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THE ANCIENT-GREEK SPECIAL PROBLEMS, AS THE QUANTIZATION MOULDS OF SPACES.

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Abstract:-

The Special Problems of E-geometry consist the, Mould Quantization, of Euclidean Geometry in it, to become → Monad, through mould of Space –Anti-space in itself, which is the material dipole in inner monad Structure as the Electromagnetic cycloidal field → Linearly, through mould of Parallel Theorem [44- 45], which are the equal distances between points of parallel and line → In Plane, through mould of Squaring the circle [46], where the two equal and perpendicular monads consist a Plane acquiring the common Plane-meter → and in Space (volume), through mould of the Duplication of the Cube [46], where any two Unequal perpendicular monads acquire the common Space-meter to be twice each other, as analytically all methods are proved and explained. [39-41]

The Unification of Space and Energy becomes through [STPL] Geometrical Mould Mechanism of Elements, the minimum Energy-Quanta, In monads → Particles, Anti-particles, Bosons, Gravity – Force, Gravity -Field, Photons, Dark Matter, and Dark-Energy, consisting Material Dipoles in inner monad Structures i.e. the Electromagnetic Cycloidal Field of monads. [39-41]

Euclid's elements consist of assuming a small set of intuitively appealing axioms, proving many other propositions. Because nobody until [9] succeeded to prove the parallel postulate by means of pure geometric logic, many self consistent non - Euclidean geometries have been discovered, based on Definitions, Axioms or Postulates, in order that none of them contradicts any of the other postulates. In [39] the only Space-Energy geometry is Euclidean, agreeing with the Physical reality, on unit $AB = \text{Segment}$ which is The Electromagnetic field of the Quantized on AB Energy Space Vector, on the contrary to the General relativity of Space-time which is based on the rays of the non-Euclidean geometries to the limited velocity of light and Planck's cavity. Euclidean geometry elucidated the definitions of geometry-content, { for Point, Segment, Straight Line, Plane, Volume, Space [S], Antispace [AS], Sub-space [SS], Cave, Space-Anti-Space Mechanism of the Six-Triple-Points-Line, that produces and transfers Points of Spaces, Anti-Spaces and Sub-Spaces in a Common Inertial Sub-Space and a cylinder, Gravity field [MFMF], Particles } and describes the Space-Energy beyond Plank's length level [Gravity Length $3,969.10^{-62} \text{ m}$], reaching the Point = $L_v = e^{i \cdot (\frac{N\pi}{2})} b = 10^{-N} = -m = 0 \text{ m}$, which is nothing and zero space. [43-46] -The Geometrical solution of the Special Problems is now presented.

Keywords:- The ancient - Greek Special Problems, The Quantization moulds of Euclidean geometry

DEFINITION OF QUANTIZATION.

Quantization is the concept (*the Process*) that any, **Physical Quantity** \rightarrow [PQ] of the objective reality (*Matter, Energy or Both*) is mapping the Continuous Analogous, *the points*, to only certain Discrete values. Quantization of Energy is done in Space-tanks, on the material points, tiny volumes and on points consisting the Equilibrium, the Opposite Twin, of Space Anti-space.

In Geometry [PQ] are the Points, *nothing*, only, transformed into Segments, Lines, Surfaces, Volumes and to any other Coordinate System such as (x,y,z) , (i,j,k) and which are all quantized.

Quantization of E-geometry is the way of Points to become \rightarrow (Segments, Anti-segments = Monads = Anti-), (Segments, Parallel-segments = Equal monads), (Equal Segments and Perpendicular-segments = Plane Vectors), (Unequal Segments twice – Perpendicular -segments = The Space Vectors = Quaternion). [46] **In Philosophy** [PQ] are the concepts of Matter and of Spirit or Materialism and Idealism.

a).. **Anaximander**, claimed that non of the elements could be, *arche* and proposed, *apeiron*, an infinitive substance from which all things are born and to which all will return.

b).. **Archimedes**, is very clear regarding the definitions, that they say nothing as to whether the things defined exist or not, but they only require to be understood. Existence is only postulated in the case where [PQ] are the Points to Segments (the magnitude = quantization). In geometry we assume Point, Segment, Line, Surface and Volume, without proving their existence, and the existence of everything else has to be proved. The Euclid's similar figures correspond to Eudoxus' theory of proportion.

c).. **Zenon**, claimed that, Belief in the existence of many things rather than, *only one thing*, leads to absurd conclusions and for, *Point and its constituents will be without magnitude*.

Considering Points in space are a distinct place even if there are an infinity of points, defines the Presented in [44] idea of *Material Point*.

d).. **Materialism or and Physicalism**, is a form of philosophical monism and holds that matter fundamental substance in nature and that all phenomena, *including mental phenomes and consciousness*, are identical with material interactions by incorporating notions of Physics such as spacetime, physical energies and forces, dark matter and so on.

e).. **Idealism**, such as those of Hegel, *ipso facto*, is an argument against materialism (*the mindindependent properties can in turn be reduced to the subjective percepts*) as such the existence of matter can only be assumed from the apparent (*perceived*) stability of perceptions with no evidence in direct experience.

Matter and Energy are necessary to explain the physical world but incapable of explaining mind and so results, *dualism*. The Reason determined in itself and its relation to the world creates the very old question as, *what is the ultimate purpose of the world* ?.

f).. **Hegel's** conceive for mind, *Idea*, defines that, mind is *arche* and is reduced to [PQ] the subjective percepts, while Materialism holds just the opposite.

In Physics [PQ] are The, Electrical charge, Energy, Light, Angular momentum, Matter which are all quantized on the microscopic level. They do not seem quantized in the macroscopic scale because the size of the steps between each possible value is so small.

a).. **De Broglie** found that, light and matter at subatomic level display characteristics of both waves and particles which move at specific speeds called Energy-levels.

b).. **Max Planck** found that, Energy and frequency of Electromagnetic radiation is quantized as relation $E = h.f$.

In Mechanics, *Kinematics* describes the motion while *Dynamics* causes the motion.

c).. **Bohr model for Electrons in free-Atoms** is the Scaled Energy levels, *for Standing-Waves* is the constancy of Angular momentum, *for Centripetal- Force in electron orbit*, is the constancy of Electric Potential, *for the Electron orbit radii*, is the Energy level structure with the Associated electron wavelengths.

d).. **Hesiod Hypothesis** [PQ] is *Chaos*, i.e. *the Primary Point* from which is quantized to *Primary Anti-Point*. [From Chaos came forth **Erebus**, *the Mechanism*, but of Night were born **Aether**, *The Gravity's dipole Field connected by the Gravity Force*, and **Day**, *Particles Anti-particles*, whom she conceived and **Bare**, *The Equilibrium of Particles Anti-particles*, in Spaces Anti-spaces, from union in love with Erebus]. [43-46]

The Special Greek Problems .

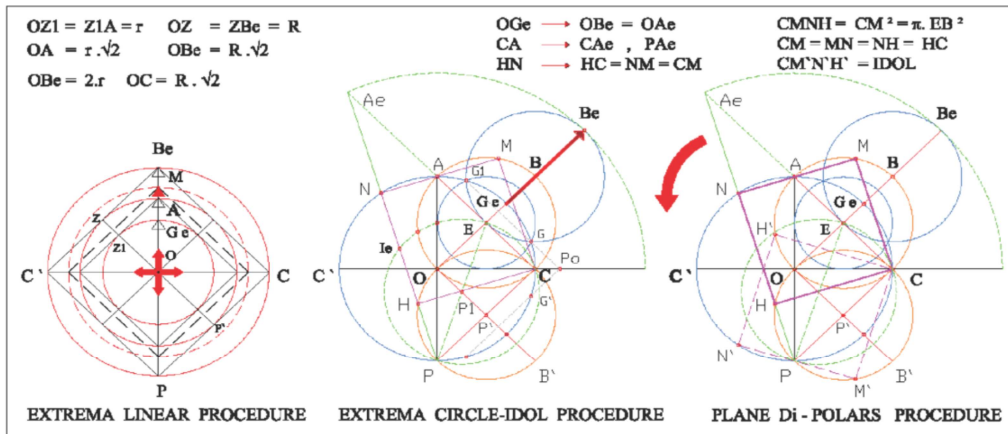
1.. The Squaring of the Circle .

The Plane Procedure Method . [45-46]

The property, of Resemblance Ratio be equal to 2 on a Square, is transferred simultaneously by the equality of the two areas, *when square is equal to the circle*, where that square is twice of the inscribed circle.

This property becomes from the linear expansion in three spaces of the inscribed (O, OGe) to the circumscribed (O, OM) circle, in a circle (O, OA) as in . F.1-(1).

1.. The Extrema method of Squaring the circle Fig.



(1)

(2)

(3)

F.1 → The steps for Squaring any circle (E, EA = EC) on diameter CA through the – Four-Polar Procedure method. Expanding Inscribed circle O, OGe → to circle O, OA → to circumscribed O, OM.

In (1) Expanding Inscribed circle O, OGe to → to circle O, OA → to circumscribed O, OM.

In (2) Expanding Inscribed square CBAO to → to square OMNH → to circumscribed CAC'P

In (3) Extrema Edge circle O, OBe to → to Ae point → to extrema square CMNH = NH² = π.EA²

Plane Procedure method is consisted of two equal and perpendicular vectors CA, CP, **the Mechanism**, where CA = CP and CA ⊥ CP, such, so that the Work produced is zero and this because each area is zero, with three conjugate Poles A, C, P related to central O, with three Pole-lines CA, CP, AP and three perpendicular Anti -Pole-lines OB, OB', OC, and

Converting the Rectilinear motion on the Mechanism, to Four - Polar Expanding rotational motion.

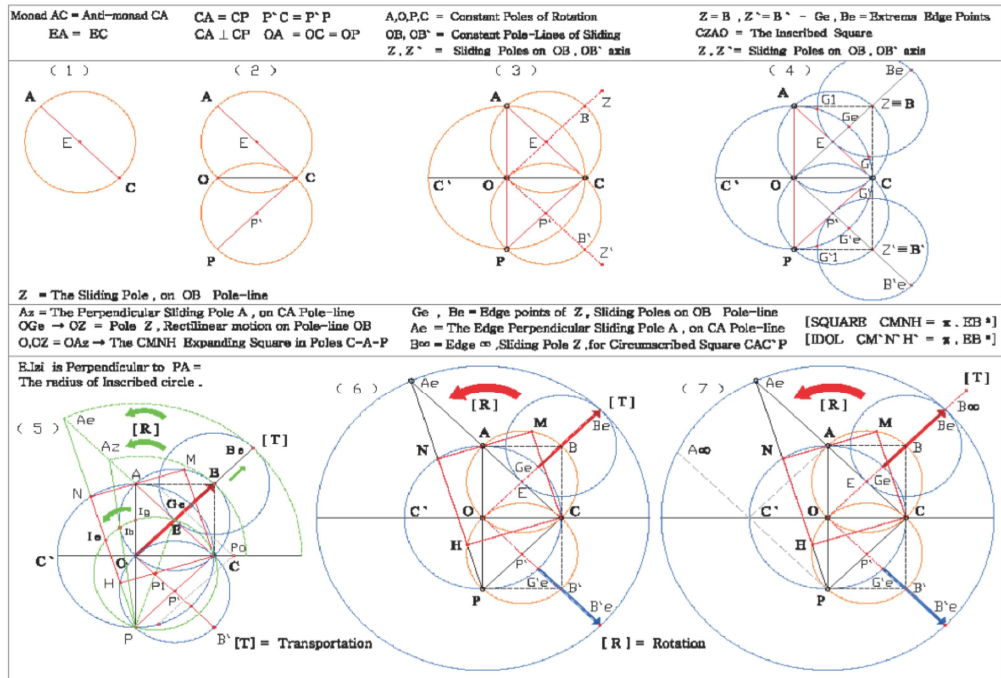
The formulated Five Conjugate circles with diameters → CA = OB, CP = OB', EBe = OB, P'Pe = OB', PoP1 = PoP2 = CA and also the circumscribed circle on them ← define **A System of infinite Changable Squares from** → CBAO to → CMNH and to → CAC'P, **through the Four - Poles of rotation**. The Geometrical construction: F.1-(2) – F.2

- 1.. Let E be the center, and CA is the diameter of any circle (E, EA = EC).
- 2.. Draw CP = CA perpendicular at point C and also the equal diameter circle (P', P'C = P'O).
- 3.. From mid-point O of hypotenuse AP as center, Draw the circle (O, OA = OP = OC) and complete squares, OCBA, OCB'P. On perpendicular diameters OB, OB' and from points B, B' draw circles (B, BE = Be), (B', B'P') intersecting (O, OA) = (O, OP) circle at double points [G, G1], [G'G'1] respectively, and OB, OB' produced at points Be, B'e, respectively.
- 4.. Draw on the symmetrical to OC axis, lines GG1 and G'G'1 intersecting OC axis at point Po. 5.. Draw the edge circle (O, OBe) intersecting CA produced at point Ae and draw PAe line intersecting the circles, (O, OA), (P', P'P) at points N-H, respectively.
- 6.. Draw line NA produced intersecting the circle (E, EA) at point M and draw Segments CM, CH and complete quadrilateral CMNH, calling it the *Space = the System*. Draw line CM' and line M'P produced intersecting circle (O, OA) at point N' and line AN' intersecting circle (E, EA) at point H', and complete quadrilateral CM'N'H', calling it *the Anti-space = Idol = Anti-System*.
- 7.. Draw the circle (P1, P1E) of diameter PE intersecting OA at point ,Ig, and (E, EA) circle at point Ib.

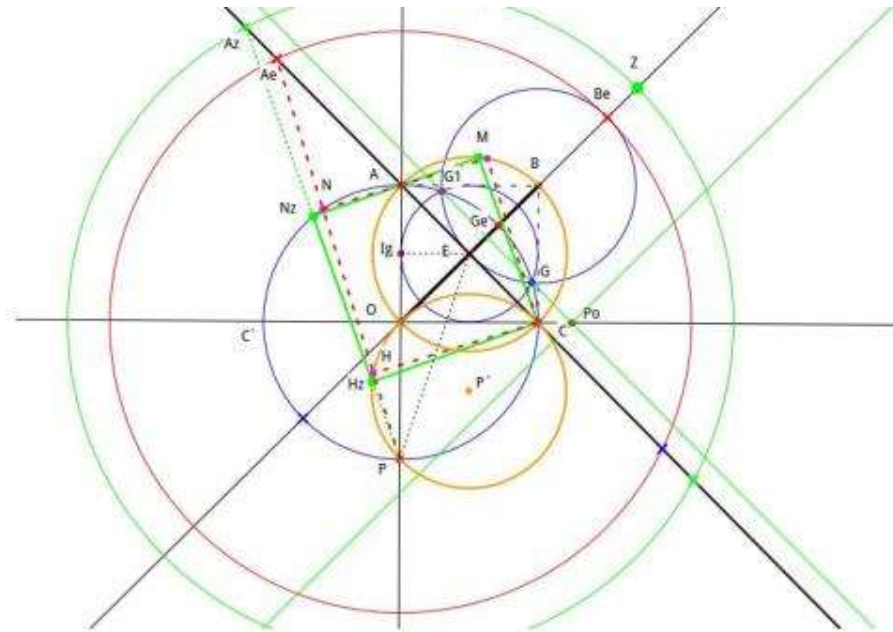
A. Show that CMNH, CM'N'H' are Squares.

