrac{a_{14}x_4(t)}{m_{14}x_4(t) + x_1(t)} + D_1(x_2(t) - x_1(t)), \\ \dot{x}_2 = x_2(t)[a_2 - a_{22}x_2(t)] + D_2(x_1(t) - x_2(t)), \\ \dot{x}_3 = x_3(t)[-a_3 + \dfrac{a_{31}x_1(t-\tau_1)}{m_{13}x_3(t-\tau_1) + x_1(t-\tau_1)}], \\ \dot{x}_4 = x_4(t)[-a_4 + \dfrac{a_{41}x_1(t-\tau_2)}{m_{14}x_4(t-\tau_2) + x_1(t-\tau_2)}], \end{cases} \tag{1.7}$$

On the other hand, in the real world, ecosystems are continuously distributed by unpredictable forces which could cause a stable system to become unstable or cause the species to fluctuate. So it is necessary to study models with control variables. Recently, there has been a lot of literature related to the study of Lotka-Volterra-type system with feedback controls [25-29]. In 2003, K. Gopalsamy et al. [25] studied the following two species competition system with feedback controls

$$\begin{cases} \dot{x}_1(t) = x_1(t)[b_1 - a_{11}x_1(t) - a_{12}x_2(t) - \alpha_1 u_1(t)], \\ \dot{x}_2(t) = x_2(t)[b_2 - a_{21}x_1(t) - a_{22}x_2(t) - \alpha_2 u_2(t)], \\ \dot{u}_1(t) = -\eta_1 u_1(t) + a_1 x_1(t), \\ \dot{u}_2(t) = -\eta_2 u_2(t) + a_2 x_2(t), \end{cases} \tag{1.8}$$

Where $b_i, a_{i,j}, \alpha_i, \eta_i, a_i (i, j = 1, 2)$ Are positive constants, $u_i(t)$ $(i = 1,2)$ are the indirect control variables, they obtained some conditions for the existence of a globally attracting positive equilibrium of the system.

In order to show that whether the feedback control variables play an essential role on the persistent property and global stability of Lotka-Volterra cooperative systems or not, Liang [28] discussed the following a system with time delays and feedback control

$$\begin{cases} \dot{x}_1 = x_1(t)[a_1 - b_1 x_1(t) + c_1 x_2(t-\tau_1) - d_1 u_1(t-\tau_2)], \\ \dot{x}_2 = x_2(t)[a_2 - b_2 x_2(t) + c_2 x_1(t-\tau_3) - d_2 u_2(t-\tau_4)], \\ \dot{u}_1 = -e_1 u_1(t) + f_1 x_1(t-\tau_5), \\ \dot{u}_2 = -e_2 u_2(t) + f_2 x_2(t-\tau_6), \end{cases} \tag{1.9}$$

By using the differential inequality and comparison principle, the condition for persistence is obtained, then by the means of conducting the Lyapunov function, it is proved that the only positive equilibrium point of the system is globally stable when the parameters of system satisfy a certain conditions.

In 2016, Wang et al. [29] studied a ratio-dependent Lotka-Volterra predator-prey model with feedback control,

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - \dfrac{a_{13}(t)x_3(t)}{a_{13}(t)x_3(t) + x_1(t)} - d_1(t)u_1(t)], \\ \dot{x}_2(t) = x_2(t)[r_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - \dfrac{a_{23}(t)x_3(t)}{a_{23}(t)x_3(t) + x_2(t)} - d_2(t)u_2(t)], \\ \dot{x}_3(t) = x_3(t)[-r_3(t) + \dfrac{a_{31}(t)x_1(t)}{a_{13}(t)x_3(t) + x_1(t)} + \dfrac{a_{32}(t)x_2(t)}{a_{23}(t)x_3(t) + x_2(t)} + d_3(t)u_3(t)], \\ \dot{u}_1(t) = e_1(t) - f_1(t)u_1(t) + q_1(t)x_1(t), \\ \dot{u}_2(t) = e_2(t) - f_2(t)u_2(t) + q_2(t)x_2(t), \\ \dot{u}_3(t) = e_3(t) - f_3(t)u_3(t) - q_3(t)x_3(t), \end{cases} \tag{1.10}$$

The authors by constructing suitable Lyapunov function and developing some new analysis techniques, some sufficientcondition which guarantees the globally attractive of positive solution for the predator-prey model is obtained.

