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KNOWLEDGE OF FLEXIBLE MENTAL COMPUTATION OF PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS UPON ENTRY TO UNIVERSITY

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Abstract:-

This study sets out to determine the knowledge of flexible mental computation of pre-service elementary mathematics teachers when enrolling at university. Knowledge of the existing flexible mental computation skills of pre-service teachers could inform teacher educators' preparation for teaching by determining the algorithms that pre-service teachers are unable to calculate. Such knowledge could also inform teacher educators as to why pre-service teachers are unable to calculate the algorithms. The study produced a domain-specific instrument in the form of a diagnostic test that teacher educators could adopt in their teaching to elicit pre-service teachers' knowledge, a gap that this study aims to bridge. A diagnostic test comprising 21 items was administered by the researcher to 51 year-one pre-service teachers who agreed to participate in the study. Findings of the study indicate that the majority of the pre-service teachers who participated in the study have a limited understanding of the magnitude of numbers, of basic number facts, of the relationship between numbers and of the relationship between basic operations to calculate flexibly. The argument of the study is that teacher educators' instruction would be relevant and successful if focused on the cognitive needs of pre-service teachers and the mathematical concepts that they are going to teach. Therefore, this study recommends that teacher educators establish pre-service teachers' knowledge prior to instruction for relevance and for the success of teaching.

1. INTRODUCTION

Existing research on pre-service elementary mathematics teachers in Namibia reports a degree of incompetency in the flexible mental computation domain among final-year pre-service teachers (Courtney-Clarke & Wessels, 2014). Evidence from a study by Vatilifa (2014) presents preservice teachers' own needs to have their mathematical content knowledge and pedagogical content knowledge improved, as they found their content knowledge insufficient for them to teach effectively. One main challenge pointed out by Vatilifa is the difficulty that pre-service teachers have in teaching a range of mathematics subject matter (2014).

Gresham's (2007) study indicates that teacher education programmes could improve pre-service teachers' knowledge of subject matter. In Namibia, however, few studies exist that propose solutions to improve pre-service teacher education. As a result, this study sets out to propose one possible solution to the problem of inadequate mathematical knowledge among pre-service teachers. In light of the aforementioned problem, we argue that, if teacher educators' mathematical instruction is informed by research on the existing knowledge of pre-service teachers in terms of their zone of actual development (ZAD), such instruction would be aligned with teachers' cognitive needs and would lead to improved knowledge (Vygotsky, 1978). By establishing the prior knowledge of pre-service teachers pertaining to flexible mental computation, awareness would be raised among teacher educators as to the type of skills that preservice teachers possess relating to flexible mental computation and how well versed they are in these skills.

Awareness of the existing knowledge of pre-service teachers illuminates teacher educators' practice in terms of planning relevant tasks to support teacher development effectively. According to Bruner (1977), meaningful activities must form a basis for further learning, as further learning is founded on existing knowledge. The argument is that "people learn as they are stretched beyond their own knowledge but only within the range that is within their grasp given what knowledge and skills they bring to the task" (Hein, 1991, p. 7). In this case, to improve preservice teachers' knowledge, teacher educators need to present the "fundamental structure of the subject . . . [as] . . . understanding fundamentals makes a subject more comprehensible" (Bruner, 1977, p. 23).

Pre-service teachers need to be able to compute algorithms mentally themselves first using a variety of strategies, as required by the elementary school curriculum, for them to teach effectively. The ability of pre-service teachers to compute effectively fosters understanding of flexible mental computation, as ". . . to teach is first to understand . . ." (Shulman, 1987, p. 14).

However, there is a gap in that the elementary teacher education curriculum for pre-service mathematics teachers in Namibia only implicitly outlines ways to improve teacher knowledge pertaining to specific conceptual knowledge, as outlined in the elementary school curriculum. There are currently no specific guidelines or frameworks to facilitate this knowledge and develop appropriate skills.