B. Show that it is an Extrema Mechanism, on **Four Poles where**, The Two dimensional Space (the Plane) is Quantized to a System of infinite **Squares** → CBAO → CMNH → CAC'P, and to **CMNH** square of side $CM = HN$, **where holds** $CM^2 = CH^2 = \pi \cdot EA^2 = \pi \cdot EO^2$

THE PROCESS OF SQUARING THE CIRCLE →



F.2 → The steps for Squaring the circle (E, EA = EC) on diameter CA through Plane Procedure Mechanism.



F.2-A → A Presentation of the Quadrature Method on Dr. Geo-Machine Macro-constructions

The Inscribed Square CBAO, with Pole-line AOP, rotates through Pole P, to the → Circle-Square CMNH with Pole-line NHP, and to the → Circumscribed Square CAC'P, with Pole-line C'PP = C'P, of the circle E, EO = EC.

Analysis: In (1) EA = EC and the unique circle (E, EA) of Segment AC, where AC, CA is monad Anti-monad. In (2) Since circles (E, EA), (P', P'P) are symmetrical to OC axis (line) then are equal (conjugate) and since they are Perpendicular so, → No work is executed for any motion ←. In (3) Points A, C, P and O are the constant Poles of Rotation, and OB, OB', OC-C A, CP, AP the Six, Pole and Anti-Pole, lines, of sliding points Z, Z', and Az, A'z, while CA, CP are the constant Pole-lines {PA, PAz, PAe, PC'}, of Rotation through pole P.

In (1) $EA = EC$ and the unique circle (E, EA) of Segment AC, where AC, CA is monad Antimonad.
 In (2) Since circles (E, EA), (P', P'P) are symmetrical to OC axis (line) then are equal (conjugate) and since they are Perpendicular so, \rightarrow No work is executed for any motion \leftarrow .

In (3) Points A, C, P and O are the constant Poles of Rotation, and OB, OB', OC - CA, CP, AP the Six, Pole and Anti-Pole, lines, of sliding points Z, Z', and Az, A'z, while CA, CP are the constant Pole-lines {PA, PAz, PAe, PC'}, of Rotation through pole P
 In (4) Circles (E, EO), (P', P'O) on diameters OB, OB' follow, My Theorem of the three circles on any Diameters on a circle, where the pair of points G, G1 and G', G'1 consist a Fix and Constant system of lines GG1 and G'G'1. When Points Z, Z' coincide with the Fix points B, B' and thus forming the inscribed Square CBAO or CZAO, (this is because point Z is at point A. The PA, Pole-line, rotates through pole P where Ge, Be, are the Edge points of the sliding poles on this Rectilinear-Rotating System.

In (5) Points $Z \equiv B, Z' \equiv B'$ on lines OB, OB', and points Az, A'z are the Sliding points while CA, CP, are the constant Pole-lines {PA, PAz, PAe, PC'}, of Rotation through pole P.

Sliding points Z, Z', Az, A'z are forming Squares CMNH, CM'N'H', and this as in Proof A-B below, where PN, AN' are the Pole-lines rotating through poles P, A, and diamesus HM passes through O. The circles (E, EO), (P', P'O) on diameters OB, OB' follow also, my Theorem of the Diameters on a circle.

In (6), Sliding poles Z, Z' being at Edge point $Ge \equiv Z$ formulates CBAO Inscribed square, at Edge point $Be, Be \equiv Z$ formulates CMNH equal square to that of circle and, at Edge point $B\infty$, formulates CAC'P square, which is the Circumscribed square.

In (7), are holding \rightarrow CBAO the Inscribed square, CMNH The equal to the (E, EO = P'O) circle square, and CAC'P the Circumscribed square.



F.3. \rightarrow Markos theorem, on any OB diameter.

Theorem: [F.1-(2)], F.3

On each diameter OEB of a circle (E, EB) we draw,

- 1.. the circumscribed circle (O, OA = OE . $\sqrt{2}$) at the edge point O as center,
2. the inscribed circle (E, OE / $\sqrt{2}$ = OA/2 = EG) at the mid-point E as center,
- 3.. the circle (B, BE = B.Be) = (E, EO) at the edge point B as center, through t

Then the three circles pass the common points G, G1, and the symmetrical to OB point G1 forming an axis perpendicular to OB, which has the Properties of the circles, where the tangent from point B to the circle (O, OA = OC) is constant and equal to $2EB^2$, and has to do with, Resemblance Ratio equal to 2.

A-B. The Common-Proof:

In F.1-(2), F.2-(5),

Angle $\angle CHP = 90^\circ$ because is inscribed on the diameter CP of the circle (P', P'P). The supplementary angle $\angle CHN = 180 - 90 = 90^\circ$.

Angle $\angle PNA = \angle PNM = 90^\circ$ because is inscribed on the diameter AP of the circle (O, OA) and Angle $\angle CMA = 90^\circ$ because is inscribed on the diameter CA of the circle (E, EA = EC).

The upper three angles of the quadrilateral CHMN are of a sum of $90+90+90 = 270$, and from the total of 360° , the angle $\angle MCH = 360 - 270 = 90^\circ$, Therefore shape CMNH is **rightangled** and exists $CM \perp CH$. Since also $CM \perp CH$ and $CA \perp CP$ therefore angle $\angle MCA = \angle HCP$.

The rightangled triangles CAM, CPH are equal because have hypotynousa $CA = CP$ and also angles $\angle CMA = \angle CHP = 90^\circ$, $\angle MCA = \angle HCP$, therefore side $CH = CM$, and **Because $CH = CM$, the rectangle CMNH is Square.** The same for Square CM'N'H'. (o.e.δ),(q.e.d).

This is the General proof of the squares on this Mechanism without any assumptions.

From the equal triangles COH, CBM angle $\angle CHO = \angle CHM = 45^\circ$ because lie on CO chord and so points H, O, M lie on line HM i.e.

Any segment $PA \rightarrow PAz \rightarrow PAe \rightarrow PC' = CA$, drawn from Pole, P, beginning from A to ∞ , intersecting the circumscribed (O,OA) circle, and the circle (P', P'P = P'C = EO = EC) at the points N, H, Formulates Squares CBAO

CMzNzHz , CAC'P respectively , which are , **The Inscribed , In-between , Circumscribed Squares , of circle (O,OE)** = (E,EO = EB) = (P,P'P) . Since angles < CAzP , HCP have their sides CAz , CP – AzP,CH perpendicular each other , then are equal so angle < PAzC = PCH = OZZm , and so point Az , is common to circle O,OZ , Poleline CA , and Pole-axis PN , where Z,Zm the perpendicular to CM .

Since PE is diameter on (P1,P1P) circle , therefore triangle E,Ig,P is right-angled and segment ,EIg, perpendicular to OA and equal to OE/√2 = OA/2 , the radius of the Inscribed circle . Since also point ,Ig, lies on PA , therefore moves on (P1, P1.O) circle and point A on CA Pole-line and since point B is on the same circle as Az then point B moves on circle E,EB .

B.. Proof : F.2-(5)

(1) Point **Z** , which moves on diameter OB produced , Beginning from Edge-point **Ge** of the first circle , Passing from center **B** of the second circle , Passing from Edge-point **Be** of the third circle , and Ending to infinite ∞ , → **Creates on the three circles** (O,OA) , (E,EO/√2) , (B,BE) , the

Changeable moving Squares

- a)..The Inscribed **CBAO** , at Z ≡ Ge
- b)..The In-between **CMzNzHz** , at Z ≡ B
- c)..The Extrema **CMNH** , at Z ≡ Be
- d)..The Circumscribed **CAC'P** . at Z ≡ B∞

(2). Through the four constant Poles A,C,P – O of the *Plane Procedure Mechanism* , pass (rotate) the Sides and Diamesus (from O) of Squares , AntiSquares.

(3). Point **Z** moving from Edge points Ge and , (forming inscribed square CBAO) , in-between points Ge-Be (forming any square CMzNzHz) , at Extrema point Be (forming that square CMNH equal to the circle) , and between points , Be - ∞ , (forming the circumscribed square CAC'P) .

(4). Point Ig , belongs to the Inscribed circle (E,EG) and it is the Rotating , expanding , Inscribed Edge point on (P1,P1P) circle to Ig,Ib,Ie and to → P point . The other two , Sliding , Edge moving points B,A slide on OB , CA , Pole-lines respectively .

A – Proof (1) :

Since BC ⊥ CO , the tangent from point B to the circle (O , OA) is equal to :

$$BC^2 = BO^2 - OC^2 = (2 \cdot EB)^2 - (EB \cdot \sqrt{2})^2 = 2$$

$$EB^2 = (2 \cdot EB) \cdot EB = (2 \cdot BG) \cdot BG \text{ and since}$$

2.BG = BG1 then **BC² = BG . BG1** , where point G1 lies on the circumscribed circle , and this means that BG produced intersects circle (O, OA) at a point G1 twice as much as BG . Since E is the mid-point of BO and also G midpoint of BG1 , so **EG** is the diamesus of the two sides BO,BG1 of the triangle BOG1 and equal to 1/2 of radius OG1 = OC , the base , and since the radius of the inscribed circle is half (1/2) of the circumscribed radius **then the circle (E , EB / √2 = OA/2) passes through point G** . Because BC is perpendicular to the radius OC of the circumscribed circle , so **BC is tangent and equal to BC² = 2 . EB² . (o.e.δ).(q.e.d) Proof F.2- (5-6) :**

The point **Z** moving on **OB** Pole-line , defines on CA , point Az as that of intersection of circle (O,OZ) and this line. Polar-line PAz defines N,H points such that CHNM rightangled is completed as Square without any more assumptions .

Following again prior A-B common proof , Angle < CHP = 90° because is inscribed on the diameter CP of the circle (P' , P'P) . The supplementary angle < CHN = 180 – 90 = 90° . Angle < PNA = PNM = 90° because is inscribed on the diameter AP of the circle (O , OA) and Angle < CMA = 90° because is inscribed on the diameter CA of the circle (E , EA = EC) . The upper three angles of the quadrilateral CHMN are of a sum of 90+90+90 = 270 , and from the total of 360° , the angle < MCH = 360 –270 = 90° , therefore shape CMNH is rightangled and exists CM ⊥ CH . Since also CM ⊥ CH and CA ⊥ CP therefore angle < MCA = HCP .

The rightangled triangles CAM , CPH are equal because have hypotynousa CA = CP and also angles < CMA = CHP = 90° , < MCA= HCP and side CH = CM therefore , rectangle CMNH is Square on **CA,CP** Mechanism , through the three constant Poles C,A,P of rotation . The same for square CM'N'H' . (o.e.δ)-(q.e.d) .

From the equal triangles COH , CBM angle < CHO = CHM = 45° and so points H,O,M lie on line HM .i.e. Diagonal **HM** of squares CMNH on Mechanism passes through central Pole O . (o.e.δ)-(q.e.d) .

The two equal and perpendicular vectors CA , CP , the *Plane Mechanism* , of the Changable Squares through the two

constant Poles C, P of rotation , is converting *the Circular motion to Four-Polar Rotational motion*.

Transferring the above property to [F.2 -(5)] then when point Z moves on OB → Point Az moves on CA and → PAz line defines on circle of diameter PE the points Iz , on circles O,OA = Circumscribed P'P'O = The Circle , and points H,N such that shapes → CHNM are all Squares between the Inscribed and Circumscribed circle .

Since Areas of above circles are \rightarrow Area of Inscribed = $\frac{1}{2} \pi.OE^2$
 Area of Circle = $1 \pi.OE^2$ = 1 Area of Circumscribed = $2 \pi.OE^2$ and those of corresponding squares , then
 one square of *Plane Mechanism* is equal to the circle , but Which one ??
 \rightarrow **That square which is formed on Extrema Case. The Plane Mechanism :**

The radius of the inscribed circle is $AB/2$ and equal to the perpendicular distance between center E and OA , so any circle of EP diameter passes through the edge-point (I_g) , and point (I_b) is the Edge common point of the two circles .

The Common Edge -Point of the three circles is (I_e) belongs to the Edge point B_e of circle (B, BE = B.B_e) , so exists ,
 Case : [1] [2] [3] [4]
 Point **Z** at \rightarrow Ge B B_e B ∞
 Point **A** at \rightarrow A A(I) A_e A ∞
 Point **I_g** at \rightarrow I_g I_z = I_b I_e P
 ↓ ↓ ↓ ↓
 Square CBAO , C_miN_iH_i , CMNH , CAC`P

i.e. Square CMNH of case [3] is equal to the circle , and $CM^2 = CH^2 = \pi \cdot EA^2 = \pi \cdot EO^2$
 On the three Circles and Lines exist \rightarrow
 a).. Circle (O, OZ = OGe) is Expanding to \rightarrow (O, OZ = OBe) Circumscribed circle , for the CBAO square .
 b).. Point (A-A_g) to \rightarrow (A-A_z) is The Expanding Pole-line A-A_z for the In-between CMzNzHz square ,
 c).. Circle (P₁, P₁.I_g) is Expanding to \rightarrow (P₁, P₁.I_b) Inscribed circle (E, E.I_g) to I_b point.
 d).. Point (P -P_g) to \rightarrow (P -P_e) is The Expanding Pole-line P -P_e for the Extrema CMNH = $\pi \cdot EA^2$ and is the square equal to the circle ,
 e).. Circle (O, OZ=OB ∞) , Pole-line (A -AZ_e = A ∞) , Pole-line (P -P_{Ie} = PP \rightarrow P) , for CAC`P square, Point N on (O, OA) , belongs to Circumscribed circle Point I_e , on circle with diameter ,PE , belongs to the Inscribed circle

(E , EI_g = EG) Point H , on (P` , P`O) , belongs to the Circle.
i.e. It was found a Mechanism where the Linearly Expanding Squares \rightarrow CBAO - CMNH - CAC`P , and circles \rightarrow (P₁, P₁E) - (B, BE) - (O, OA) , which are between the

Inscribed and Circumscribed ones , are Polarly - Expanded as Four - Polar Squares . The problem is in two dimensions determining an edge square between the inscribed and the circumscribed circle . A quick measure for radius $r = 2694$ m gives side of square 4775 m and $\pi = 3,1416048 \rightarrow 11/10/2015$

Segments CM = CM` is the Plane
Procedure Quantization of radius EC = EO in Euclidean Geometry , through this Mould (The Plane Procedure
Method is called so , because it is in two dimensions \rightarrow CA CP) as this happens also in Cube mould for the three dimensions of the spaces , which is a Geometrical machine for constructing Squares and Anti Squares and that one equal to the circle . This is the Plane Quantization of , E - Geometry , i.e. The Area of square CMNH is equal to that of one of the five conjugate circles , or
CM² = $\pi \cdot CE^2$, and System with number π to be a constant .