To eliminate the influence of the patch diffusion, ratio-dependent and feedback control on existence of periodic solution, Xie and Weng in [26]considered the following predator–prey model with patch–diffusion,ratio-dependent and feedback control,

$$\begin{cases} \dot{x}_1(t) = x_1(t)[b_1(t) - a_1(t)x_1(t) - \dfrac{k_1(t)y_1(t)}{c(t)y(t) + x_1(t)}] + D_1(t)(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = x_2(t)[b_2(t) - a_2(t)x_2(t) - \beta_1(t)u_1(t)] + D_2(t)(x_1(t) - x_2(t)), \\ \dot{y}(t) = y(t)[-r(t) - a_3(t)y(t) + \dfrac{k_2(t)x_1(t-\tau_1)}{c(t)y(t-\tau_1) + x_1(t-\tau_1)} - \beta_2(t)u_2(t)], \\ \dot{u}_1(t) = -\eta_1(t)u_1(t) + \xi_1(t)x_2(t-\tau_2), \\ \dot{u}_2(t) = -\eta_2(t)u_2(t) + \xi_2(t)y(t-\tau_3), \end{cases} \tag{1.11}$$

By developing some new analysis methods, the existence of at least one positive periodic solution for this model is proved. However, to the best of the authors' knowledge, there are few scholars who study the three species multi-delay Lotka-Volterra and ratio-dependent predator- prey model with feedback controls and prey diffusion. So, based on system (1.7) and motivated by the above works, in this paper, we propose and investigate the following three species multi-delay and ratio-dependent predator-prey model with feedback controls and prey diffusion

$$\begin{cases}\dot{x}_1(t)=x_1(t)[r_1(t)-a_{11}(t)x_1(t)-\dfrac{a_{13}(t)x_3(t)}{b_{13}(t)x_3(t)+x_1(t)}-\dfrac{a_{14}(t)x_4(t)}{b_{14}(t)x_4(t)+x_1(t)}\\ \qquad -d_1(t)u_1(t)]+D_1(t)(x_2(t)-x_1(t)),\\ \dot{x}_2(t)=x_2(t)[r_2(t)-a_{22}(t)x_2(t)]+D_2(t)(x_1(t)-x_2(t)),\\ \dot{x}_3(t)=x_3(t)[-r_3(t)+\dfrac{a_{31}(t)x_1(t-\tau_1)}{b_{13}(t)x_3(t)+x_1(t-\tau_1)}-a_{34}(t)x_4(t-\tau_2)+d_2(t)u_2(t)],\\ \dot{x}_4(t)=x_4(t)[-r_4(t)+\dfrac{a_{41}(t)x_1(t-\tau_3)}{b_{14}(t)x_4(t)+x_1(t-\tau_3)}-a_{43}(t)x_3(t-\tau_4)+d_3(t)u_3(t)],\\ \dot{u}_1(t)=e_1(t)-f_1(t)u_1(t)+q_1(t)x_1(t),\\ \dot{u}_2(t)=e_2(t)-f_2(t)u_2(t)-q_2(t)x_3(t),\\ \dot{u}_3(t)=e_3(t)-f_3(t)u_3(t)-q_3(t)x_4(t),\end{cases} \tag{1.12}$$

