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A DECOMPOSITION OF COMPLETE BIPARTITE 4-UNIFORM HYPERGRAPH $K^4_{7,7}$ INTO LOOSE CYCLES

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Abstract:-
A k-uniform hypergraph H is a pair (V,ε), where $V = \{v_1, v_2, \dots, v_n\}$ is a set of n vertices and ε is a family of k-subset of V called hyperedges. We consider the problem of constructing a decomposition for complete bipartite uniform hypergraph into loose cycles.

Keywords:-*Hypergraph, Loose Cycle, Decomposition*

INTRODUCTION

A decomposition of graph $G = (V, E)$ is a partition of the edge-set E. The problem of constructing decomposition is a long-standing and well-studied one in graph theory; in particular, for the complete graph K_n, it was solved in the 1890s by Walecki who showed that K_n has a Hamiltonian decomposition if and only if n is odd, while if n is even K_n has a decomposition into Hamiltonian cycles and a perfect matching. As many problem in graph theory, it seems natural to attempt a generalization to hypergraphs. Indeed, the notion of Hamiltonian was first generalized to uniform hypergraph by Berge in his book [1]. His definition of a Hamiltonian cycle in a hypergraph $H = (V,E)$ is a sequence $(v_0,e_1,v_1,e_2,\cdots,v_{n-1},e_n,v_0)$, where $V = \{v_0,v_1,\cdots,v_n\}$, and $e_1,e_2,\cdots,e_n$ are distinct elements of E, such that the hyperedges e_i contains both v_{i+1} and $v_i (mod\ n)$. The study of decomposition of complete 3-uniform hypergraph into cycles of this type was begun by Bermond et al in the 1970s [3] and completed by Verrall in 1994[6]. A k-uniform hypergraph H is a pair (V,ε), where $V = \{v_1,v_2,\cdots,v_n\}$ is a set of n vertices and ε is a family of k-subset of V called hyperedges. If ε consists of all k-subsets of V, then H is a complete k-uniform hypergraph on n vertices and denoted by K_n^k. At the same time we may refer a vertex v_i to v_{i+n}. A cycle of length l of H is a sequence of the form $(v_1,e_1,v_2,e_2,\cdots,v_l,e_l,v_1)$, where $v_1,v_2,\cdots,v_l$ are distinct vertices and $e_1,e_2,\cdots,e_l$ are k-hyperedges of H, satisfying the following conditions: (i) $v_i,v_{i+1} \in e_i$, $1 \le i \le l$, where addition on the subscripts is module n; (ii) e_i 6= e_j for i 6= j. This cycle is known as a Berge cycle, having been introduced in [1]. A decomposition of H into cycles is a partition of the hyperedges of H into cycles length (l_1, $l2$, $\cdots$, lm).

In this paper, we consider the decomposition of bipartite complete 4-uniform hypergraph $K_{q,q}^k$ for $q = 7, k = 4$.

1 Main results

Before giving the decomposition of hypergraph, we need some definitions about hypergraphs.

Definition 1 A loose cycle is of length l is a hyperpraph made of l hyperedges e_1, $e2$, $\cdots$, el such that, for any i,j, if $e_i \cap e_j$ 6= $\emptyset$, then $|e_i \cap e_j| = t < k$.

Definition 2 A bipartite hypergraph is a pair (V,ε), where $V = V_1 \cup V_2$, $V_1 \cap V_2 = \emptyset$, and for any $e_i \in \varepsilon$, $e_i \cup V_1$ 6= $\emptyset$ and $e_i \cup V_1$ 6= $\emptyset$.

Theorem 1 Bipartite complete 4-uniform hypergraph $K_{7,7}^4$ could be decomposed into loose cycles length 7 and 14, where for any two hyperedges e_i,e_j in a loose cycle, if $e_i \cap e_j$ 6= $\emptyset$, then $|e_i \cap e_j| = 2$.

Proof. We construct the decomposition of $K_{7,7}^4$. Let $V_1 = \{1,2,3,4,5,6,7\}$, $V_2 = \{\bar{1},\bar{2},\bar{3},\bar{4},\bar{5},\bar{6},\bar{7}\}$. We discuss it in two types.