B - Proof (1) :
 Since circle (O, OGe) intersects CA vector at point A forming the inscribed square CBAO , the circle (O, OZ) is intersecting CA at point Az forming square CMzNzHz then edge circle (O, OBe) intersecting CA at point Ae is forming square CMNH equal to the circle . The circle (O, OB ∞) intersecting CA at point A ∞ (and Poleline PAz becomes PC` which is parallel to PA) is forming the circumscribed square CAC`P .

B - Proof (2) :
 Since PE is diameter on (P₁, P₁P) circle , therefore triangle E.I_g.P is right-angled and ,EI_g perpendicular to OA and equal to $OE/\sqrt{2} = OA/2$, to the Inscribed circle . Since also point ,I_g lies on PA , therefore moves on (P₁, P₁O) circle and point A on CA Pole-line and since point B is on the same circle as Az then point B moves on OB Pole-line .

Remarks :
 Since Monads $AC = ds = 0 \rightarrow \infty$ are simultaneously (*actual infinity*) and (*potential infinity*) in Complex number form , this defines that the infinity exists also between all points which are not coinciding , and **ds** comprises any two edge points with imaginary part , for where this property differs between the infinite points between edges . This property of monads shows the link between space and energy which Energy is between the points and Space on points. In plane and on solids , energy is spread as the Electromagnetic field in surface . The position and the distance of points , can be

calculated between the points and so to *perform independent Operations* (Divergence, Gradient , Curl , Laplacian) on points .

This is the Vector relation of Monads , $ds = CA$,(or , as Complex Numbers in their general form $w = a + b.i = discrete\ and\ continuous$) , and which is the Dual Nature of Segments = monads in Plane, to be discrete and continuous). Their monad – meter in Plane , and in two dimensions is CM , the analogous length , in the above Mechanism of the Squaring the circle with monad the diameter of the circle .

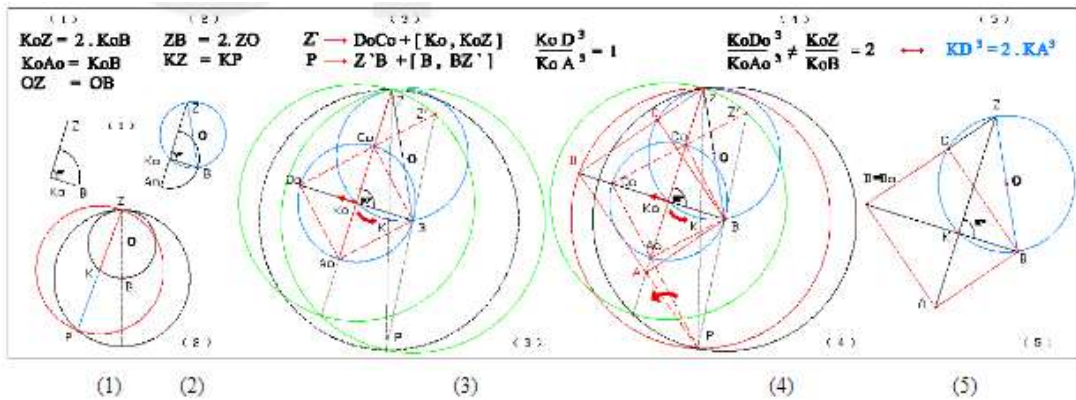
Monad is $ds = CA = OB$, the diameter of the circle (E , EA) with CBAO Square , on the Expanding by Transportation and Rotation Mechanism which is $\rightarrow \{Circumscribed\ circle\ (O,OA) - Inscribed\ circle\ (E , EG = E.Ig) - Circle\ (B, BE)\} \leftarrow In\ extended\ moving\ System \rightarrow \{OB\ Pole-line - CA\ Pole-line - Circle\ (P1,P1.B = P1.Ig)\}$,and Is quantized to CMNH square. A deeper analysis for , Mechanics and Physics , concerning the Theorem of the three circles and applications , in [51]

2.. The Duplication of the Cube , Or the Problem of the two Mean Proportionals .

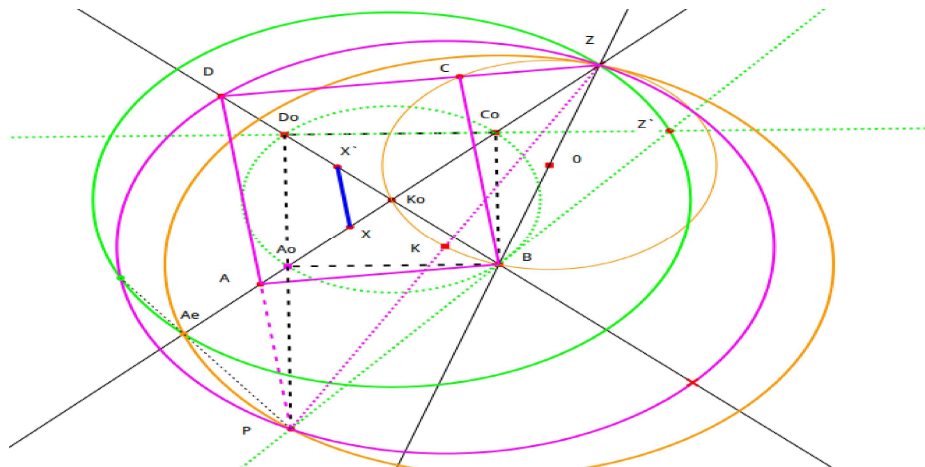
The Extrema method for the Duplication of the cube ? [44-45]

This problem is in three dimensions as this first was by Archytas proposed by determining a certain point as the intersection of three surfaces , a right cone , a cylinder , a tore or anchoring with inner diameter nil . Because of the three master meters where is holding the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (continuous analogy) in all Spaces , the solution of this problem , as well as that of squaring the circle , is linearly transformed .

The geometrical construction in F-4 : The Presentation of the method on Dr-Geo machine for macro-constructions in F.4-A



F.4-A.→ A Presentation of the Dublication Method on Dr.Geo-Machine Macro -constructions



→ BCDA Is the In-between Quadrilateral , on (K,KZ)Extrema-circle, and on KoZ-KoB Extrema-lines of common poles Z , P, mechanism. The Initial Quadrilateral BCoDoAo , with Pole-lines DoAoP -DoCoZ' , rotates through Pole P and the moveable Pole Z' on Z'Zarc, to the → Extreme Quadrilateral BCDA through Pole-lines DAP -DCZ with point Do , sliding on BKoDo Pole-line

F.4-A. → A Presentation of the Duplication Method on Dr.Geo - Machine Macro - constructions

→ BCDA Is the In-between Quadrilateral , on (K,KZ) Extrema-circle , and on KoZ-KoB Extrema – lines of common poles Z , P , mechanism . *The Initial Quadrilateral BCoDoAo , with Pole-lines DoAoP - DoCoZ` , rotates through Pole P and the moveable Pole Z` on Z`Z arc , to the → Extreme*

Quadrilateral BCDA through Pole-lines DAP - DCZ with point Do , sliding on BKoDo Pole-line .

F.4-A → BCoDoAo Is the initial Basic Quadrilateral ,square , on KoZ , KoB Extrema-lines mechanism.

Draw Line segment KoZ to be perpendicular to its half segment KoB or as $KoZ = 2.KoB \square KoB$ and the circle (O, BZ/2) of diameter BZ . Line segment ZKo produced to KoAo = KoB (or and $KoXo \neq KoB$) is forming the Isosceles right-angled triangle AoKoB.

Draw segments BCo , AoDo equal to BAo and be perpendicular to AoB such that points Co , Do, meet the circle (Ko,KoB) in points Co , Do respectively, and thus forming the inscribed square BCoDoAo . Draw circle (Ko ,KoZ) intersecting line DoCo produced at point Z` and draw the circle (B , BZ) intersecting diameter Z`B , produced at point P (the Pole) .

Draw line ZP intersecting (O, OZ) circle at point K , and draw the circle (K , KZ) intersecting line BDo produced at point D .

Draw line DZ intersecting (O, OZ) circle at point C and Complete Rectangle CBAD on the diamesus BD .

Show that this is an Extrema Mechanism on where **The Three dimensional Space KoA → is Quantized to KoD as** $\rightarrow KoD^3 = 2. KoA^3$.

Analysis :

In (1) $KoZ = 2.KoB$ and $KoAo = KoB$, $KoB \square KoZ$ and $KoZ / KoB = 2$.

In (2) Circle (B, BZ) with radius twice of circle (O, OZ) is **the extrema** case where circles with radius $KZ = KP$ are formulated and are the locus of all moving circles on arc BK as in F4-(2) , F.5 In (3) Inscribed square BCoDoAo passes through middle point of KoZ so $CoKo = CoZ$ and since angle $\angle ZCoO = 90^\circ$, then segment $OCo \parallel BKo$ and $BKo = 2.OCo$. Since radius OB of circle (O,OB = OZ) is $\frac{1}{2}$ of radius OZ of circle (B,BZ = 2.BO) then ,D, is **Extrema** case where circle (O,OZ) is the **locus of the centers** of all circles (Ko, KoZ) , (B, BZ) moving on arc ,KoB, as this was proved. All circles **centered on this locus** are common to circle (Ko,KoZ) and (B,BZ) separately.

The only case of being together is the common point of these circles which is their common point P , where then → **centered circle exists on the Extrema edge , ZP diameter**. In (4) , F4-(4) Initial square AoBCoDo , **Expands and Rotates** through point B , while segment DoCo limits to DC , where **extrema point Z`** moves to Z . Simultaneously , the circle of radius KoZ moves to circle of radius BZ on the locus of $\frac{1}{2}$ chord KoB . Since angle $\angle Z`DoAoP$ is always 90° so , exists on the diameter Z`P of circle (B, BZ`) and is the limit point of chord DoAo of the rotated square BCoDoAo , and not surpassing the common point Z . Rectangle BAoDoCo in angle $\angle PDoZ`$ is expanded to Rectangle BADC in angle $\angle PDZ$ by existing on the two limit circles (B,BZ` = BP) and (Ko, KoZ) and point Do by sliding to D .

On arc KoB of these limits is **centered circle on ZP diameter** , i.e. **Extrema** happens to → **the common Pole of rotation through a constant circle centered on KoB arc** , and since point Do is the intersection of circle (Ko , KoB = KoDo) which limit to D , therefore the intersection of the common circle (K , KZ = KP) and line KoDo denotes that extrema point , where the expanding line DoCoZ` with leverarm DoAoP **is rotating through Pole P** , and limits to line DCZ , **and , Point P is the common Pole of all circles on arc ,KoB, for the Expanding and simultaneously rotating Rectangles** .

In (5) rectangle BCDA formulates the two right-angled perpendicular triangles ADZ ,ADB which solve the problem. **Segments KoD , KoAo = KoB are the two Quantized magnitudes in Space (volume) such that Euclidean Geometry Quantization becomes through the Mould of Doubling of the Cube . [This is the Space Quantization of E-Geometry**

i.e. The cube of Segment KoD is the double magnitude of KoA cube , or monad $KoD^3 = 2$ times the monad KoA^3] . About Poles in [5] . Proof : F.4. (3)-(4).

1.. Since $KoZ = 2.KoB$ then $(KoZ / KoB) = 2$, and since angle $\angle ZKoB = 90^\circ$ then BZ is the diameter of circle (O,OZ) and angle $\angle ZKoB = 90^\circ$ on diameter ZB 2.. Since angle $\angle ZKoAo = 180^\circ$ and angle $\angle ZKoB = 90^\circ$ therefore angle $\angle BKoAo = 90^\circ$ also .

□

3.. Since BKo ZKo then Ko is the midpoint of chord on circle (Ko, KoB) which passes through Rectangle (square) BAoDoCo . Since angle $\angle ZDP = 90^\circ$ (because exists on diameter ZP) and since also angle $\angle BCZ = 90^\circ$ (because exists on diameter ZB) therefore triangle BCD is right-angled and BD the diameter .

Since Expanding Rectangles BAoDoCo , BADC rotate through Pole , P , then points Ao , A lie on circles with BDo , BD diameter , therefore point D is common to BDo line and (K , KZ = KP) circle , and BCDA is Rectangle . F.4-(2)

i.e. Rectangle BCDA possess AKo ⊥ BD and DCZ line passing through point Z .

The Unsolved Ancient - Greek Problems of E-geometry

4.. From right angle triangles ADZ , ADB that XX1 // AD , as in Fig.6 -(4) . we ha
 , KoA is → (KoX)² = (KoA)² . (XX1 / AD) such *All comments are left to the readers* , 30/8/2015.

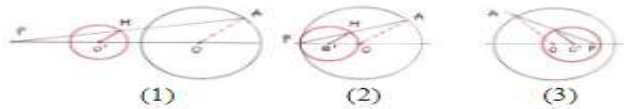
$$\begin{aligned} \Delta ADZ &\rightarrow KD^2 = KA \cdot KZ && \dots (a) \\ \Delta ADB &\rightarrow KA^2 = KD \cdot KB && \dots (b) \end{aligned}$$

and by division (a) / (b) then →

$$\frac{KD^2}{KA \cdot KZ} = \frac{KD^2}{KA \cdot KZ} \cdot \frac{KD^3}{KD^3} = \frac{KD^3}{KZ} = 2$$

(1) (2) (3)

³ 2.KoA³ . *This is one of the basic and Perpendicular - segments = Plane Vectors* ,
Geometrical Euclidean Geometry Moulds , which
 (Non-equal Segments and twice-Perpendicular-
create the METERS of monads → where Linear segments = The Space Vectors = Quaternion) , by

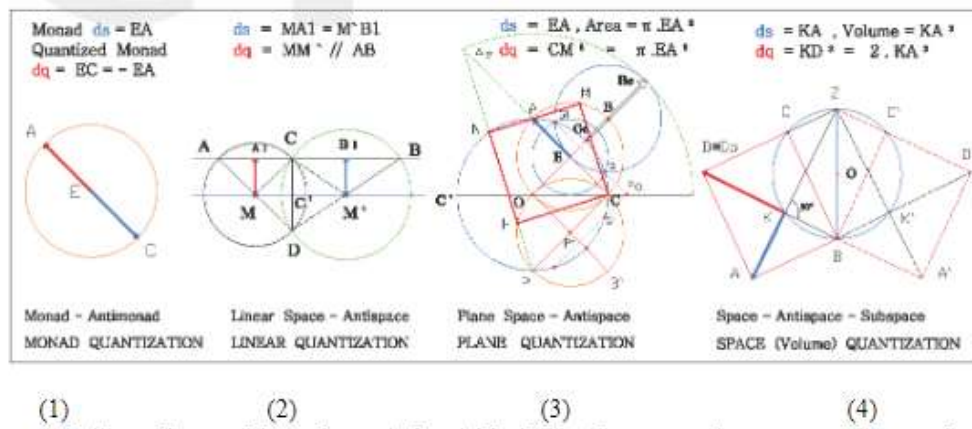


F.5. → For any point A on , and P Out-On-In circle [O, OA] and O'P = O'O , exists O'M = OA / 2 .