This article provides guidance on the type of test items that teacher educators could use to establish the flexible mental computation skills of pre-service teachers. Teacher educators could also use this to establish the types of algorithms that pre-service teachers are expected to teach but are likely to struggle with when calculating mentally, as found in this study.

2. Literature review

The flexible mental computation of whole numbers forms the basis of the elementary school mathematics curriculum in Namibia. The National Council of Teachers of Mathematics (2000) emphasises the fundamental role of whole number computation in the elementary school phase. By the end of Grade 3, for example, elementary school learners are expected to develop competency in the four basic operations involving the addition, subtraction, multiplication and division of positive whole numbers below 500 using their own calculation strategies. Flexible mental computation refers to the ability to perform calculations in your head using your own invented strategies, without any external support like a calculator or paper and a pencil and without performing the standard algorithm in your head.

Before learners are taught to calculate mentally, the syllabus for elementary school mathematics expects teachers to enhance the number concept development of learners to illuminate their ability to invent their own calculation strategies. Concrete objects, pictures and diagrams are used for learners to understand the magnitude of numbers and so to lay the foundation for flexible mental computation skills development.

The development of the flexible mental computation skills of learners is necessary despite the availability of calculators. The reason for this is that both pre-service teachers and learners need to make sense of solutions provided by calculators by verifying solutions themselves mentally. This is practical, since there are indeed algorithm teachers and learners who compute flexibly without using a calculator or a paper and a pencil strategy.

Flexible mental computation provides both teachers and learners "cognitive freedom" (Carpenter, 1974, p. 127) through independence from calculators or rote-learned (Parrish, 2010) paper and pencil strategies to compute algorithms, while viewing mathematics as meaningful, easy and interesting (McIntosh, 2004b). Another benefit of flexible mental computation is that it increases both teachers' and learners' confidence in doing mathematics and facilitates the learning of written algorithms.

Thorndike (1921) emphasises that learning in one area leads to learning in another area and that practise in one activity improves performance in the next activity due to improved understanding. However, cognitive freedom from calculators and from paper and pencil methods to solve algorithms needs to manifest itself both in teacher educators and in pre-service teachers first before learners can develop this.

Inadequate mathematical content knowledge among pre-service teachers and in-service teachers is evident in studies conducted both nationally and internationally. A study conducted among final-year pre-service teachers in Namibia, for example, indicates incompetency in pre-service teachers to compute algorithms mentally (Courtney-Clarke & Wessels, 2014). Similarly, findings of other studies conducted in Namibia indicate inadequate teacher content knowledge to teach mathematics (Kasanda, 2005; Nambira, Kapenda, Tjipueja & Sichombe, 2009). As shown by Kajander (2010, p. 228), teachers who participated in that study were ". . . found to be initially weak in conceptual understanding of basic mathematics concepts as needed for teaching" but that ". . . a strong focus on specialised mathematical concepts significantly improved pre-service teachers' understandings, but only to a minimally acceptable level". A further finding on preservice teachers' knowledge comes from Jakimovik's (2014) study, which showed a very low degree of mathematics content knowledge by pre-service teachers upon entry to university.

Similar to the need for enhanced content knowledge articulated in Vatilifa's (2014) study by former pre-service teachers who are currently teaching are concerns about mathematics content raised by elementary teachers internationally. In a study by Hart, Oesterle and Swars (2013), elementary teachers stressed that the kind and degree of mathematics content offered by teacher courses was on a very advanced level and was irrelevant to the mathematics content intended for the teaching of elementary school learners. This shows the urgency for teacher preparation programmes to focus more on developing the understanding of mathematical concepts that preservice teachers are expected to teach before they are taught advanced college mathematics (Ma, 2010; Shulman, 1987). Therefore, the success of mathematics teacher education rests on appropriate teacher educators' teaching practice.