Type 1 First of all, we arrange 35 hyperedges as following:

123,345,567,712,234,456,671
135,572,246,613,357,724,461
147,736,625,514,473,362,251
124,457,713,346,672,235,561
245,571,134,467,723,356,612

Where every line is a cycle of length 7, after adding a vertex $\bar{k} \in V_2$ to every hyperedge,

$123\bar{k}, 345\bar{k}, 567\bar{k}, 712\bar{k}, 234\bar{k}, 456\bar{k}, 671\bar{k}$
$135\bar{k}, 572\bar{k}, 246\bar{k}, 613\bar{k}, 357\bar{k}, 724\bar{k}, 461\bar{k}$
$147\bar{k}, 736\bar{k}, 625\bar{k}, 514\bar{k}, 473\bar{k}, 362\bar{k}, 251\bar{k}$
$124\bar{k}, 457\bar{k}, 713\bar{k}, 346\bar{k}, 672\bar{k}, 235\bar{k}, 561\bar{k}$
$245\bar{k}, 571\bar{k}, 134\bar{k}, 467\bar{k}, 723\bar{k}, 356\bar{k}, 612\bar{k}$

Now we have $\frac{\binom{7}{3}\binom{7}{1}}{7} = 35$ cycles of length 7.

Then by exchanging the positions for i and $\bar{i} (i = 1,2,\cdots,7)$, we have another $\frac{\binom{7}{1}\binom{7}{3}}{7} = 35$ cycles of length 7.

Type 2 For convenience, we apply

$$\begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_1 \\ j_1 & j_2 & j_3 & j_4 & j_5 & j_6 & j_7 & j_1 \end{pmatrix} \text{ or } \begin{pmatrix} i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 \\ j_1 & j_2 & j_3 & j_4 & j_5 & j_6 & j_7 \end{pmatrix}$$

To express cycle of length 7, where $i_1i_2i_3i_4i_5i_6i_7$ is a permutation of V_1, $j_1j_2j_3j_4j_5j_6j_7$ is a permutation of V_2, four adjacent elements in the upper and lower sides represent an hyperedge. Based on the above symbols,

$$\begin{pmatrix}1&2&3&4&5&6&7\\ \bar{1}&\bar{2}&\bar{3}&\bar{4}&\bar{5}&\bar{6}&\bar{7}\end{pmatrix}, \begin{pmatrix}1&2&3&4&5&6&7\\ \bar{2}&\bar{3}&\bar{4}&\bar{5}&\bar{6}&\bar{7}&\bar{1}\end{pmatrix}, \begin{pmatrix}1&2&3&4&5&6&7\\ \bar{3}&\bar{4}&\bar{5}&\bar{6}&\bar{7}&\bar{1}&\bar{2}\end{pmatrix},$$

$$\begin{pmatrix}1&2&3&4&5&6&7\\ \bar{4}&\bar{5}&\bar{6}&\bar{7}&\bar{1}&\bar{2}&\bar{3}\end{pmatrix}, \begin{pmatrix}1&2&3&4&5&6&7\\ \bar{5}&\bar{6}&\bar{7}&\bar{1}&\bar{2}&\bar{3}&\bar{4}\end{pmatrix}, \begin{pmatrix}1&2&3&4&5&6&7\\ \bar{6}&\bar{7}&\bar{1}&\bar{2}&\bar{3}&\bar{4}&\bar{5}\end{pmatrix},$$

$$\begin{pmatrix}1&2&3&4&5&6&7\\ \bar{7}&\bar{1}&\bar{2}&\bar{3}&\bar{4}&\bar{5}&\bar{6}\end{pmatrix}, \begin{pmatrix}1&3&5&7&2&4&6\\ \bar{1}&\bar{3}&\bar{5}&\bar{7}&\bar{2}&\bar{4}&\bar{6}\end{pmatrix}, \begin{pmatrix}1&3&5&7&2&4&6\\ \bar{3}&\bar{5}&\bar{7}&\bar{2}&\bar{4}&\bar{6}&\bar{1}\end{pmatrix},$$