2.1 The Quantization of E-Geometry , { Points , Segments , Lines , Planes , and the Volumes } , to its moulds F-6 .

Quantization of E-geometry is the Way of Points to become as → (Segments , Anti-segments = Monads = Anti-monads) , (Segments , Parallel-segments = Equal monads) , (Equal Segments and Perpendicular - segments = Plane Vectors) , (Non-equal Segments and twice-Perpendicular-segments = The Space Vectors = Quaternion) , by defining the mould of quantization .

The three Ways of quantization are → for Monads = Material points , the mould is the Cycloidal Curl Electromagnetic field , for Lines the mould is that of Parallel Theorem with the least constant distance , for Plane the mould is the Squaring of the circle and , for Space is the mould of the Duplication of cube . All methods in , F- 6 below .



F.6. → The Point , Linear , Plane , Space (volume) Mould for E-geometry Quantization , of monad EA to Anti-monad EC – of AB line to Parallel line MM' - of AE Radius to the CM side of Square of KA Segment to KD Cube Segment .

The numeric METERS of Quantization of any material monad $ds = AB$ are as → In any point A , happens through Mould in itself (The material point as a → ± dipole) in [43]

In monads $ds = AC$, happens through Mould in itself for two points (The material dipole in inner monad Structure as the Electromagnetic Cycloidal field which equilibrium in dipole by the Anti-Cycloidal field as in [43]). **For monad $ds = EA$ the quantized and Anti-monad is $dq = EC = \pm EA$**

Remark : The two opposite signs of monads EA , EC represent the two Symmetrical equilibrium monads of Space-Antispace , the Geometrical dipole AC on points A,C which consist space AC as in F6 - (1)

Linearly , happens through Mould of Parallel

Theorem , where for any point M not on $ds = \pm AB$, the Segment $MA1 = \text{Segment } M'B1 = \text{Constant}$. F6 - (1-2)

Remark : The two opposite signs of monads represent the two Symmetrical monads in the Geometrical machine of the equal and Parallel monads [$MM' // AB$ where $MA1 \square AB$, $M'B1 \square AB$ and $MA1 = M'B1$] which are \rightarrow The Monad $MA1 - \text{Antimonad } M'B1$, or \rightarrow The Inner monad $MA1$ Structure - The Inner Anti monad structure $M'B1 = -MA1 = \text{Idle}$, and { The Space = line AB , Anti-space = the Parallel line MM' = constant } .

The Parallel Axiom is no-more Axiom because this has been proved as a Theorem [9-32-38-44].

Plainly , happens through Mould of Squaring of the circle , where for any monad $ds = CA = CP$, the Area of square $CMNH$ is equal to that of one of the five conjugate circles and $\pi = \text{constant}$, or as $CM^2 = \pi \cdot CE^2$.

On monad $ds = EA = EC$, the Area = $\pi \cdot EC^2$ and the quantized Anti-monad $dq = CM^2 = \pm \pi \cdot EC^2$. F6-(3)

Remark : The two opposite signs represent the two Symmetrical squares in Geometrical machine of the equal and perpendicular monads [$CA \square CP$, and $CA = CP$] , which are \rightarrow The Square $CMNH - \text{Antisquare } CM'N'H'$, or \rightarrow The Space - Idle = Anti-space .

In Mechanics this property of monads is very useful in Work area , where two perpendicular vectors produce Zero Work . {Space = square $CMNH$, Anti-space = Anti-square $CM'N'H'$ } .

In three dimensional Space , happens through Mould Doubling of the Cube , where for any monad $ds = KA$, the Volume or , The cube of a segment KD is the double the volume of KA cube , or monad $KD^3 = 2 \cdot KA^3$. On monad $ds = KA$ the Volume = KA^3 and the quantized Anti-monad , $dq = KD^3 = \pm 2 \cdot KA^3$. F6-(4)

Remark : The two opposite signs represent the two Symmetrical Volumes in Geometrical machine of triangles [$\Delta ADZ \square \Delta ADB$] , which are \rightarrow The cube of a segment KD is the double the volume of KA cube - The Anti-cube of a segment $K'D'$ is the double the Anti-volume of $K'A'$ cube ,

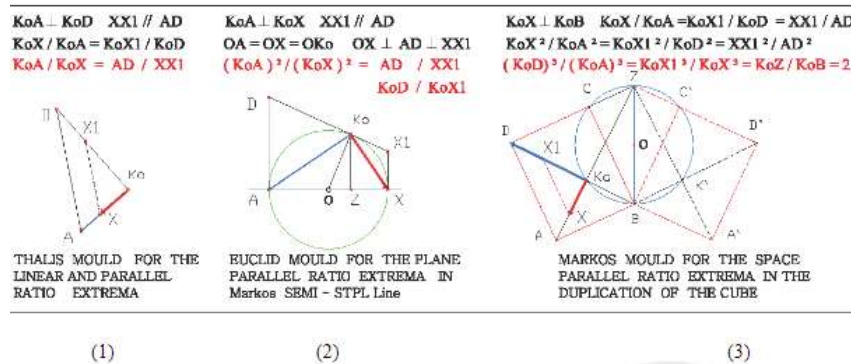
Monad $ds = KA$, the Volume = KA^3 and the quantized Anti-monad $dq = KD^3 = \pm 2 \cdot KA^3$. {The Space = the cube KA^3 , The Anti-space = the Anti-cube KD^3 } .

In Mechanics this property of Material monads is very useful in the Interactions of the Electromagnetic Systems where Work of two perpendicular vectors is Zero .

{ Space = Volume of KA , Anti-space = Anti - Volume of KD , and this in applied to Dark-matter , Energy in Physics } . [43]

Radiation of Energy is enclosed in a cavity of the tiny energy volume λ , (which is the cycloidal wavelength of monad) with perfect and absolute reflecting boundaries where this cavity may become infinite in every direction and thus getting in maxima cases (the edge limits) the properties of radiation in free space .

The electromagnetic vibrations in this volume is analogous to vibrations of an Elastic body (Photo-elastic stresses in an elastic material [18]) in this tiny volume , and thus Fringes are a superposition of these standing (stationary) vibrations .[41]



Above are analytically shown , the Moulds (The three basic Geometrical Machines) of Euclidean Geometry which create the METERS of monads

Linearly is the Segment MA1 , In Plane the square CMNH , and in Space is volume KD³ in all Spaces , Anti-spaces and Sub-spaces .

This is the Euclidean Geometry Quantization in points to its constituents , i.e. the

- 1.. METER of Point A is the Material Point A , the ,
- 2.. METER of line is the discrete Segment ds = AB = monad = constant , the
- 3.. METER of Plane is that of circle on Segment = monad , which is the Square equal to the area of the circle , and the
- 4.. METER of Volume is that of Cube on any Segment = monad , which is the Double Cube of Segment and Thus is the measuring of the Spaces , Anti-spaces and Sub-spaces in this cosmos . markos 11/9/2015

2.2 The Three Master - Meters in One , for E-geometry Quantization , F-7

Master - meter is the linear relation of the Ratio , (continuous analogy) of geometrical magnitudes , of all Spaces and Anti-spaces in any monad .This is so because of the , extrema - ratio - meters .

Saying **master-meters** , we mean That the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (continuous analogy) in all Spaces , in one in two in three dimensions, as this happens to the Compatible Coordinate Systems as these are the Rectangular [x,y,z] , [i,j,k] , the Cylindrical and Spherical Polar . The position and the distance of points can be then calculated between the points , and thus to **perform independent Operations** (Divergence , Gradient , Curl , Laplacian) on points only .This property issues on material points and monads .

This is permitted because , Space is quaternion and is composed of Stationary quantities , the position $\vec{r}(t)$ and the kinematic quantities , the velocity $\rightarrow \vec{v} = dr/dt$ and acceleration $\rightarrow \vec{a} = d\vec{v}/dt = d^2r/dt^2$. Kinematic quantities are also the tiny Energy volume caves (**cycloid is length , λ , the Space of velocity \vec{v} , and \vec{a} consist in gravity's field the infinite Energy dipole Tanks in where energy is conserved**) . In this way all operations on edge points are possible and applicable .
Remarks :

In F7-(1) ,The Linear Ratio , for Vectors , begins from the same Common point Ko , of the two concurring and Non-equal , Concentrical and Co-parallel Direction monads KoX –KoA and becomes KoX1- KoD .

In F7-(2) ,The Linear Ratio , for Plane , begins from the same Common point Ko , of the two Non-equal , Concentrical and Co-perpendicular Direction monads.

Proof :

Segment KoA \square KoX because triangle AKoX is rightangled triangle and KoZ \square AX . Radius OKo = OA = OX . Since DA , X1X are also perpendicular to AX , therefore KoZ //X1X//DA. According to Thales theorem ratio (ZA/ZX) = (KoD/KoX1) and since tangent DA = DKo and X1Ko =X1X then AZ / ZX = DA/ XX1 . From Pythagorean theorem (Lemma 6) \rightarrow KoA² / KoX² = (AZ/ZX) = (DA/XX1) = (KoD/KoX1) i.e. The ratio of the two squares KoA² , KoX² are proportional to line segments KoD,KoX1 (o.e.δ).

In F7-(3) ,The Linear Ratio ,for Volume , begins from the same Common point Ko , of the two Non-equal , Concentrical and Co-perpendicular Direction monads.

In (1) \rightarrow Segment KoA \square KoD , Ratio KoX / KoA = KoX1 / KoD , and Linearly (in one dimension) the Ratio of KoA / KoX = AD / XX1 i.e. in Thales linear mould [XX1 // AD] , **Linear Ratio of Segments XX1 , AD is , constant and Linear , and it is the Master key Analogy of the two Segments , monads .**

In (2) \rightarrow Segment KoA \square KoX , OKo =OA= OX and since OX1 , OD are diameters of the two circles then KoD = AD , KoX1 = XX1 , and Linearly (in one dimension) the Ratio of KoA / KoX = AD / XX1 , in Plane (in two dimensions) the Ratio [KoA]² / [KoX]² = AD / XX1 ,

i.e. in Euclid's Plane mould [KoA \square KoX] , **The Plane Ratio square of Segments – KoA , KoX - is constant and Linear , and for any Segment KoX on circle (O,OKo) exists KoA such that , \rightarrow KoA² / KoX² = AD / XX1 = KoD / KoX1 \leftarrow i.e. the Square Analogy of the sides in any rectangle triangle AKoX is linear to Extrema Semi-segments AD , XX1 or to KoD , KoX1 monads , i.e.**

the mapping of the continuous analog segment KoX to the discrete segment KoA. In (3) \rightarrow Segment KoB \square KoX , OKo = OB = OZ and since XX1 // AD , then KoA / KoD = KoX / KoX1 = AD / XX1 , and Linearly (in one dimension) the Ratio of KoA / KoX = AD / XX1 and in Space (Volume) (in three dimensions) the Ratio [KoA]³ / [KoD]³ = [KoX / KoX1]³ = 1/2 . i.e. in Euclid's Plane mould [KoA // KoX , KoD // KoX1] , **Volume Ratio of volume**

Segments – KoA , KoD - , is constant and Linear , and for any Segment KoX exists KoX1 such that $\rightarrow KoX1^3 / KoX^3 = 2 \leftarrow$

i.e. the Duplication of the cube.

In F-7 , The *three* dimensional Space [KoA \square KoD \square Ko...], where $XX1 \parallel AD$, The *two* dimensional Space [KoA \square KoX] , where $XX1 \parallel AD$, The *one* dimensional Space [$XX1 \parallel AD$] , where $XX1 \parallel AD$, is constant and Linearly Quantized in each dimension.

i.e. All dimensions of Monads coexist linearly in *Segments – monads* separately (*they are the units of the three dimensional axis x,y,z - i , j , k -*) and consequently in Volumes , Planes , Lines , Segments , and Points of Euclidean geometry, which are all the one point only and which is nothing. For more in [49-50] . 25/9/2015 At the beginning of the article it was referred to Geometers scarcity from which instigated to republish this article and to locate the weakness of proving these Axioms which created the Non -Euclid geometries and which deviated GR in Space-time confinement .Now is more referred ,

- a). There is not any Paradoxes of the infinite because is clearly defined what is a Point and what is a Segment .
- b). **The Algebra of constructible numbers and number Fiels is an Absurd theory** based on groundless Axioms as the fields are , and with directed non-Euclid orientations which must be properly revised .
- c). **The Algebra of Transcental numbers has been devised to postpone the Pure geometrical thought** , which is the base of all sciences , by changing the base - field of the geometrical solutions to Algebra as base . The Pythagorians discovered the existence of the incommensurable of the diagonal of a square in relation to its side without giving up the base of it , which is the geometrical logic.
- d). All theories concerning **the Unsolvability of the Special Greek problems are based on Cantor’s shady proof** , < that the totality of All algebraic numbers is denumerable > and not edified on the geometrical basic logic which is the foundations of all Algebra .

The problem of Doubling the cube F.3 , as that of the Trisection of any angle , is a Mechanical problem and could not be seen differently and the proposed Geometrical solutions is clearly exposed to the critic of all readers .

All trials for Squaring the circle are shown in [46] and the set questions will be answered on the Changeable System of the two Expanding squares ,*Translation* [T] and *Rotation* [R] . The solution of Squaring the circle using the Plane Procedure method is now presented in F.1,2 , and consists an , *Overthrow* , to all existing theories in Geometry , Physics and Philosophy .

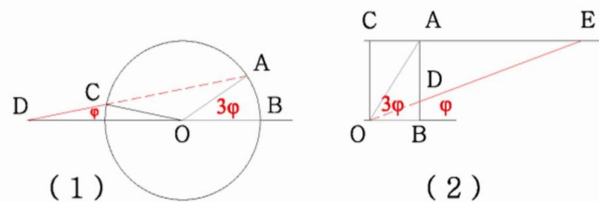
e). Geometry is the base of all sciences and it is the reflective logic from the objective reality and which is nature .

3.. The Trisection of Any Angle .

Because of the three *master-meters* , where is holding the Ratio of two or three geometrical magnitudes , is such that they have a linear relation (*a continuous analogy*) in all Spaces , the solution of this problem , as well as of those before , is linearly transformed .