Pre-service mathematics teacher instruction must be founded on the prior knowledge of preservice teachers to be effective. Teacher educators are therefore urged to establish the diverse degrees of comprehension of mathematics concepts by pre-service teachers (Bolden, Barmby &
Harries, 2013). Establishing pre-service teachers' existing knowledge is important, as prior knowledge can, at times, entail misconceptions, which could be difficult to correct and, in turn, become a hindrance to further learning (Thorndike, 1921). As mentioned, Vygotsky (1978) refers to existing knowledge as the ZAD. To maximise learning, teacher educators need to identify preservice teachers' ZAD, meaning that which pre-service teachers can do without anyone's support. Teacher educators need to determine pre-service teachers' existing knowledge upon entry to university or before discussing a particular mathematics concept.

Teacher educators therefore need to identify ways to ascertain pre-service teachers' existing knowledge. One possible way of establishing pre-service teachers' existing understanding of and skills in flexible mental computation is to administer a diagnostic test similar to the one used in this study or to use a modified version of the test.

Once existing skills are established, teacher educators can then adopt the hypothetical learning trajectory as informed by the conceptual framework of flexible mental computation skills development of Heirdsfield (2002) to hone the flexible mental computation skills of pre-service teachers. The use of frameworks for teaching has, in the past, proven successful in developing the mathematical knowledge of pre-service teachers (Hartnett, 2007; Star & Stylianides, 2013).

Whitacre and Nickerson's (2006) intervention study shows that the use of a domain-specific hypothetical learning trajectory can improve the mental calculation skills of pre-service elementary teachers and help them to understand what they are going to teach. A domainspecific hypothetical learning trajectory specifies the mathematical concepts to be learned, specific tasks to be given and the relevant teaching material to be used. The framework incorporates aspects relating to appropriate ways to teach learners at the elementary stage and supports pre-service teachers to understand that learners at the elementary level learn "by doing and experiencing" mathematics (Kesicioğlu, 2015, p. 97). In this case, teacher educators need to ensure that pre-service teachers investigate the mathematics and curriculum that they are expected to teach (Ma, 2010). However, a domain-specific framework for teaching and learning can be developed only once the existing knowledge of pre-service teachers is established.

As mentioned, for teacher education instruction to improve the flexible mental computation skills of pre-service teachers, it has to begin with what teachers already know about flexible mental computation. Once established, teacher educators can then extend the existing knowledge of pre-service teachers with appropriate tasks that are meaningful to them (Bruner, 1977). According to Hein (1991), effective learning is that which goes beyond what pre-service teachers already understand and focuses on the basic organisation of the subject (Bruner, 1977). Awareness of the existing knowledge of pre-service teachers should inform planning in terms of where teaching should start and of creating activities that are within the level of the flexible mental computation of teachers. Knowledge of pre-service teachers' ZAD can provide teacher educators with a guide that prevents them from designing tasks that are too challenging, causing teachers to opt out of learning mathematics, or that are too easy, resulting in boredom. Understanding what pre-service teachers know about flexible mental computation and how they acquired the knowledge is helpful, as existing knowledge can be right or wrong and needs to be corrected when found to be inappropriate? Comprehension of the existing level of understanding and skills of pre-service teachers should thus inform planning.

3. Methods

This article is part of a major study that was conducted to explore the flexible mental computation skills, beliefs and experiences of pre-service elementary mathematics teachers upon entry to university. The methodology of this study was informed by the interpretive paradigm to allow the researcher to perceive the world through the insights and experiences of pre-service teachers (Creswell, 2013). The design of the study was informed by the philosophy of realistic mathematics education, an instructional theory for mathematics, which originated in the Netherlands that focuses on a particular mathematics area (Van den Heuvel-Panhuizen & Drijvers, 2014). A qualitative research design was used to provide detailed understanding of the level of flexible mental computation of individual pre-service teachers (Creswell, 2015). A real world classroom environment formed the site where the qualitative data were collected through a diagnostic test (Wang & Hannafin, 2005); identifying the cognitive needs of pre-service teachers in a real classroom setting enabled the adoption of solutions that are realistic and empirical in nature (Given, 2008). The setting for the study was a public university satellite campus situated in one of the 13 regions of Namibia.