Where $x_i(t)$ $(i=1,2)$ denote the prey density in the i-th patch, $x_j(t)$ $(j=3,4)$ represents the predator density, predator species is confined to patch 1 while the prey species can disperse between two patches; $r_i(t)$ $(i=1,2)$ denote the intrinsic growth rate of the prey species, $r_j(t)$ $(j=3,4)$ are the death rate of the predators; $a_{11}(t), a_{22}(t)$ denote the internal competitive coefficient of the first species, $a_{13}(t), a_{14}(t)$ are shows the ratio of prey by predator, $a_{31}(t), a_{41}(t)$ represents the nutrient absorption ratio of predator after predation, $a_{34}(t), a_{43}(t)$ are the competitive coefficient of species $x_3(t)$ and $x_4(t)$; $D_i(t)$ $(i=1,2)$ are the dispersion rate of prey species, $u_i(t)$ $(i=1,2,3)$ are the feedback control terms; furthermore $r_i(t), a_{ii}(t), d_i(t), e_i(t), f_i(t)$, $q_i(t)(i=1,2,3)$ and $a_{13}(t), a_{14}(t), b_{13}(t), b_{14}(t), a_{31}(t), a_{41}(t), a_{34}(t), a_{43}(t), D_1(t), D_2(t)$ are continuous, bounded and strictly positive functions on $[0,+\infty)$, $\tau_1, \tau_2, \tau_3, \tau_4$ are positive constants.

Due to biological interpretation of system (1.12), it is reasonable to consider only positive solution of (1.12), in other words, to take admissible initial conditions

$$\begin{aligned} &x_i(t)=\phi_i(t),\ \phi_i(0)>0,\ t\in[-\tau,0],\ i=1,2,3,\\ &u_i(t)=\phi_i(t),\ \varphi_i(0)>0,\ t>0,\ i=1,2,3.\end{aligned} \tag{1.13}$$

where $\tau=\max\{\tau_1,\tau_2,\tau_3,\tau_4\}$. Obviously, the solutions of system (1.12) with the initial values (1.13) are positive for all $t\geq 0$. By developing a new analysis technique, the sufficient conditions are established for the permanence of the predator-prey model in this paper.

2. Permanence

For a continuous bounded function $g\ t\ (\ )$ defined on $[t_0, \square\square)$, we set

$$g^m=\sup\{g(t)\,|\,t_0<t<\infty\},\quad g^l=\inf\{g(t)\,|\,t_0<t<\infty\}.$$

.

Definition 2.1. System (1.12) is called permanent, if there exist positive constants $M_i, N_j, m_i, n_j (i=1,2,3,4, j=1,2,3)$ and T, such that $m_i \le x_i(t) \le M_i, n_j \le u_j(t) \le N_j$, $(i=1,2,3,4, j=1,2,3)$ for any positive solution $Z(t)=(x_1(t),x_2(t),x_3(t),u_1(t),u_2(t),u_3(t))$ of system (1.12) as $t>T$.

As a direct corollary of Lemma 2.1 of Chen [30], we have

Lemma 2.1. If $a>0, b>0$ and $\dot{x} \ge b-ax$, when $t \ge 0$ and $x(0)>0$, we have

$$\liminf_{t\to+\infty} x(t) \ge b/a.$$

If $a>0, b>0$ and $\dot{x} \le b-ax$, when $t \ge 0$ and $x(0)>0$, we have

$$\limsup_{t\to+\infty} x(t) \le b/a.$$

As a direct corollary of Lemma 2.2 of Chen [30], we have

Lemma 2.2. If $a>0, b>0$ and $\dot{x} \ge x(b-ax)$, when $t \ge 0$ and $x(0)>0$, we have

$$\liminf_{t\to+\infty} x(t) \ge b/a.$$

If $a>0, b>0$ and $\dot{x} \le x(b-ax)$, when $t \ge 0$ and $x(0)>0$, we have

$$\limsup_{t\to+\infty} x(t) \le b/a.$$

For the system (1.12), we let

$$M_1 = M_2 = \max\{\frac{r_1^m}{a_{11}^l}, \frac{r_2^m}{a_{22}^l}\},\ N_1 = \frac{e_1^m + q_1^m M_1}{f_1^l},\ N_2 = \frac{e_2^m}{f_2^l},\ N_3 = \frac{e_3^m}{f_3^l},$$

$$M_3 = \frac{a_{41}^m + d_3^m N_3 - r_4^l}{a_{43}^l} \exp[(a_{31}^m + d_2^m N_2 - r_3^l)\tau_4],$$