The present method is Plane method , *i.e. straight lines and circles* , as the others and is not required the use of conics or some other equivalent .

Archimedes and Pappus proposal was instinctly right .



F.8. \rightarrow (1) Archimedes , (2) Pappus Method

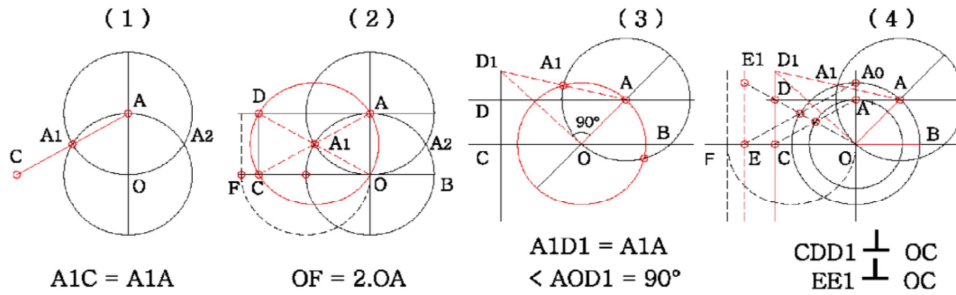
The Present method :

It is based on the Extrema geometrical analysis of the mechanical motion of shapes related to a system of poles of rotation .The classical solutions by means of conics , or reduction to a , $\nu\epsilon\upsilon\sigma\iota\varsigma$, is a part of Extrema method . This method changes the Idle between the edge cases and *Rotates* it through constant points , *The Poles* , [11] .

The basic triangle AOD1 is such that angle OD1A=30° and rotating through pole O . The three edge positions ,

- a). Angle AOB = 90° when $OC \equiv -OB$ and then point D1 is on OB axis ,
- b). Angle AOB = 0 - 90° when $OC \equiv OD1$ and then point D1 is perpendicular to OC axis ,
- c). Angle AOB = 0 when $OC \equiv O$ and then point D1 is perpendicular to OB axis.

This moving geometrical mechanism acquires common circles and constant common poles of rotation which are defined with initial ones . This geometrical motion happens between the Extrema cases referred above . The steps of the basic Rotating Triangle AOD1 between the extrema cases $AOB=180^\circ$, $AOB=0$.



F.9. → The proposed Contemporary Trisection method.

We extend Archimedes method as follows : a. F.9.-(2). Given an angle $\angle AOB = \angle AOC = 90^\circ$

- 1.. Draw circle (A , $AO = OA$) with its center at the vertex A intersecting circle (O , $OA = AO$) at the points A1 , A2 respectively .
- 2.. Produce line AA1 at C so that $A1C = A1A = AO$ and draw $AD \parallel OB$.
- 3.. Draw CD perpendicular to AD and complete rectangle AOCD .
- 4.. Point F is such that $OF = 2 . OA$ b. F.9.(3-4) . Given an angle $\angle AOB < 90^\circ$

- 1.. Draw AD parallel to OB .
- 2.. Draw circle (A , $AO = OA$) with its center] at the vertex A intersecting circle (O , $OA = AO$) at the points A1 , A2 .
- 3.. Produce line AA1 at D1 so that $A1D1 = A1A = OA$.
- 4.. Point F is such that $OF = 2 . OA = 2.OAo$
- 5.. Draw CD perpendicular to AD and complete rectangle A'OCD .
- 6.. Draw AoE Parallel to A'C at point E (or sliding E on OC) .
- 7.. Draw AoE' parallel to OB and complete rectangle AoOEE1 .
- 8.. In F10 - (1-2-3) , Draw AF intersecting circle (O , OA) at point F1 and insert on AF segment F1F2 equal to $OA \rightarrow F1F2 = OA$. 9.. Draw AE intersecting circle (O , OA) at point E1 and insert on AE segment E1E2 equal to $OA \rightarrow E1E2 = OA = F1F2$.

To show that :

- a). For all angles equal to 90° Points C and E are at a constant distance $OC = OA . \sqrt{3}$ and $OE = OAo . \sqrt{3}$, from vertices O , and also $A'C \parallel AoE$.
- b). The geometrical locus of points C , E is the perpendicular CD , EE1 line on OB .
- c). All equal circles with their center at the vertices O , A and radius $OA = AO$ have the same geometrical locus $EE1 \perp OE$ for all points A on AD , or All radius of equal circles drawn at the points of intersection with its Centers at the vertices O , A and radius $OA = AO$ lie on CD , EE1 .
- d). Angle $\angle D1OA$ is always equal to 90° and angle AOB is created by rotation of the rightangled triangle AOD1 through vertex O .
- e). Angle $\angle AOB$ is created in two ways , by constructing circle (O , $OA = OAo$) and by sliding , of point A1 on line A1D Parallel to OB from point A1 , to A .
- f). Angle $\angle AOB$ is created in two ways , by constructing circle (O , $OA = OAo$) and by sliding , of point A' on line A'D Parallel to OB from point A' , to A .
- g). The rotation of lines AE , AF (minimum and maximum edge positions) on circle (O , $OA = OAo$) from point E to point F which lines intersect circle (O , OA) at the points E1 , F1 respectively , fixes a point G on line EF and a point G1 common to line AG and to the circle (O , OA)

such that $G G1 = OA$.

Proof :

a) .. F.9 .(1 - 2)

Let OA be one-dimensional Unit perpendicular to OB such that angle $\angle AOB = \angle AOC = 90^\circ$
 Draw the equal circles (O , OA) , (A , AO) and let points A1 , A2 be the points of intersection . Produce AA1 to C . Since triangle AOAI has all sides equal to OA ($AA1 = AO = OA1$) then it is an equilateral triangle and angle $\angle A1AO = 60^\circ$ Since Angle $\angle CAO = 60^\circ$ and $AC = 2 . OA$ then triangle ACO is right-angled and angle $\angle AOC = 90^\circ$, and so the angle $\angle ACO = 30^\circ$.
 Complete rectangle AOCD , and angle $\angle ADO = 180 - 90 - 60 = 30^\circ = \angle ACO = 90^\circ / 3 = 30^\circ$ From Pythagoras theorem $AC^2 = AO^2 + OC^2$ or $OC^2 = 4.OA^2 - OA^2 = 3 . OA^2$ and

$$OC = OA \cdot \sqrt{3}.$$

For $OA = OA_0$ then $AOE = 2 \cdot OA_0$ and $OE = OA_0 \cdot \sqrt{3}$.

Since $OC/OE = OA/OA_0 \rightarrow$ then line CA' is parallel to EA_0 .

b) .. F.9.(3 - 4)

Triangle OAA_1 is isosceles, therefore angle $\angle A_1AO = 60^\circ$. Since $A_1D_1 = A_1O$, triangle D_1A_1O is isosceles and since angle $\angle OA_1A = 60^\circ$, therefore angle $\angle OD_1A = 30^\circ$ or, Since $A_1A = A_1D_1$ and angle $\angle A_1AO = 60^\circ$ then triangle AOD_1 is also right-angle triangle and angles $\angle D_1OA = 90^\circ$, $\angle OD_1A = 30^\circ$. Since the circle of diameter D_1A passes through point O and also through the foot of the perpendicular from point D_1 to AD , and since also $\angle ODA = \angle ODA' = 30^\circ$, then this foot point coincides with point D , therefore the locus of point C is the perpendicular CD_1 on OC . For $AA_1 > A_1D_1$, then D_1 is on the perpendicular D_1E on OC .

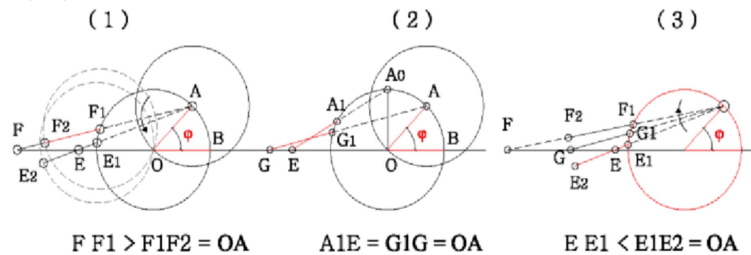
Since the Parallel from point A_1 to OA passes through the middle of OD_1 , and in case where $AOB = AOC = 90^\circ$ through the middle of AD , then the circle with diameter D_1A passes through point D which is the base point of the perpendicular, i.e.

The geometrical locus of points C , or E , is the perpendicular CD , EE_1 on OB . c) .. F.9.(3 - 4)

Since $A_1A = A_1D_1$ and angle $\angle A_1AO = 60^\circ$ then triangle AOD_1 is a right-angle triangle and angle $\angle D_1OA = 90^\circ$.

Since angle $\angle A-D_1-O$ is always equal to 30° and angle D_1-O-A is always equal to 90° , therefore angle $\angle AOB$ is created by the rotation of the right-angled triangle $A-O-D_1$ through vertex O .

Since tangent through A_0 to circle (O, OA') lies on the circle of half radius OA then this is perpendicular to OA and equal to $A'A$. (F.8)



F.10. \rightarrow *The three cases of the Sliding segment $OA = F_1F_2 = E_1E_2$ between a line OB and a circle (O, OA) between the Maxima - edge cases F_1F_2, E_1E_2 or F, E points. F-9 On AF, AE lines exists :*

$FF_1 > OA$ $GG_1 = OA$, $A_1E = OA_0$ $EE_1 < OA$ $F_2F_1 = OA$ $A_1E = OA_0$, $EA_1 = OA$ $E_1E_2 = OA$

d) .. F.9(4) - (F.10)

Let point G be sliding on OB between points E and F where lines AE, AG, AF intersect circle (O, OA) at the points E_1, G_1, F_1 respectively where then exists $FF_1 > OA$, $GG_1 = OA$, $EE_1 < OA$.

Points E, F are the limiting points of rotation of lines AE, AF (because then for angle $\angle AOB = 90^\circ \rightarrow A_1C = A_1A = OA, A_1A_0 = A_1E = OA_0$ and for angle $\angle AOB = 0^\circ \rightarrow OF = 2 \cdot OA$). Exists also $E_1E_2 = OA, F_1F_2 = OA$ and point G_1 common to circle (O, OA) and on line AG such that $GG_1 = OA$. AE Oscillating to AF passes through AG so that $GG_1 = OA$ and point G on sector EF . When point G_1 of line AG is moving

(rotated) on circle $(E_2, E_2E_1 = OA)$ and Point G_1 of G_1G is stretched on circle (O, OA) then $G_1G \neq OA$. A position of point G_1 is such that, when $GG_1 = OA$ point G lies on line EF .

When point G_1 of line AG is moving (rotated) on circle $(F_2, F_2F_1 = OA)$ and point G_1 of G_1G is stretched on circle (O, OA) then length $G_1G \neq OA$.

A position of point G_1 is such that, when $GG_1 = OA$ point G lies on line EF without stretching.

For both opposite motions there is only one position where point G lies on line OB and is not needed point G_1 of GA to be stretched on circle (O, OA) .

This position happens at the common point P , of the two circles which is their point of intersection. At this point P exists only rotation and is not needed G_1 of GA to be stretched on circle (O, OA) so that point G to lie on line EF .

This means that point P lies on the circle (G, GG1 = OA), or GP = OA. Point A of angle < BOA is verged through two different and opposite motions, i.e.

1.. From point A' to point Ao where is done a parallel translation of CA' to the new position EAo, this is for all angles equal to 90°, and from this position to the new position EA by rotating EAo to the new position EA having always the distance E1E2 = OA. This motion is taking place on a circle of center E1 and radius E1E2.

2.. From point F, where OF = 2.OA, is done a parallel translation of A'F to FAo, and from this position to the new position FA by rotating FAo to FA having always the distance F1F2 = OA.

The two motions coexist again on a point P which is the point of intersection of the circles (E2, E2E1 = OA) and (F2, F2F1 = OA). f) .. (F.9 .3 - 4) - (F.10 -3) Remarks – Conclusions :

1.. Point E1 is common of line AE and circle (O, OA) and point E2 is on line AE such that E1E2 = OA and exists E1E1 < E2E1. Length E1E2 = OA is stretched, moves on EA so that point E2 is on EF. Circle (E, E1E1 < E2E1 = OA) cuts circle (E2, E2E1 = OA) at point E1. There is a point G1 on circle (O, OA) such that G1G = OA, where point G is on EF, and is not needed G1G to be stretched on GA where then, circle (G, GG1 = OA) cuts circle (E2, E2E1 = OA) at a point P.

2.. Point F1 is common of line AF and circle (O, OA) and point F2 is on line AF such that F1F2 = OA and exists F1F1 > F2F1.

Segment F1F2 = OA is stretched, moves on FA so that point F2 is on FE. Circle (F, F1F1 > F2F1 = OA) cuts circle (F2, F2F1 = OA) at point F1. There is a point G1 on circle (O, OA) such that G1G = OA, where point G is on FE, and is not needed G1G to be stretched on OB where then circle (G, GG1 = OA) cuts circle (F2, F2F1 = OA) at a point P.

3.. **When point G is at such position on EF that GG1 = OA, then point G must be at A COMMON, to the three lines EE1, GG1, FF1, and also to the three circles (E2, E2E1 = OA), (G, GG1 = OA), (F2, F2F1 = OA)** This is possible at the common point P of Intersection of circle (E2, E2E1 = OA) and (F2, F2F1 = OA) and since GG1 is equal to OA without G1G be stretched on GA, then also GP = OA.