$$\begin{pmatrix}1&3&5&7&2&4&6\\ \bar{5}&\bar{7}&\bar{2}&\bar{4}&\bar{6}&\bar{1}&\bar{3}\end{pmatrix}, \begin{pmatrix}1&3&5&7&2&4&6\\ \bar{7}&\bar{2}&\bar{4}&\bar{6}&\bar{1}&\bar{3}&\bar{5}\end{pmatrix}, \begin{pmatrix}1&3&5&7&2&4&6\\ \bar{2}&\bar{4}&\bar{6}&\bar{1}&\bar{3}&\bar{5}&\bar{7}\end{pmatrix},$$

$$\begin{pmatrix}1&3&5&7&2&4&6\\ \bar{4}&\bar{6}&\bar{1}&\bar{3}&\bar{5}&\bar{7}&\bar{2}\end{pmatrix}, \begin{pmatrix}1&3&5&7&2&4&6\\ \bar{6}&\bar{1}&\bar{3}&\bar{5}&\bar{7}&\bar{2}&\bar{4}\end{pmatrix}, \begin{pmatrix}1&4&7&3&6&2&5\\ \bar{1}&\bar{4}&\bar{7}&\bar{3}&\bar{6}&\bar{2}&\bar{5}\end{pmatrix},$$

$$\begin{pmatrix}1&4&7&3&6&2&5\\ \bar{4}&\bar{7}&\bar{3}&\bar{6}&\bar{2}&\bar{5}&\bar{1}\end{pmatrix}, \begin{pmatrix}1&4&7&3&6&2&5\\ \bar{7}&\bar{3}&\bar{6}&\bar{2}&\bar{5}&\bar{1}&\bar{4}\end{pmatrix}, \begin{pmatrix}1&4&7&3&6&2&5\\ \bar{3}&\bar{6}&\bar{2}&\bar{5}&\bar{1}&\bar{4}&\bar{7}\end{pmatrix}$$

$$\begin{pmatrix}1&4&7&3&6&2&5\\ \bar{6}&\bar{2}&\bar{5}&\bar{1}&\bar{4}&\bar{7}&\bar{3}\end{pmatrix}, \begin{pmatrix}1&4&7&3&6&2&5\\ \bar{2}&\bar{5}&\bar{1}&\bar{4}&\bar{7}&\bar{3}&\bar{6}\end{pmatrix}, \begin{pmatrix}1&4&7&3&6&2&5\\ \bar{5}&\bar{1}&\bar{4}&\bar{7}&\bar{3}&\bar{6}&\bar{2}\end{pmatrix},$$

Are 21 cycles of length 7?

We apply

$$\begin{pmatrix}i_1&i_2&i_3&i_4&i_5&i_6&i_7&\cdots\cdots\\ j_1&j_2&j_3&j_4&j_5&j_6&j_7&\cdots\cdots\end{pmatrix}$$

To express cycle of length 14. Based on the symbol above,

$$\begin{pmatrix}1&2&4&5&7&1&3&4&6&7&2&3&5&6\\ \bar{1}&\bar{3}&\bar{4}&\bar{6}&\bar{7}&\bar{2}&\bar{3}&\bar{5}&\bar{6}&\bar{1}&\bar{2}&\bar{4}&\bar{5}&\bar{7}\end{pmatrix}$$

$$\begin{pmatrix}1&2&4&5&7&1&3&4&6&7&2&3&5&6\\ \bar{2}&\bar{4}&\bar{5}&\bar{7}&\bar{1}&\bar{3}&\bar{4}&\bar{6}&\bar{7}&\bar{2}&\bar{3}&\bar{5}&\bar{6}&\bar{1}\end{pmatrix}$$

$$\begin{pmatrix}1&2&4&5&7&1&3&4&6&7&2&3&5&6\\ \bar{3}&\bar{5}&\bar{6}&\bar{1}&\bar{2}&\bar{4}&\bar{5}&\bar{7}&\bar{1}&\bar{3}&\bar{4}&\bar{6}&\bar{7}&\bar{2}\end{pmatrix}$$