$$M_4 = \frac{a_{31}^m + d_2^m N_2 - r_3^l}{a_{34}^l} \exp[(a_{41}^m + d_3^m N_3 - r_4^l)\tau_2],$$

$$m_1 = \frac{r_1^l - a_{13}^m/b_{13}^l - a_{14}^m/b_{14}^l - d_1^m N_1 - D_1^m}{a_{11}^m},\ m_2 = \frac{r_2^l - D_2^m}{a_{22}^m},$$

$$n_1 = \frac{e_1^l + q_1^l m_1}{f_1^m},\ n_2 = \frac{e_2^l - q_2^m M_3}{f_2^m},\ n_3 = \frac{e_3^l - q_3^m M_4}{f_3^m},$$

$$m_3 = \frac{d_3^l n_3 - r_4^m}{a_{43}^m} \exp[(d_2^l n_2 - r_3^m - a_{34}^m M_4)\tau_4],$$

$$m_4 = \frac{d_2^l n_2 - r_3^m}{a_{34}^m} \exp[(d_3^l n_3 - r_4^m - a_{43}^m M_3)\tau_2].$$

Proof. According to the first and the second equations of system (1.12), we define $W_1(t)=\max\{x_1(t),x_2(t)\}$ and calculate the upper right derivative of $W_1(t)$ along the positive solution of system (1.12), we have that

Theorem 2.1. Assume that the system (1.12) satisfies the initial conditions (1.13) and following conditions

$$
\begin{aligned}
&(H_1)\ r_4^l < a_{41}^m + d_3^m N_3,\ (H_2)\ r_3^l < a_{31}^m + d_2^m N_2 \\
&(H_3)\ r_1^l > a_{13}^m/b_{13}^l + a_{14}^m/b_{14}^l + d_1^m N_1 + D_1^m, \\
&(H_4)\ r_2^l > D_2^m,\ (H_5)\ e_2^l > q_2^m M_3, \\
&(H_6)\ e_3^l > q_3^m M_4,\ (H_7)\ d_3^l n_3 > r_4^m,\ (H_8)\ d_2^l n_2 > r_3^m.
\end{aligned}
$$

then the system (1.12) is permanent.

(P1) if $x_1(t)\ge x_2(t)$, then

$$
\begin{aligned}
D^+W_1(t) = \dot{x}_1(t) &= x_1(t)[r_1(t) - a_{11}(t)x_1(t) - \frac{a_{13}(t)x_3(t)}{b_{13}(t)x_3(t)+x_1(t)} - \frac{a_{14}(t)x_4(t)}{b_{14}(t)x_4(t)+x_1(t)} \\
&\quad - d_1(t)u_1(t)] + D_1(t)(x_2(t)-x_1(t)) \\
&\le x_1(t)[r_1(t)-a_{11}(t)x_1(t)] \le x_1(t)[r_1^m - a_{11}^l x_1(t)] = W_1(t)[r_1^m - a_{11}^l W_1(t)].
\end{aligned}
$$

(P2) if $x_1(t)\le x_2(t)$, then

$$
\begin{aligned}
D^+W_1(t) = \dot{x}_2(t) &= x_2(t)[r_2(t)-a_{22}(t)x_2(t)] + D_2(t)(x_1(t)-x_2(t)) \\
&\le x_2(t)[r_2(t)-a_{22}(t)x_2(t)] \le x_2(t)[r_2^m - a_{22}^l x_2(t)] = W_1(t)[r_2^m - a_{22}^l W_1(t)]
\end{aligned}
$$

It follows from (P1) and (P2) that

$$
D^+W_1(t) \le W_1(t)[r_i^m - a_{ii}^l W_1(t)],\ i=1,2. \tag{2.1}
$$

By (2.1) we can derive

(A) If $W_1(0)=\max\{x_1(0),x_2(0)\}\le M_1$, then $\max\{x_1(t),x_2(t)\}\le M_1,\ t\ge 0$.