4.. In additional, for point G1 :

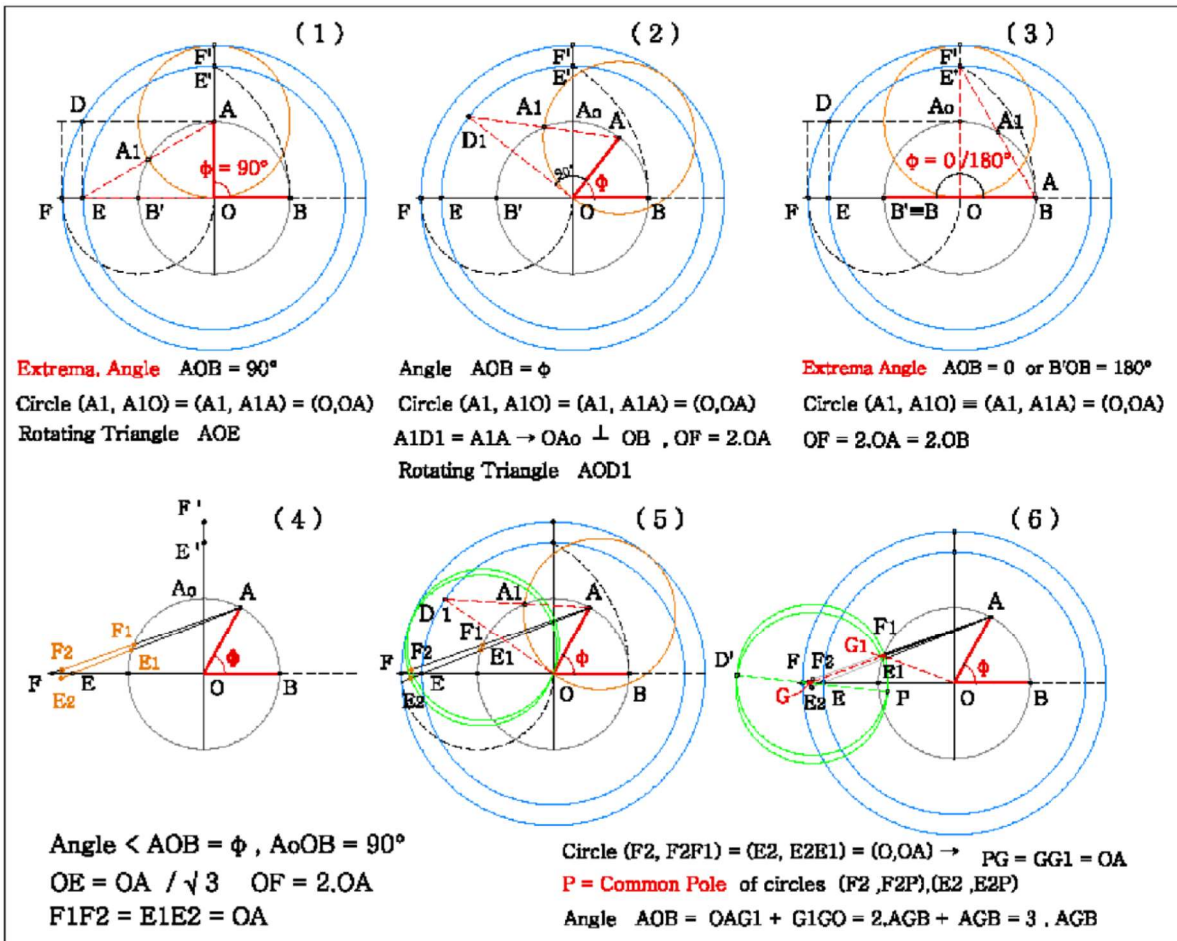
a.. Point G1, from point E1, moving on circle (E2, E2E1 = OA) formulates AE1E such that E1E = G1G < OA, for G moving on line GA. There is a point on circle (E2, E2E1 = OA) such that GG1 = OA.

b.. Point G1, from point F1, moving on circle (F2, F2F1 = OA) formulates AF1F such that F1F = GG1 > OA, for G moving on line GA. There is a point on circle (F2, F2F1 = OA) such that GG1 = OA.

c.. Since for both Opposite motions there is a point on the two circles that makes GG1 = OA then point say P, is common to the two circles. d.. Since for both motions at point P exists GG1 = OA then circle (G, GG1 = OA) passes through point P, and since point P is common to the three circles, then fixing point P as the common to the two circles (E2, E2E1 = OA), (F2, F2F1 = OA), then point G is found as the point of intersection of circle (P, PG = OA) and line EF. This means that the common point P of the three circles is constant to point P of the three circles and is constant to this motion.

e.. Since, happens also the motion of a constant Segment on a line and a circle, then it is Extrema Method of the moving Segment as stated. The method may be used for part or Blocked figures either sliding or rotating. In our case, the Initial triangle forming 1/3 angle is formulating in all cases the common pole, P, of the three circles.

From all above the geometrical trisection of any angle is as follows, Fig.11.



F. 11 → The extrema Geometrical method of the Trisection of any angle $\angle AOB$

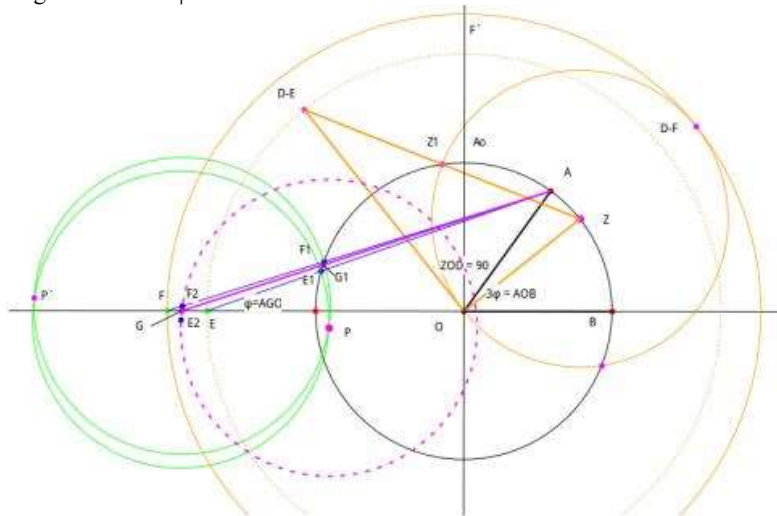
In F.11- (1) Basic triangle $\triangle AOD1$ defines E point such that angle $\angle AEO = 30^\circ = \angle AOB/3$.

In F.11- (2) Basic triangle $\triangle AOD1$ defines $D1$ point such that angle $\angle AD1O = 30^\circ = \angle AOB/3$.

In F.11- (3) Basic triangle $\triangle AOD1$ defines E' point such that angle $\angle AE'O = 30^\circ$, and it is the Extrema Case for angles $\angle AOB = 0^\circ, \angle B'OB = 180^\circ$.

In F.11- (4) The two Edge cases (1), (3) issue for any angle $\angle AOB = \phi^\circ$.

In F.11- (5) The two circles corresponding to Edge cases (1), (3) issuing for any angle $\angle AOB = \phi^\circ$. In F.11- (6) The three circles corresponding to Edge cases (1), (3) defining the common pole ,P, of the rotational system, which issues for any angle $\angle AOB = \phi^\circ$.



In F.11- A From Initial position of triangle AOC , where angle AOB = 90° and Segment A1C = OA , to the Final position of triangle , where angle AOB = BOB = 0° and AOB = B'OB = 180° , through the Extrema position between edge- cases of triangle ZOD where AOB = φ° and GG1 = GP = G1O .

f. The steps of Trisection of any angle $\angle AOB = 90^\circ \rightarrow 0^\circ$ F.11-[1-6]

- 1.. Draw circles (O , OA) , (A , AO) , intersected at A1 point .
- 2.. Draw OAo ⊥ OB where point Ao is on the circle (O , OA) and circle (Ao , AoE = 2OA) which intersects line OB at the point E .
3. Fix point F on line OB such that $OF = 2 \cdot OA$
- 4.. Draw lines AF , AE intersecting circle (O,OA) at points F1 , E1 respectively .
- 5.. On lines F1A , E1A fix points F2 , E2 such that F1F2 = OA and E1E2 = OA .
- 6.. Draw circles (F2 , F2F1 = OA) , (E2 , E2E1 = OA) and fix point P as their common point of intersection
- 7.. Draw circle (P , PG = OA) intersecting line OB at point G and draw line GA intersecting circle (O , OA) at point G1 .

Then Segment GG1 = OA , and angle $\angle AOB = 3 \cdot \angle AGB$.

Proof :

- 1.. Since point P is common to circles (F2 , F2F1 = OA) , (E2 , E2E1 = OA) , then $PG = PF2 = PE2 = OA$ and line AG between AE , AF intersects circle (O,OA) at the point G1 such that GG1 = OA . (F10.1 -2) - (F.11-5)
- 2.. Since point G1 is on the circle (O , OA) and since GG1 = OA then triangle GG1O is isosceles and angle $\angle AGO = \angle G1OG$. 3.. The external angle of triangle GG1O is $\angle AG1O = \angle AGO + \angle G1OG = 2 \cdot \angle AGO$.
- 4.. The external angle of triangle GOA is angle $\angle AOB = \angle AGO + \angle OAG = 3 \cdot \angle AGO$.

Therefore angle $\angle AGB = (1/3) \cdot (\angle AOB)$ (o.ε.δ.) Analysis :

Since angle $\angle D1OA$ is always equal to 90° then angle AOB is created by rotation of the right-angled triangle AOD1 through vertex O . The circle (A , AO = A1O) and triangle AOD1 consists the geometrical Mechanism which creates the maxima at positions from , AOE , to AoOE and to BOF' triangles , on (O , OE = √3.OA) , (O , OF = 2.OA) circles.

In (1) Angle AOB = 90° , AE = 2.OA = OF , and point A1 common to circles (O , OA) , (A , AO) define point E on OB line such that A1E = OA . This happens for the extrema angle AOB = 90° .

In (2) Angle is , $0 < \angle AOB < 90^\circ$, AE = 2.OA and point A1 common to circles (O,OA) , (A,AO) defines point D1 on (O,OE = √3.OA) circle such that A1D1 = OA and on (O , OF = 2.OA) circle at point Df .

In (3) Angle $\angle AOB = 0$ or $\angle B'OB = 180^\circ$, AE =

2.OA = BB' and point A1 common to (O , OA) , (A , AO) circles define point E on OAo line such that $E \equiv E'$, where then point $D \equiv F'$. This happens for the extrema angle AOB = 0 or 90° .

In (4-5) where angle is , $0 < \angle AOB < 90^\circ$, and

Segments F1F2 = E1E2 = OA the equal circles (F2,F2F1) , (E2,E2E1) define the common point P .

Since this geometrical formulation exists on Extrema edge angles , 0 and 90° , then this point is constant to this formulation , and this point as centre of a radius OA circle defines the extrema geometrical locus on it.

In (6) Since angle AOB is , $0 \rightarrow 90^\circ$, and point

P is constant , and this because extrema circle (P,PG = OA) where G on OB line , then is defining (G,GG1) circle on GA segment such that point G1 , to be the common point of segment AG and circles (O , OA) , (G,GG1) .

4.. The Parallel Postulate , Axiom , is a Theorem .

The Parallel Postulate. F.12

General : Axiom or Postulate is a statement checked if it is true and is ascertained with logic (the experiences of nature as objective reality).

Theorem or Proposition is a non-main statement requiring a proof based on earlier determined logical properties.

Definition is an initial notion without any sensible definition given to other notions.

Definitions, Propositions or Postulates created Euclid geometry using the geometrical logic which is that of nature, the logic of the objective reality.

Using the same elements it is possible to create many other geometries but the true uniting element is the before referred.

4.1. The First Definitions (D) , of Terms in Geometry but the true uniting D1: A point is that which has no part (Position).

D2: A line is a breathless length (for straight line, the whole is equal to the parts) . D3: The extremities of lines are points (equation).

D4: A straight line lies equally with respect to the points on itself (identity).

D: A midpoint C divides a segment AB (of a straight line) in two. $CA = CB$ any point C divides all straight lines through this in two.

D: A straight line AB divides all planes through this in two.

D: A plane ABC divides all spaces through this in two .

4.2. Common Notions (Cn) Cn1: Things which equal the same thing also equal one another.

Cn2: If equals are added to equals, then the wholes are equal.

Cn3: If equals are subtracted from equals, then the remainders are equal.

Cn4: Things which coincide with one another, equal one another.

Cn5: The whole is greater than the part.

4.3. The Five Postulates (P) for Construction P1. To draw a straight line from any point A to any other point B .

P2. To produce a finite straight line AB continuously in a straight line.

P3. To describe a circle with any centre and distance. P1, P2 are unique.

P4. That, all right angles are equal to each other.

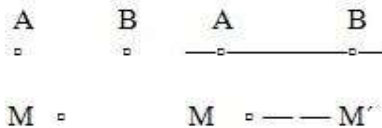
P5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, if produced indefinitely, meet on that side on which are the angles less than the two right angles, or (for three points on a plane) . Three points consist a Plane .

P5a. The same is plane's postulate which states that, from any point M, not on a straight line AB, only one line MM' can be drawn parallel to AB.

Since a straight line passes through two points only and because point M is the third then the parallel postulate it is valid on a plane (three points only).

AB is a straight line through points A, B , AB is also the measurable line segment of line AB , and M any other point . When $MA+MB > AB$ then point M is not on line AB . (differently if $MA+MB = AB$, then this answers the question of why any line contains at least two points) , i.e. for any point M on line AB where is holding

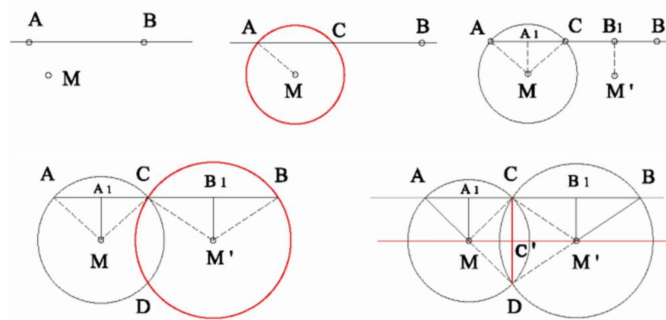
$MA+MB = AB$, meaning that line segments MA,MB coincide on AB , is thus proved from the other axioms and so D2 is not an axiom . \rightarrow To prove that , one and only one line MM' can be drawn parallel to AB.



F.12. \rightarrow The three points (in a Plane).

In F.13 , in order to prove the above Axiom is necessary to show :

- a..The parallel to AB is the locus of all points at a constant distance h from the line AB, and for point M is MA_1 ,
- b..The locus of all these points is a straight line.



F.13. \rightarrow The Parallel Method Step 1

Draw the circle (M, MA) be joined meeting line AB in C. Since $MA = MC$, point M is on midperpendicular of AC. Let A_1 be the midpoint of AC, (it is $A_1A+A_1C = AC$ because A_1 is on the straight line AC). Triangles MAA_1 , MCA_1 are equal because the three sides are equal, therefore angle $\angle MA_1A = \angle MA_1C$ (CN1) and since the sum of the two angles $\angle MA_1A + \angle MA_1C = 180^\circ$ (CN2, 6D) then angle $\angle MA_1A = \angle MA_1C = 90^\circ$.(P4) so, MA_1 is the minimum fixed distance h of point M to AC.