$$\begin{pmatrix}1&2&4&5&7&1&3&4&6&7&2&3&5&6\\ \bar{4}&\bar{6}&\bar{7}&\bar{2}&\bar{3}&\bar{5}&\bar{6}&\bar{1}&\bar{2}&\bar{4}&\bar{5}&\bar{7}&\bar{1}&\bar{3}\end{pmatrix}$$

$$\begin{pmatrix}1&2&4&5&7&1&3&4&6&7&2&3&5&6\\ \bar{5}&\bar{7}&\bar{1}&\bar{3}&\bar{4}&\bar{6}&\bar{7}&\bar{2}&\bar{3}&\bar{5}&\bar{6}&\bar{1}&\bar{2}&\bar{4}\end{pmatrix}$$

$$\begin{pmatrix}1&2&5&6&2&3&6&7&3&4&7&1&4&5\\ \bar{1}&\bar{4}&\bar{5}&\bar{1}&\bar{2}&\bar{5}&\bar{6}&\bar{2}&\bar{3}&\bar{6}&\bar{7}&\bar{3}&\bar{4}&\bar{7}\end{pmatrix}$$

$$\begin{pmatrix}1&2&5&6&2&3&6&7&3&4&7&1&4&5\\ \bar{2}&\bar{5}&\bar{6}&\bar{2}&\bar{3}&\bar{6}&\bar{7}&\bar{3}&\bar{4}&\bar{7}&\bar{1}&\bar{4}&\bar{5}&\bar{1}\end{pmatrix}$$

$$\begin{pmatrix}1&2&5&6&2&3&6&7&3&4&7&1&4&5\\ \bar{3}&\bar{6}&\bar{7}&\bar{3}&\bar{4}&\bar{7}&\bar{1}&\bar{4}&\bar{5}&\bar{1}&\bar{2}&\bar{5}&\bar{6}&\bar{2}\end{pmatrix}$$

$$\begin{pmatrix}1&2&5&6&2&3&6&7&3&4&7&1&4&5\\ \bar{4}&\bar{7}&\bar{1}&\bar{4}&\bar{5}&\bar{1}&\bar{2}&\bar{5}&\bar{6}&\bar{2}&\bar{3}&\bar{6}&\bar{7}&\bar{3}\end{pmatrix}$$

$$\begin{pmatrix}1&2&6&7&4&5&2&3&7&1&5&6&3&4\\ \bar{1}&\bar{5}&\bar{6}&\bar{3}&\bar{4}&\bar{1}&\bar{2}&\bar{6}&\bar{7}&\bar{4}&\bar{5}&\bar{2}&\bar{3}&\bar{7}\end{pmatrix}$$

$$\begin{pmatrix}1&2&6&7&4&5&2&3&7&1&5&6&3&4\\ \bar{2}&\bar{6}&\bar{7}&\bar{4}&\bar{5}&\bar{2}&\bar{3}&\bar{7}&\bar{1}&\bar{5}&\bar{6}&\bar{3}&\bar{4}&\bar{1}\end{pmatrix}$$

$$\begin{pmatrix}1&2&6&7&4&5&2&3&7&1&5&6&3&4\\ \bar{3}&\bar{7}&\bar{1}&\bar{5}&\bar{6}&\bar{3}&\bar{4}&\bar{1}&\bar{2}&\bar{6}&\bar{7}&\bar{4}&\bar{5}&\bar{2}\end{pmatrix}$$

$$\begin{pmatrix}1&2&7&1&6&7&5&6&4&5&3&4&2&3\\ \bar{1}&\bar{6}&\bar{7}&\bar{5}&\bar{6}&\bar{4}&\bar{5}&\bar{3}&\bar{4}&\bar{2}&\bar{3}&\bar{1}&\bar{2}&\bar{7}\end{pmatrix}$$

$$\begin{pmatrix}1&2&7&1&6&7&5&6&4&5&3&4&2&3\\ \bar{2}&\bar{7}&\bar{1}&\bar{6}&\bar{7}&\bar{5}&\bar{6}&\bar{4}&\bar{5}&\bar{3}&\bar{4}&\bar{2}&\bar{3}&\bar{1}\end{pmatrix}$$