(B) If $W_1(0)=\max\{x_1(0),x_2(0)\}> M_1$, take appropriate $\alpha>0$, we have the following three possibilities:

(a) $W_1(0)=x_1(0)>M_1,\ (x_1(0)>x_2(0))$;

(b) $W_1(0)=x_2(0)>M_1,\ (x_1(0)<x_2(0))$;

(c) $W_1(0)=x_1(0)=x_2(0)>M_1$.

If (a) holds, then there exists $\varepsilon>0,\ t\in[0,\varepsilon)$, we have $W_1(t)=x_1(t)>M_1$, then we get

$$D^+W_1(t) = \dot{x}_1(t) \le a_{11}^l W_1(t)[\frac{r_1^m}{a_{11}^l} - W_1(t)] \le -\alpha < 0.$$

If (b) holds, then there exists $\varepsilon > 0$, $t \in [0, \varepsilon)$, and $W_1(t) = x_2(t) > M_1$, and also we have

$$D^+W_1(t) = \dot{x}_2(t) \le a_{22}^l W_1(t)[\frac{r_2^m}{a_{22}^l} - W_1(t)] \le -\alpha < 0.$$

If (c) holds, a similar argument in (a) and (b) shows that

$$D^+W_1(t) = \dot{x}_i(t) \le a_{ii}^l W_1(t)[\frac{r_i^m}{a_{ii}^l} - W_1(t)] \le -\alpha < 0,\ i = 1 \text{ or } 2.$$

From what we have been discussed above, we can conclude that if $W_1(0) > M_1$, then W_1 is strictly monotone decreasing with speed at least α, so there exists $T_1 > 0$ such that if $t \ge T_1$, we have $W_1(t) = \max\{x_1(t), x_2(t)\} \le M_1$. Which is

$$\limsup_{t\to+\infty} x_1(t) \le M_1 = M_2 = \max\{\frac{r_1^m}{a_{11}^l}, \frac{r_2^m}{a_{22}^l}\}. \tag{2.2}$$

$$\limsup_{t\to+\infty} x_2(t) \le M_1 = M_2 = \max\{\frac{r_1^m}{a_{11}^l}, \frac{r_2^m}{a_{22}^l}\}. \tag{2.3}$$

According to the fifth equation of system (1.12), we have

$$\begin{aligned} \dot{u}_1(t) &= e_1(t) - f_1(t)u_1(t) + q_1(t)x_1(t) \\ &\le e_1^m - f_1^l u_1(t) + q_1^m M_1, \end{aligned} \tag{2.4}$$

By **Lemma 2.1**, we have

$$\limsup_{t\to+\infty} u_1(t) \le \frac{e_1^m + q_1^m M_1}{f_1^l} = N_1. \tag{2.5}$$

From the sixth and seventh of system (1.12), we have

$$\begin{aligned} \dot{u}_2(t) &= e_2(t) - f_2(t)u_2(t) - q_2(t)x_3(t) \\ &\le e_2(t) - f_2(t)u_2(t) \le e_2^m - f_2^l u_2(t), \end{aligned} \tag{2.6}$$

$$\begin{aligned} \dot{u}_3(t) &= e_3(t) - f_3(t)u_3(t) - q_3(t)x_4(t) \\ &\le e_3(t) - f_3(t)u_3(t) \le e_3^m - f_3^l u_3(t), \end{aligned} \tag{2.7}$$

By **Lemma 2.1**, we can get

$$\limsup_{t\to+\infty} u_2(t) \le \frac{e_2^m}{f_2^l} = N_2. \qquad ($$

$$\limsup_{t\to+\infty} u_3(t) \le \frac{e_3^m}{f_3^l} = N_3. \qquad ($$

For the third and fourth equations of system (1.12),

$$\dot{x}_3(t) = x_3(t)[-r_3(t) + \frac{a_{31}(t)x_1(t-\tau_1)}{b_{13}(t)x_3(t) + x_1(t-\tau_1)} - a_{34}(t)x_4(t-\tau_2) + d_2(t)u_2(t)]$$
$$\le x_3(t)[-r_3(t) + a_{31}(t) - a_{34}(t)x_4(t-\tau_2) + d_2(t)u_2(t)] \qquad (2.10)$$
$$\le x_3(t)[-r_3^l + a_{31}^m - a_{34}^l x_4(t-\tau_2) + d_2^m N_2],$$