Step 2

Let B_1 be the midpoint of CB,(it is $B_1C+B_1B = CB$ because B_1 is on the straight line CB) and draw $B_1M' = h$ equal to A_1M on the midperpendicular from point B_1 to CB. Draw the circle (M' , $M'B = M'C$) intersecting the circle (M, $MA = MC$) at point D.(P3) Since $M'C = M'B$, point M' lies on mid-perpendicular of CB. (CN1)

Since $M'C = M'D$, point M' lies on midperpendicular of CD . (CN1) Since $MC = MD$, point M lies on midperpendicular of CD . (CN1) Because points M and M' lie on the same midperpendicular (This mid-perpendicular is drawn from point C' to CD and it is the midpoint of CD) and because only one line MM' passes through points M, M' then line MM' coincides with this mid-perpendicular (CN4)

Step 3

a. Because $MA1 \perp AC$ and also $MC' \perp CD$ then
 $\angle < A1MC' = A1CC'$. (Cn 2, Cn3, E.I.15)
 Because $M'B1 \perp CB$ and also $M'C' \perp CD$ then
 $\angle < B1M'C' = B1CC'$. (Cn2, Cn3, E.I.15)

b. The sum of angles $A1CC' + B1CC' = 180^\circ = A1MC' + B1M'C'$. (6.D), and since Point C' lies on straight line MM' , therefore the sum of angles in shape $A1B1M'M$ are $\angle MA1B1 + \angle A1B1M' + [\angle B1M'M + \angle M'MA1] = 90^\circ + 90^\circ + 180^\circ = 360^\circ$ (Cn2), i.e. The sum of angles in a Quadrilateral is 360° and in Rectangle all equal to 90° . (m)

c. The right-angled triangles $MA1B1, M'B1A1$ are equal because $A1M = B1M'$ and $A1B1$ common, therefore side $A1M' = B1M$ (Cn1). Triangles $A1MM', B1M'M$ are equal because have the three sides equal each other, therefore angle $\angle A1MM' = \angle B1M'M$, and since their sum is 180° as before (6D), so angle $\angle A1MM' = \angle B1M'M = 90^\circ$ (Cn2).

d. Since angle $\angle A1MM' = \angle A1CC'$ and also angle $\angle B1M'M = \angle B1CC'$ (P4), therefore the three quadrilaterals $A1CC'M, B1CC'M', A1B1M'M$ are Rectangles (CN3).

From the above three rectangles and because all points (M, M' and C') equidistant from AB , this means that $C'C$ is also the minimum equal distance of point C' to line AB or, $h = MA1 = M'B1 = CD / 2 = C'C$ (Cn1) Namely, line MM' is perpendicular to segment CD at point C' and this line coincides with the mid-perpendicular of CD at this point C' and points M, M', C' are on line MM' . Point C' equidistant, h , from line AB , as it is for points M, M' , so the locus of the three points is the straight line MM' , so the two demands are satisfied, ($h = C'C = MA1 = M'B1$ and also $C'C \perp AB, MA1 \perp AB, M'B1 \perp AB$). (o.e.d.) –(q.e.d) e. The right-angle triangles $A1CM, MCC'$ are equal because side $MA1 = C'C$ and MC common so angle $\angle A1CM = \angle C'MC$, and the Sum of angles $\angle C'MC + \angle MCB1 = \angle A1CM + \angle MCB1 = 180^\circ$

4.4 The Succession of Proofs

- 1.. Draw the circle (M, MA) be joined meeting line AB in C and let $A1, B1$ be the midpoint of CA, CB .
- 2.. On mid-perpendicular $B1M'$ find point M' such that $M'B1 = MA1$ and draw the circle ($M', M'B = M'C$) intersecting the circle ($M, MA = MC$) at point D .
- 3.. Draw mid-perpendicular of CD at point C' .
- 4.. To show that line MM' is a straight line passing through point C' and it is such that $MA1 = M'B1 = C'C = h$, i.e. a constant distance h from line AB or, also The Sum of angles $\angle C'MC + \angle MCB1 = \angle A1CM + \angle MCB1 = 180^\circ$

4.5. Proofed Succession

- 1.. The mid-perpendicular of CD passes through points M, M' .
- 2.. Angle $\angle A1MC' = \angle A1MM' = \angle A1CC'$, Angle $\angle B1M'C' = \angle B1M'M = \angle B1CC' < \angle A1MC' = \angle A1CC'$ because their sides are perpendicular among them i.e.

$$MA1 \perp CA, MC' \perp CC'$$

a.. In case $\angle A1MM' + \angle A1CC' = 180^\circ$ and $\angle B1M'M + \angle B1CC' = 180^\circ$ then $\angle A1MM' = 180^\circ - \angle A1CC'$, $\angle B1M'M = 180^\circ - \angle B1CC'$, and by summation $\angle A1MM' + \angle B1M'M = 360^\circ - \angle A1CC' - \angle B1CC'$ or Sum of angles $\angle A1MM' + \angle B1M'M = 360^\circ - (\angle A1CC' + \angle B1CC') = 360^\circ - 180^\circ = 180^\circ$

3.. The sum of angles $\angle A1MM' + \angle B1M'M = 180^\circ$ because the equal sum of angles $\angle A1CC' + \angle B1CC' = 180^\circ$, so the sum of angles in quadrilateral $MA1B1M'$ is equal to 360° .

4.. The right-angled triangles $MA1B1, M'B1A1$ are equal, so diagonal $MB1 = M'A1$ and since triangles $A1MM', B1M'M$ are equal, then angle $\angle A1MM' = \angle B1M'M$ and since their sum is 180° , therefore angle $\angle A1MM' = \angle B1M'M = 90^\circ$

5.. Since angle $\angle A1CC' = \angle B1CC' = 90^\circ$, then quadrilaterals $A1CC'M, B1CC'M'$ are rectangles and for the three rectangles $MA1CC', CB1M'C', MA1B1M'$ exists $MA1 = M'B1 = C'C$

6.. The right-angled triangles $MCA1, MCC'$ are equal, so angle $\angle A1CM = \angle C'MC$ and since the sum of angles $\angle A1CM + \angle MCB1 = 180^\circ$ then also $\angle C'MC + \angle MCB1 = 180^\circ \rightarrow$ which is the second to show, as this problem has been set at first by Euclid.

All above is a Proof of the Parallel postulate due to the fact that the parallel postulate is dependent of the other four axioms (*now is proved as a theorem from the other four*).

Since line segment AB is common to ∞ Planes and only one Plane is passing through point M (Plane ABM from the three points A, B, M), then the Parallel Postulate is valid for all Spaces which have this common Plane, as Spherical, n -dimensional geometry Spaces. It was proved that it is a necessary logical consequence of the others axioms, agree also

with the Properties of physical objects, $d + 0 = d$, $d * 0 = 0$, now is possible to decide through mathematical reasoning, that the geometry of the physical universe is Euclidean. Since the essential difference between Euclidean geometry and the two non-Euclidean geometries, Spherical and hyperbolic geometry, is the nature of parallel line, i.e. the parallel postulate so, << **The consistent System of the – Non - Euclidean geometry - have to decide the direction of the existing mathematical logic** >>.

The above consistency proof is applicable to any line Segment AB on line AB, (segment AB is the first dimensional unit, as $AB = 0 \rightarrow \infty$), from any point M not on line AB, [$MA + MB > AB$ for three points only which consist the Plane. For any point M between points A, B is holding $MA + MB = AB$ i.e. from two points M, A or M, B passes the only one line AB. A line is also continuous (P1) with points and discontinuous with segment AB [14], which is the metric defined by non- Euclidean geometries, and it is the answer to the cry about the < crisis in the foundations of Euclid geometry >

4.1. **A Line Contains at Least Two Points, is Not an Axiom Because is Proved as Theorem**

Definition D2 states that for any point M on line AB is holding $MA + MB = AB$ which is equal to < segment MA + segment MB is equal to segment AB > i.e. the two lines MA, MB coincide on line AB and thus this postulate is proved also from the other axioms, thus D2 is not an axiom, which form a system self-consistent with its intrinsic realworld meaning. F.12-13.

5. Conclusions.

1. **Quadrature.**

The exact Numeric Magnitude of number π , may be found only by numeric calculations. All magnitudes exist on the < **Plane Formation Mechanism of the first dimensional unit AB** > as geometrical elements consisting, **the Steady**

Formulation, (The Plane System of the Isosceles Right-angle triangle ACP with the three Circles on the sides) and **the moving and Changeable Formulation of the twin, System-Image**, (This Plane Perpendicular System of Squares, Antisquares is such that, *the Work produced in a closed area to be equal to zero*).

Starting from this logic of correlation upon Unit, we can control *Resemblance Ratio* and construct all Regular Polygons on the unit Circle as this is shown in the case of squares. On this **System** of these three circles (The Plane Procedure Mechanism which is a Constant System) is created also, a *continues* and, a *not continues* Symmetrical Formation, the changeable System of the Regular Polygons, and the **Image** (Changeable System of Regular anti-Polygons) **Idol**, as much this in **Space** and also in **Time**, and was proved that in this Constant System, *the Rectilinear motion of the Changeable Formation is Transformed into a twin Symmetrically axial - centrifugal Pole rotation (this is the motion on System)*.

The conservation of the Total Impulse and Momentum, as well as the conservation of the Total Energy in this Constant System with all properties included, exists in this Empty Space of the un-dimensional point Units of mechanism. All the forgoing referred can be shown (maybe presented) with a Ruler and a Compass, or can be seen, live, on any Personal Computer. The method is presented on Dr.Geo machine.

The theorem of *Hermit-Lindeman* that number π , is not algebraic, is based on the theory of Constructible numbers and number fields (*on number analysis*) and not on the < *Euclidean Geometrical origin-Logic on unit elements basis* > The mathematical reasoning (*the Method*) is based on the restrictions imposed to seek the solution < i.e. *with a ruler and a compass* >. By extending Euclid logic of Units on the Unit circle *to unknown and now proved Geometrical unit elements*, thus the settled age-old question for the unsolved problems is now approached and continuously standing solved. All Mathematical interpretation and the relative Philosophical reflections based on the theory of the non-solvability must properly revised.

2. **Duplication.**

This problem follows the three dimensional dialectic logic of ancient Greek, *Αναξίμανδρος*, [« τὸ μὴ ὄν, ὄν γίγνεται » The Non-existent Exists when is done, ‘The Non-existent becomes and never is’], where **the geometrical magnitudes**, have a linear relation (continuous analogy) in all Spaces as, in one in two in three dimensions, as this happens to the *Compatible Coordinate Systems*.

The Structure of Euclidean geometry is such [8] that it is a Compact Logic where **Non - Existent** is found everywhere, and **Existence, monads**, is found and is done everywhere. In Euclidean geometry points do not exist, but their position and correlation is doing geometry. The universe cannot be created, because it is continuously becoming and never is. According to Euclidean geometry, and since the position of points (*empty Space*) creates the geometry and Spaces, Zenon Paradox is the first concept of Quantization.

In terms of Mechanics, Spaces Mould happen through *Mould of Doubling the Cube*, where for any monad $ds = KoA$ and analogous to KoD , *the Volume or The cube of segment KoD is the double the volume of KoA cube, or monad*

$KoD^3 = 2.KoA^3$. This is one of the basic Geometrical Euclidean Geometry Moulds , which create the METERS of monads which \rightarrow **Linear** is the Segment MA1 , **Plane** is the square CMNH equal to the circle , and in **Space** is volume KoD^3 , in all Spaces , Anti-spaces and Sub -spaces of monads \leftarrow i.e The Expanding square BAoDoCo is Quantized to BADC Rectangle by Translation to point Z` , and by Rotation through point P , (the Pole of rotation) . The Constructing relation between any segments KoX , KoA is $\rightarrow (KoX)^3 = (KoA)^3$. (XX1 /AD) as in F.7

3.. Trisection.

This problem follows the two dimensional logic, where , **the geometrical magnitudes and their unique circle** , have a linear relation (continuous analogy) in all Spaces as , in one in two in three dimensions , and as this happens to Compatible Coordinate Systems , happens also in Circle-arcs . The Compact-Logic-Space-Layer exists in Units , (**The case of 90° angle**) , where then we may find a new machine that produces the 1/3 of angles as in F.11. Since angles can be produced from any monad OB , and this because monad can formulate a circle of radius OB , and any point A on circle can then formulate angle $\angle AOB$, therefore the logic of continuous analogy issues also and on OA radius equal to OB .

4.. Parallel line.

A line (two points only) is not a great circle (three not coinciding points) , so anything built on this logic is a mislead false .

The fact that the sum of angles on any triangle is 180° is springing for the first time, in article (Rational Figured numbers or Figures) [9] .

This admission of two or more than two parallel lines, instead of one of Euclid's, does not prove the truth of the admission. The same to Euclid's also, until the present proved method . Euclidean geometry does not distinguish , Space from time because time exists only in its deviation - Plank's length level -, neither Space from Energy - because Energy exists as quanta on any first dimensional Unit AB , which as above connects the only two fundamental elements of Universe , that of points or Sector = Segment = Monad = Quaternion , and that of Energy. [23]-[39].

The proposed Method in articles , based on the prior four axioms only , proofs , (not using any admission but a pure geometric logic under the restrictions imposed to seek the solution) that , through point M on any Plane ABM (three points only that are not coinciding and which consist the Plane) , passes only one line of which all points equidistant from AB as point M , i.e. the right is to Euclid Geometry.

The what is needed for conceiving the alterations from Points which are nothing , to segments ,

i.e. quantization of points as , *the discretizing = monads = quaternion* , to lines , plane and volume , is the acquiring and having Extrema knowledge .

In Euclidean geometry the inner transformations exist as **pure** Points , segments , lines , Planes , Volumes, etc. as the Absolute geometry is (*The Continuity of Points*) , automatically transformed through the three basic Moulds (*the three Master moulds and Linear transformations exist as one Quantization*) to Relative external transformations , which exist as the , **material** , Physical world of matter and energy (*Discrete of Monads*) . [43]

The new Perception connecting the Relativistic Time and Einstein's Energy , is Now Refining Time and Dark-matter Force , clearly proves That Big-Bang have Never been existed .

In [17-45-46] is shown the most important **Extrema Geometrical Mechanism in this Cosmos** , that of STPL lines , that produces and composite , All the opposite space Points from Spaces to Anti-Spaces and Sub -Spaces in a Common Circle , *it is the SubSpace* , to lines or to Cylinder .

This extrema mould is a Transformation , i.e. a Geometrical Quantization Mechanism , \rightarrow

for the Quantization of Euclidean geometry, *points* , to the Physical world , **to Physics** , and is based on the following geometrical logic , Since Primary point ,A, is nothing and without direction and it is the only Space , and this point to exist , *to be* , at any other point ,B, which is not coinciding with point ,A, then on this couple exists the Principle of

Virtual Displacements $W = \int_A^B P \cdot ds$

$= 0$ or $[ds \cdot (P_A + P_B) = 0]$, i.e. for any $ds > 0$ Impulse $P = (P_A + P_B) = 0$ and Work $[ds \cdot (P_A + P_B) = 0]$, Therefore , Each Unit $AB = ds > 0$, exists by this Inner Impulse (P) where $P_A + P_B = 0$.

i.e. The Position and Dimension of all Points which are connected across the Universe and that of Spaces , exists , because of this equilibrium Static Inner Impulse and thus show the EnergySpace continuum . Applying the above logic on any monad = *quaternion* ($s + \bar{v} \cdot \mathbb{I}i$) , where , s = the real part and $\bar{v} \cdot \mathbb{I}i$ the imaginary part of quaternion so , Thrust of two equal and opposite quaternion is the , Action of these quaternions which is ,

$(s + \bar{w}.i) \cdot (s + \bar{w}.i) = [s + \bar{w}.i]^2 = s^2 + |\bar{w}|^2.i^2 + 2|s|\bar{w}.i = s^2 - |\bar{w}|^2 + 2|s|\bar{w}.i = [s^2] - [|\bar{w}|^2] + [2\bar{w}.|s|.i]$
 where,

$[+s^2] \rightarrow s^2 = (w.r)^2$, \rightarrow is the real part of the new quaternion which is, the positive Scalar product, of Space from the same scalar product, s, s with $1/2, 3/2, \dots$, spin and this because of, w , and which represents the massive, Space, part of quaternion.

