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FERMAT'S LAST THEOREM IS EQUIVALENT TOBEAL'S CONJECTURE

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Abstract:-

It is proved in this paper that (1) Fermat's Last Theorem: If π is an odd prime, there are no relatively prime positive integers x,y,z satisfying the equation $z^\pi = x^\pi+y^\pi$, and (2) Beal's Conjecture :The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x,y,z with μ,ξ and ν odd primes at least 3. It is also proved that these two statements, (1) and (2), are equivalent.

(1) (Fermat's Last Theorem): If π is an odd prime, there are no relatively prime positive integers x, y, z satisfying the equation $z^\pi = x^\pi + y^\pi$,

(2) (Beal's conjecture:) The equation $z^\xi = x^\mu + y^\nu$ has no solution in relatively prime positive integers x, y, z with μ, ξ and ν odd primes at least 3.

See [1], [2] and [3] for history of these problems.

First, **the proof of (1)**. To prove that, if π is an odd prime, then $z^\pi \neq x^\pi + y^\pi$ for relatively prime positive positive integers x, y, z. Edwards [1] has proved that $z^4 \neq x^4 + y^4$ for relatively prime positive integers x, y and z.

Suppose $z^\pi = x^\pi + y^\pi$ for relatively prime positive integers x, y, z.

We claim the following:

$$x + y - z \equiv 0 \ (mod\ \pi),$$

And

$$(x + y)^\pi - Zp \equiv 0 \ (mod\ \pi^2)$$

To prove the above claims:

Note that by expanding $(x + y - z)^\pi$ using binomial expansion,

$$(x+y-z)^\pi - ((x+y)^\pi - z^\pi) = \sum_{k=1}^{\pi-1} C(\pi, k)\,(x + y)^{\pi-k}\,(-z)^k, \qquad (1)$$

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Again, using binomial expansions for $(x + y)^\pi$ and $((x + y - z) + z)^\pi$, we have,

$$(x + y)^\pi - Zp - (x + y - z)^\pi \equiv 0 \ (mod\ \pi).(2)$$

The right hand side of equation (2) is divisible by π and hence the left hand side is divisible by π. The expansion of $(x + y)^\pi - Zp$ shows that $(x + y)^\pi - Zp$ is divisible by π and hence $(x + y - z)^\pi$ is divisible by π.

Thus

$$x + y - z \equiv 0 \ (mod\ \pi). \qquad (3)$$

So,

$(x + y - z)^\pi \equiv 0 \ (mod\ \pi^\pi)$;

And

$$(x + y)^\pi - Zp \equiv 0 \ (mod\ \pi^2). \qquad (4)$$

In view of equations (3) and (4), equation (1) gives that

$$z \equiv 0 \ (mod\ \pi) \qquad (5)$$

and

$$x + y \equiv 0 \ (mod\ \pi). \qquad (6)$$

Hence, in view of equation (3),

$$z^\pi - x^\pi - y^\pi = (x + y)^\pi - x^\pi - y^\pi$$

$$= \sum_{k=1}^{\pi-1} C(\pi, k) x^{\pi-k} y^k \equiv 0 \ (mod\ \pi^\pi). \qquad (7)$$

So,

$$y \equiv 0 \ (mod\ \pi) \qquad (8)$$

and

$$x \equiv 0 \ (mod\ \pi) \qquad (9).$$

Thus we get $x \equiv 0 \ (mod\ \pi), y \equiv 0 \ (mod\ \pi)$ and $z \equiv 0 \ (mod\ \pi)$.
Hence x,y,z are not relatively prime and thus the proof of Fermat's Last Theorem.
Now, consider **Beal's conjecture.** Assume Fermat's Last Theorem and let

$$\xi, \mu, v, \geq 3.$$

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Then,

$(z^{\xi})^{\pi}$ 6= $(x^{\mu})^{\pi} + (y^{v})^{\pi}$

Suppose that $z^{\xi} = x^{\mu} + y^{v}$, for any x,y and z.

Then $(z^{\xi})^{\xi} = (x^{\xi})^{\mu} + (y^{\xi})^{v}$, replacing x,y and z with x^{ξ}, y^{ξ} and z^{ξ}. Hence $(z^{\xi})^{\xi} = (x^{\mu})^{\xi} + (v^{v})^{\xi}$. As in the proof of Fermat's Last Theorem, it can be shown that each x^{μ}, y^{v} and z^{ξ} is divisible by ξ. Therefore, each x,y and z is divisible by ξ, which implies that x,y and z are not relatively prime. Thus Fermat's Last Theorem implies Beal's conjecture.
For the converse, take, $\xi = \mu = v = \pi$, an odd prime. Thus the proof of the equivalence is complete.

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