$$\dot{x}_4(t) = x_4(t)[-r_4(t) + \frac{a_{41}(t)x_1(t-\tau_3)}{b_{14}(t)x_4(t) + x_1(t-\tau_3)} - a_{43}(t)x_3(t-\tau_4) + d_3(t)u_3(t)]$$
$$\le x_4(t)[-r_4(t) + a_{41}(t) - a_{43}(t)x_3(t-\tau_4) + d_3(t)u_3(t)] \qquad (2.11)$$
$$\le x_4(t)[-r_4^l + a_{41}^m - a_{43}^l x_3(t-\tau_4) + d_3^m N_3].$$

Let $x_3(\bar{t})$ be the maximum of $x_3(t)$, then from (2.10) and we have

$$0 = \dot{x}_3(\bar{t}) \le x_3(\bar{t})[-r_3^l + a_{31}^m - a_{34}^l x_4(\bar{t}-\tau_2) + d_2^m N_2], \qquad (2.12)$$

By the initial conditions (1.13), we can obtained that $x_3(t) > 0$, from (2.12), we will get

$$x_4(\bar{t}-\tau_2) \le \frac{a_{31}^m + d_2^m N_2 - r_3^l}{a_{34}^l}. \qquad (2.13)$$

Let $x_4(\bar{t})$ is the maximum of $x_4(t)$, from (2.11) we can obtain

$$0 = \dot{x}_4(\bar{t}) \le x_4(t)[-r_4^l + a_{41}^m - a_{43}^l x_3(\bar{t}-\tau_4) + d_3^m N_3], \qquad (2.14)$$

With initial conditions $x_4(t) > 0$, and from (2.14), we can get

$$x_3(\bar{t}-\tau_4) < \frac{a_{41}^m + d_3^m N_3 - r_4^l}{a_{43}^l}. \qquad (2.15)$$

Integrating from $\bar{t}-\tau_4$ to $\bar{t}$ on both sides of (2.10), we have

$$\ln[\frac{x_3(\bar{t})}{x_3(\bar{t}-\tau_4)}] \le \int_{\bar{t}-\tau_4}^{\bar{t}} (-r_3^l + a_{31}^m + d_2^m N_2)dt \qquad (2.16)$$
$$\Rightarrow x_3(\bar{t}) \le x_3(\bar{t}-\tau_4)\exp[(a_{31}^m + d_2^m N_2 - r_3^l)\tau_4],$$

By (2.15) and (2.16), if (H_1) holds, we have

$$\limsup_{t\to+\infty} x_3(t) \le x_3(\bar{t}) \le \frac{a_{41}^m + d_3^m N_3 - r_4^l}{a_{43}^l} \exp[(a_{31}^m + d_2^m N_2 - r_3^l)\tau_4] = M_3. \tag{2.17}$$

Similarly, integrate from $\bar{t}-\tau_2$ to $\bar{t}$ on both sides of (2.11), we derive that

$$\begin{aligned} &\ln[\frac{x_4(\bar{t})}{x_4(\bar{t}-\tau_2)}] \le \int_{\bar{t}-\tau_2}^{\bar{t}} (-r_4^l + a_{41}^m + d_3^m N_3) dt \\ &\Rightarrow x_4(\bar{t}) \le x_4(\bar{t}-\tau_2)\exp[(a_{41}^m + d_3^m N_3 - r_4^l)\tau_2], \end{aligned} \tag{2.18}$$

From (2.13) and (2.18), if (H_2) holds, we can obtain

$$\limsup_{t\to+\infty} x_4(t) \le x_4(\bar{t}) \le \frac{a_{31}^m + d_2^m N_2 - r_3^l}{a_{34}^l} \exp[(a_{41}^m + d_3^m N_3 - r_4^l)\tau_2] = M_4. \tag{2.19}$$