$[-s^2] \rightarrow -|\bar{w}|^2 = -|\bar{w}.i|^2 = -[|\bar{w}|.|i|]^2 = -(w.r)^2 \rightarrow$ is the always, the negative Scalar product, of Anti-space from the dot product of \bar{w}, i vectors, with $-1/2, -3/2, \dots$, spin and this because of, $-w$, and which represents the massive, Anti-Space, part of quaternion.

$[2i] \rightarrow 2.|s| \times |\bar{w}.i| = 2|wr|.i = 2.(w.r)^2 \rightarrow$ is a vector of, the velocity vector product, from the cross product of \bar{w}, i vectors with double angular velocity term giving $1, 3, 5$, spin and this because of, $\pm w$, in inner structure of monads, and represents the, Energy Quanta, of the Unification of the Space and Anti-Space through the Energy (Work) part of quaternion. A wider analysis is given in articles [40-43].

When a point, A , is quantized to point, B , then becomes the line segment $AB = \text{vector } AB = \text{quaternion } [AB]$ and is the closed system, $A B$, **and since** also from the law of conservation of energy, *it is the first law of thermodynamics*, which states that the energy of a closed system remains constant, therefore *neither increases nor decreases without interference from outside*, and so the total amount of energy in this closed system, AB , in existence has always been the same, **Then** the Forms that this energy takes are constantly changing. This is the unification of this Physical world of, *Matter and Energy*, and that of Euclidean Geometry which are, *Points, Segments, Planes and Volumes*. For more in [48]

The three Moulds (i.e. The three Geometrical Machines) of Euclidean Geometry which create the METERS of monads and which are, *Linear* for a perpendicular Segment, *Plane* for the Square equal to the circle on Segment, *Space* for the Double Volume of initial volume of the Segment, *the volume of the sphere is related to Plane*, and exist on Segment in Spaces, Anti-spaces and Sub-spaces.

This is the Euclidean Geometry Quantization to its constituents (i.e. Geometry in its moulds).

The analogous happens when E-Geometry is Quantized to Space and Energy monads [48].

METER of Points A is the Point A , the METER of line is the Segment $ds = AB = \text{monad} = \text{constant}$ and equal to monad, or to the perpendicular distance of this segment to the set of two parallel lines between points A, B , the METER of Plane is that of circle on Segment = monad and which is that Square equal to the circle, the METER of Volume is that of Cube, on Segment = monad which is equal to the Double Cube of the Segment and Measures all the Spaces, the Anti-spaces and the Subspaces in this cosmos. Generally is more referred,

- a). There is **not any Paradoxes of the infinite** because is clearly defined what is a Point and what is a Segment.
- b). **The Algebra of constructible numbers and number Fields is an Absurd theory** based on groundless Axioms as the fields are, and with directed non-Euclid orientations which must be properly revised.
- c). **The Algebra of Transcendental numbers has been devised to postpone the Pure geometrical thought**, which is the base of all sciences, by changing the base-field of solutions to Algebra as base. Pythagorians discovered the existence of the incommensurable of the diagonal of a square in relation to its side without giving up the base, which is the geometrical logic.
- d). All theories concerning **the Unsolvability of the Special Greek problems are based on Cantor's shady proof**, < that the totality of All algebraic numbers is denumerable > and not edified on the geometrical basic logic which is the foundations of all Algebra. The problem of Doubling the cube $F-4$, as that of the Trisection of any angle $F-11$, is a Mechanical problem and could not be seen differently and the proposed Geometrical solutions is clearly exposed to the critic of all readers.

All trials for Squaring the circle are shown in [46] and the set questions will be answered on the Changeable System of the two Expanding squares, *Translation [T] and Rotation [R]*. The solution of Squaring the circle using the Plane Procedure method is now presented in $F-1, 2$, and consists an, *Overthrow*, to all existing theories in Geometry, Physics and Philosophy.

- e). Geometry is the base of all sciences and it is the reflective logic from the objective reality and which is nature.

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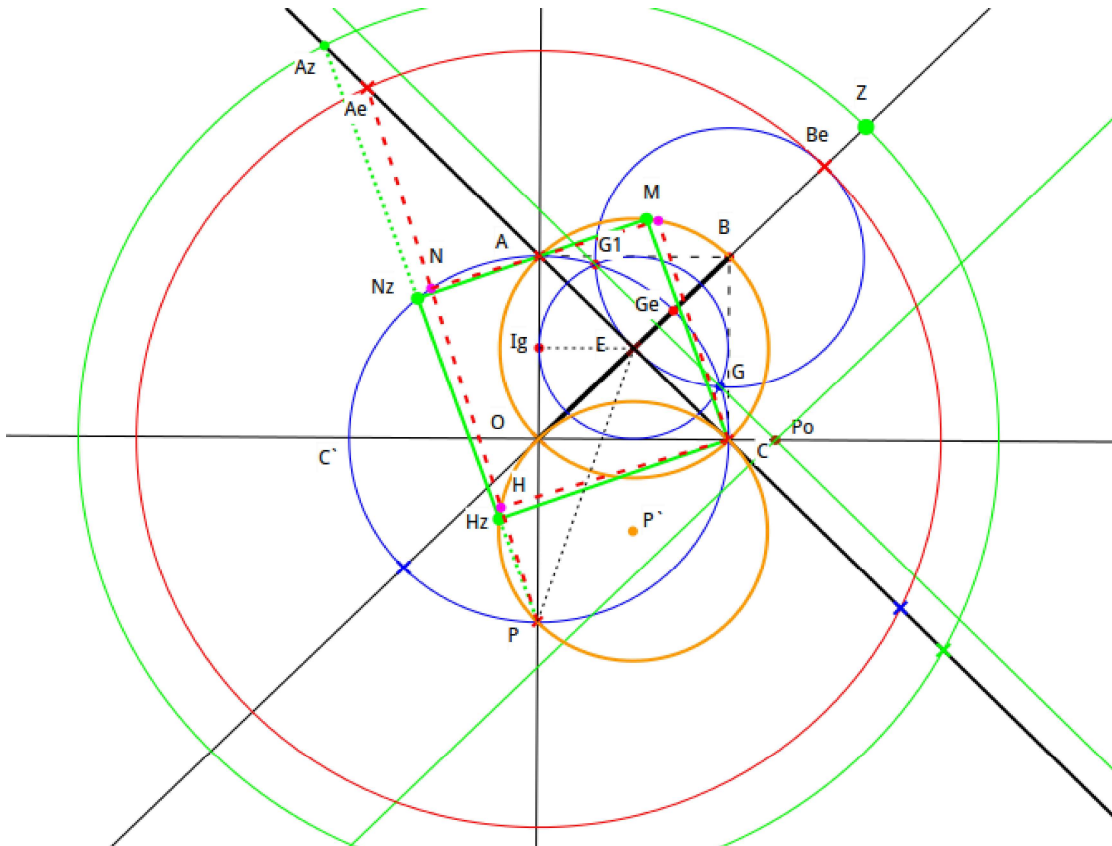
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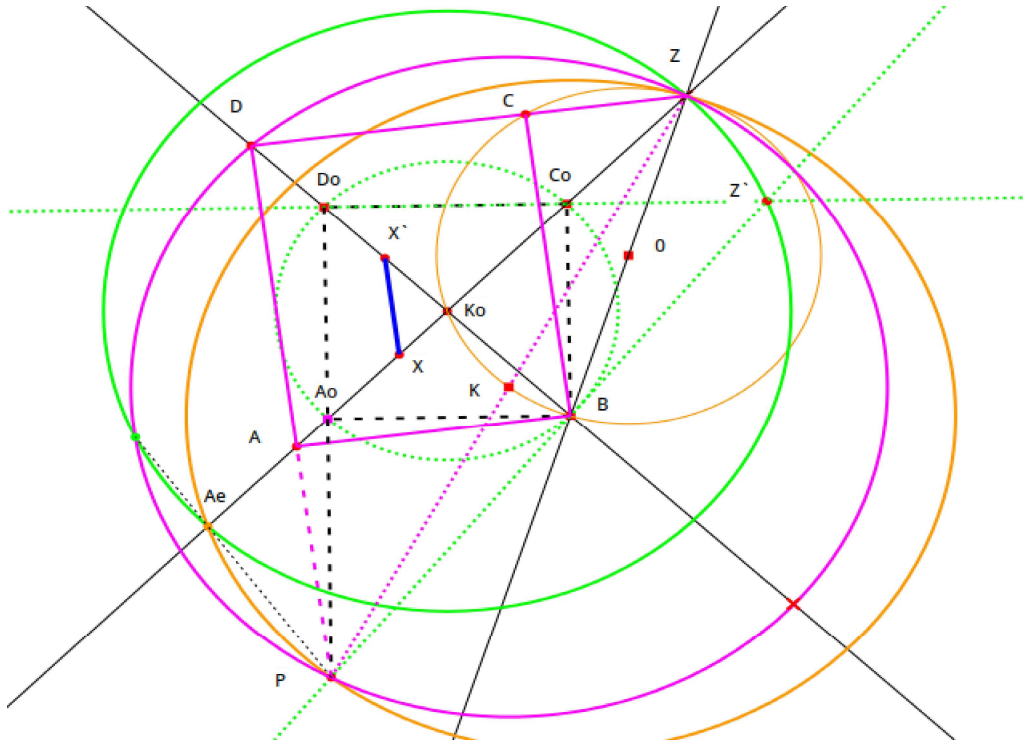
All methods presented on , Dr.Geo-machine
Macro-constructions



F.2-A → *A Presentation of the Quadrature Method on Dr. Geo-Machine Macro - constructions .*

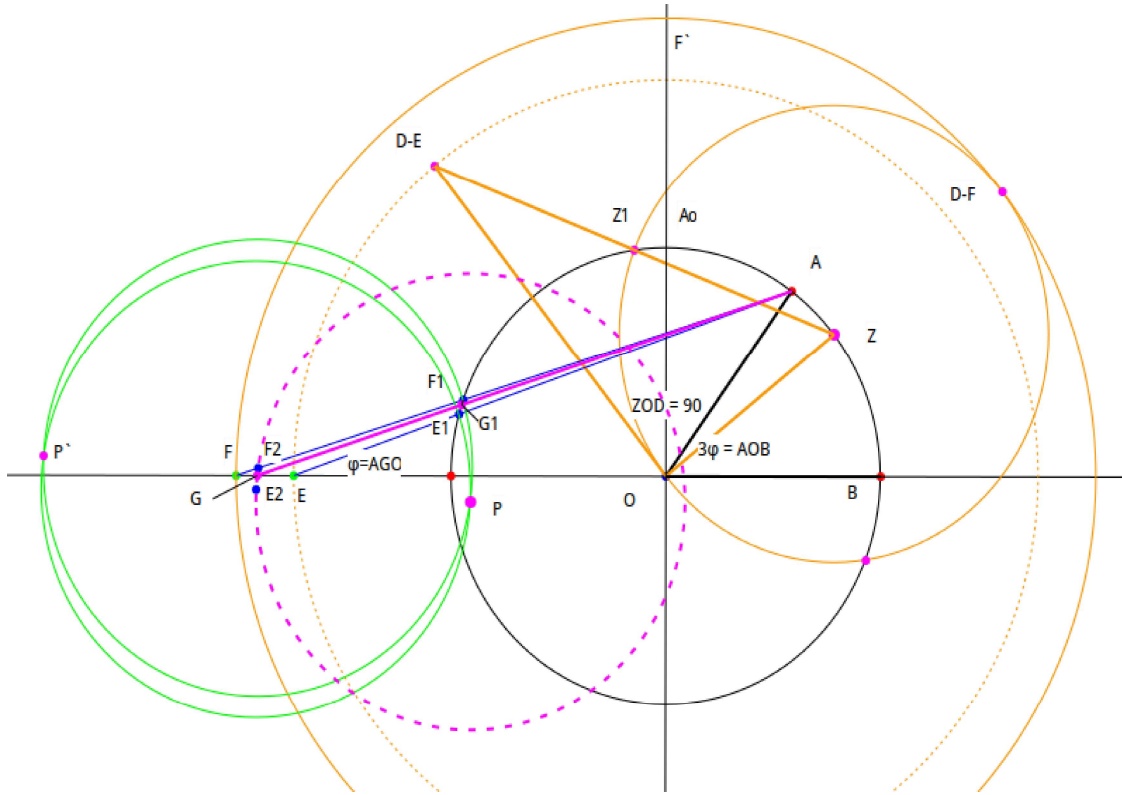
The Inscribed Square CBAO , with Pole-line AOP , rotates through Pole P , to the → Circle-CMNH with Pole-line NHP , and to the → Circumscribed Square CAC'P , with Pole-line of the circle E , EO = EC and at position Be , AeNHP Pole-line formulates square which is the Squaring of the circle .

Square
 $C'PP = C'P$,
 $CMNH = \pi \cdot EO^2$



F.4-A. → A Presentation of the Dublication Method on Dr.Geo - Machine Macro - constructions

BCDA Is the In-between Quadrilateral , on (K,KZ) Extrema-circle , and on KoZ-KoB Extrema – lines of common poles Z , P , mechanism . The Initial Quadrilateral BCoDoAo , with Pole-lines DoAoP - DoCoZ' , rotates through Pole P and the moveable Pole Z` on Z'Z arc , to the → Extreme Quadrilateral BCDA through



Pole-lines DAP - DCZ with point Do, sliding on BKOdo Pole-line, and at point D then $KD^3 = 2.KoA^3$ which is the Dublication of the Cube .

F.11-A. → Presentation of the Trisection Method on Dr. Geo - Machine Macro - constructions .

From Initial position of triangle AOC , where angle AOB = 90° and Segment A1C = OA , to the Final position of triangle , where angle AOB = BOB = 0° and AOB = B'OB = 180 ° , through the Extrema position between edge- cases of triangle ZOD where AOB = φ ° and GG1 = GP = G1O , and at point G then G1G = G1O = OA which is the Trisection of angle AOB .