$$\begin{pmatrix}1&3&6&1&4&6&2&4&7&2&5&7&3&5\\ \bar{1}&\bar{4}&\bar{6}&\bar{2}&\bar{4}&\bar{7}&\bar{2}&\bar{5}&\bar{7}&\bar{3}&\bar{5}&\bar{1}&\bar{3}&\bar{6}\end{pmatrix}$$

$$\begin{pmatrix}1&3&6&1&4&6&2&4&7&2&5&7&3&5\\ \bar{2}&\bar{5}&\bar{7}&\bar{3}&\bar{5}&\bar{1}&\bar{3}&\bar{6}&\bar{1}&\bar{4}&\bar{6}&\bar{2}&\bar{4}&\bar{7}\end{pmatrix}$$

$$\begin{pmatrix}1&3&6&1&4&6&2&4&7&2&5&7&3&5\\ \bar{3}&\bar{6}&\bar{1}&\bar{4}&\bar{6}&\bar{2}&\bar{4}&\bar{7}&\bar{2}&\bar{5}&\bar{7}&\bar{3}&\bar{5}&\bar{1}\end{pmatrix}$$

$$\begin{pmatrix}1&3&6&1&4&6&2&4&7&2&5&7&3&5\\ \bar{4}&\bar{7}&\bar{2}&\bar{5}&\bar{7}&\bar{3}&\bar{5}&\bar{1}&\bar{3}&\bar{6}&\bar{1}&\bar{4}&\bar{6}&\bar{2}\end{pmatrix}$$

$$\begin{pmatrix}1&3&7&2&6&1&5&7&4&6&3&5&2&4\\ \bar{1}&\bar{5}&\bar{7}&\bar{4}&\bar{6}&\bar{3}&\bar{5}&\bar{2}&\bar{4}&\bar{1}&\bar{3}&\bar{7}&\bar{2}&\bar{6}\end{pmatrix}$$

$$\begin{pmatrix}1&3&7&2&6&1&5&7&4&6&3&5&2&4\\ \bar{2}&\bar{6}&\bar{1}&\bar{5}&\bar{7}&\bar{4}&\bar{6}&\bar{3}&\bar{5}&\bar{2}&\bar{4}&\bar{1}&\bar{3}&\bar{7}\end{pmatrix}$$

$$\begin{pmatrix}1&3&7&2&6&1&5&7&4&6&3&5&2&4\\ \bar{3}&\bar{7}&\bar{2}&\bar{6}&\bar{1}&\bar{5}&\bar{7}&\bar{4}&\bar{6}&\bar{3}&\bar{5}&\bar{2}&\bar{4}&\bar{1}\end{pmatrix}$$

Are 21 cycles of length 14? It is easy to verify for any two cycles above, there is no common hyperedge between these two cycles. Furthermore, $|K^4_{7,7}| = 2\binom{7}{3}\binom{7}{1} + \binom{7}{2}\binom{7}{2} =$ [(2×35) ×7]+ [(21×7) + (21×14)], where the parentheses in the first parenthesis is the number of cycles of length 7 for type 1, two 21 of second parentheses are the numbers of cycles of length 7, 14 for type 2 respectively.

We have completed the decomposition of hypergraphs $K^4_{7,7}$.

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References

[1]. C. Berge, Graphs and Hypergraphs (North-Holland, Amsterdam, 1979).
[2]. J. C. Bermond, Hamiltonian decompositions of graphs, directed graphs and hypergraphs, Ann. Discrete Math. 3 (1978) 21-28.
[3]. H. Verrall, Hamilton decompositions of complete 3-uniform hypergraphs, Discrete Math. 132 (1994) 333-348
[4]. Jian-fang Wang, the theoretical base of hypergraphs(in Chinese), Beijing: Higher Education Press, 2006, 7
[5]. Ronald Fagin, Degrees of Acyclicity for Hypergraphs and Relational Database Schemes, Journal of the Assoeiauon for Computing Machinery, Vol 30, No 3, 1983,7: 514-550
[6]. Duchet, P. Hypergraphs, Handbook of Combinatorics. The MIT Press, 381-423, 1995
[7]. Jian-fang Wang, The Information Hypergraph Theory, Beijing: Science Press, 2008