On the other hand, from the first equations of system (1.12), we have

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)[r_1(t) - a_{11}(t)x_1(t) - \frac{a_{13}(t)x_3(t)}{b_{13}(t)x_3(t)+x_1(t)} - \frac{a_{14}(t)x_4(t)}{b_{14}(t)x_4(t)+x_1(t)} \\ &\quad -d_1(t)u_1(t)] + D_1(t)(x_2(t)-x_1(t)) \\ &\ge x_1(t)[r_1(t) - a_{11}(t)x_1(t) - a_{13}(t)/b_{13}(t) - a_{14}(t)/b_{14}(t) - d_1(t)u_1(t) - D_1(t)] \\ &\ge x_1(t)[r_1^l - a_{11}^m x_1(t) - a_{13}^m/b_{13}^l - a_{14}^m/b_{14}^l - d_1^m N_1 - D_1^m], \end{aligned} \tag{2.20}$$

By **Lemma 2.2,** if (H_3) holds, we have

$$\liminf_{t\to+\infty} x_1(t) \ge \frac{r_1^l - a_{13}^m/b_{13}^l - a_{14}^m/b_{14}^l - d_1^m N_1 - D_1^m}{a_{11}^m} = m_1. \tag{2.21}$$

For the second equation of system (1.12)

$$\begin{aligned} \dot{x}_2(t) &= x_2(t)[r_2(t) - a_{22}(t)x_2(t)] + D_2(t)(x_1(t)-x_2(t)) \\ &\ge x_2(t)[r_2(t) - a_{22}(t)x_2(t) - D_2(t)] \ge x_2(t)[r_2^l - a_{22}^m x_2(t) - D_2^m], \end{aligned} \tag{2.22}$$

From assumption (H_4) and by **Lemma 2.2**, we can get

$$\liminf_{t\to+\infty} x_2(t) \ge \frac{r_2^l - D_2^m}{a_{22}^m} = m_2. \tag{2.23}$$

By the fifth equation of system (1.12), we have

$$\begin{aligned} \dot{u}_1(t) &= e_1(t) - f_1(t)u_1(t) + q_1(t)x_1(t) \\ &\ge e_1^l - f_1^m u_1(t) + q_1^l m_1, \end{aligned} \tag{2.24}$$

By **Lemma 2.1**, we have

$$\liminf_{t\to+\infty} u_1(t) \geq \frac{e_1^l + q_1^l m_1}{f_1^m} = n_1. \tag{2.25}$$

For the sixth and the seventh equations of system (1.12)

$$\begin{aligned}\dot{u}_2(t) &= e_2(t) - f_2(t)u_2(t) - q_2(t)x_3(t)\\ &\geq e_2^l - f_2^m u_2(t) - q_2^m M_3,\end{aligned} \tag{2.26}$$

$$\begin{aligned}\dot{u}_3(t) &= e_3(t) - f_3(t)u_3(t) - q_3(t)x_4(t)\\ &\geq e_3^l - f_3^m u_3(t) - q_3^m M_4,\end{aligned} \tag{2.27}$$

If (H_5) and (H_6) holds, by **Lemma2.1**, we have

$$\liminf_{t\to+\infty} u_2(t) \geq \frac{e_2^l - q_2^m M_3}{f_2^m} = n_2. \tag{2.28}$$

$$\liminf_{t\to+\infty} u_3(t) \geq \frac{e_3^l - q_3^m M_4}{f_3^m} = n_3. \tag{2.29}$$

According to the third equations and the fourth of system (1.12), we have

$$\begin{aligned}\dot{x}_3(t) &= x_3(t)[-r_3(t) + \frac{a_{31}(t)x_1(t-\tau_1)}{b_{13}(t)x_3(t) + x_1(t-\tau_1)} - a_{34}(t)x_4(t-\tau_2) + d_2(t)u_2(t)]\\ &\geq x_3(t)[-r_3(t) - a_{34}(t)x_4(t-\tau_2) + d_2(t)u_2(t)] \geq x_3(t)[-r_3^m - a_{34}^m x_4(t-\tau_2) + d_2^l n_2],\end{aligned} \tag{2.30}$$

$$\begin{aligned}\dot{x}_4(t) &= x_4(t)[-r_4(t) + \frac{a_{41}(t)x_1(t-\tau_3)}{b_{14}(t)x_4(t) + x_1(t-\tau_3)} - a_{43}(t)x_3(t-\tau_4) + d_3(t)u_3(t)]\\ &\geq x_4(t)[-r_4(t) - a_{43}(t)x_3(t-\tau_4) + d_3(t)u_3(t)] \geq x_4(t)[-r_4^m - a_{43}^m x_3(t-\tau_4) + d_3^l n_3],\end{aligned} \tag{2.31}$$

Let $x_3(\underline{t}), x_4(\underline{t})$ are the minimum values of $x_3(t)$ and $x_4(t)$, by (2.30) and (2.31), we get

$$0 = \dot{x}_3(\underline{t}) \geq x_3(t)[-r_3^m - a_{34}^m x_4(\underline{t} - \tau_2) + d_2^l n_2], \tag{2.32}$$

$$0 = \dot{x}_4(\underline{t}) \geq x_4(t)[-r_4^m - a_{43}^m x_3(\underline{t} - \tau_4) + d_3^l n_3], \tag{2.33}$$

From the initial conditions $x_3(t) > 0$ and (2.32), we have

$$x_4(\underline{t} - \tau_2) \geq \frac{d_2^l n_2 - r_3^m}{a_{34}^m}. \tag{2.34}$$

Similarly, with initial conditions $x_4(t) > 0$ and (2.33), then we obtain

$$x_3(\underline{t} - \tau_4) \geq \frac{d_3^l n_3 - r_4^m}{a_{43}^m}. \qquad (2.35)$$

Integrating both sides of (2.30) on the interval $[\underline{t} - \tau_4, \underline{t}]$, we have

$$\begin{aligned} &\ln[\frac{x_3(\underline{t})}{x_3(\underline{t} - \tau_4)}] \geq \int_{\underline{t}-\tau_4}^{\underline{t}} (-r_3^m - a_{34}^m M_4 + d_2^l n_2) dt \\ &\Rightarrow x_3(\underline{t}) \geq x_3(\underline{t} - \tau_4) \exp[(d_2^l n_2 - r_3^m - a_{34}^m M_4)\tau_4], \end{aligned} \qquad (2.36)$$

From (2.35) and (2.36), if (H_7) holds, we derive that

$$\liminf_{t \to +\infty} x_3(t) \geq x_3(\underline{t}) \geq \frac{d_3^l n_3 - r_4^m}{a_{43}^m} \exp[(d_2^l n_2 - r_3^m - a_{34}^m M_4)\tau_4] = m_3. \qquad (2.37)$$

By a similar argument, integrate from $\underline{t} - \tau_2$ to $\underline{t}$ on both sides of (2.31), we have

$$\begin{aligned} &\ln[\frac{x_4(\underline{t})}{x_4(\underline{t} - \tau_2)}] \geq \int_{\underline{t}-\tau_2}^{\underline{t}} (-r_4^m - a_{43}^m M_3 + d_3^l n_3) dt \\ &\Rightarrow x_4(\underline{t}) \geq x_4(\underline{t} - \tau_2) \exp[(d_3^l n_3 - r_4^m - a_{43}^m M_3)\tau_2], \end{aligned} \qquad (2.38)$$

Similarly, by (2.34) and (2.38) if (H_8) holds, we can get

$$\liminf_{t \to +\infty} x_4(t) \geq x_4(\underline{t}) \geq \frac{d_2^l n_2 - r_3^m}{a_{34}^m} \exp[(d_3^l n_3 - r_4^m - a_{43}^m M_3)\tau_2] = m_4. \qquad (2.39)$$

From (2.2), (2.3), (2.5), (2.8), (2.9), (2.17), (2.19) and (2.21), (2.23), (2.25), (2.28), (2.29), (2.37), (2.39), this ends the proof of Theorem 2.1.

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