All 51 of the first-year pre-service teachers who enrolled for the bachelor degree course in lower primary education were approached to participate in the study. A test comprising 21 items was administered by the researcher, the test items covering the four basic operations involving the addition, subtraction, multiplication and division of numbers within the one to three-digit number range and one four-digit algorithm. The test was designed using Heirdsfield's (2002) conceptual framework for flexible mental computation, Mcintosh, Reys, & Reys (1992)'s framework for basic number sense and Hartnett (2007)'s framework for mental computation strategy.
The data were analysed using the IBMM SPSS statistics version 25, valid until September 2018. The data were first analysed to determine the individual performance of pre-service teachers in all the 21 items. Performance per item was then established to identify the type of algorithms that pre-service teachers could not calculate mentally.
The study adhered to the guidelines of Stellenbosch University's ethics committee, the study being granted ethical clearance, and permission was received from University of Namibia Management for the study to be conducted. The objectives of the research were clarified to the participants and informed consent from the participants was obtained. Assurance of the proper use of the research findings, anonymity, confidentiality, honesty and respect prevailed.

4. Diagnostic test as a tool for measuring flexible mental computation

Table 1 below presents the test items that were used to determine the flexible mental computation skills of pre-service teachers. The test item categorisation and explanation are provided immediately after Table 1.

Pre-intervention test	
Pre-service teachers' mental computation skills	
Test items	
1	5 + 7
2	19 + 15
3	23 + 18 + 37
4	23 - 16
5	151 - 98
6	563 - 292
7	4 × 50
8	32 × 15
9	16 × 25
10	16 ÷ 4
11	120 ÷ 6
12	14 × 8
13	59 + □ = 82
14	85 - □ = 67
15	□ - 38 = 89
16	192 ÷ 8
17	□ ÷ 15 = 20
18	15 × □ = 45
19	250 × 2
20	2 024 - 1 999
21	799 - 51

Table 1: Test items used to determine flexible mental computation skills

The test items covered all four basic operations. Direct and indirect addition, subtraction, multiplication and division algorithms of positive whole numbers formed the test. The direct addition algorithms consisted of 5 + 7, 19 + 15 and 23 + 18 + 37, indirect addition consisted of 85 - □ = 67 and □ - 38 = 89, direct subtraction consisted of 23 - 16, 151 - 98, 563 – 292, 799 – 51 and 2 024 – 1 999, indirect subtraction consisted of 59 + □ = 82, direct multiplication consisted of 4 × 50, 250 × 2, 32 × 15, 16 × 25 and 14 × 8, indirect multiplication consisted of □ ÷ 15 = 20, direct division consisted of 120 ÷ 6, 16 ÷ 4 and 192 ÷ 8 and indirect division consisted of 15 × □ = 45. Test items involving direct and indirect addition, subtraction, multiplication and division were used to determine the number sense, the understanding of number relationships and the understanding of the relationships of basic operations, commutative property and reversibility of pre-service teachers.

5. Results

The findings of this study are presented in two tables. The overall individual performance of each pre-service teacher in relation to the performance of the class is presented in Table 2 and the class performance in each test item is presented in Table 3.

5.1 Pre-service teachers' individual performance in all 21 items of the test

Diagnostic test					
Mark out of 100					
		Frequency	Percent	Valid percent	Cumulative percent
Valid	14	1	2.0	2.0	2.0
	19	1	2.0	2.0	3.9
	24	1	2.0	2.0	5.9
	29	2	3.9	3.9	9.8
	33	4	7.8	7.8	17.6
	38	4	7.8	7.8	25.5
	43	7	13.7	13.7	39.2
	48	6	11.8	11.8	51.0
	52	9	17.6	17.6	68.6
	57	7	13.7	13.7	82.4
	62	3	5.9	5.9	88.2
	67	2	3.9	3.9	92.2
	76	1	2.0	2.0	94.1
	81	2	3.9	3.9	98.0
	86	1	2.0	2.0	100.0
	Total	51	100. 0	100.0	

Table 2: Individual performance in all 21 items

The test results presented in Table 2 indicate that 26 pre-service teachers (51%) mentally calculated less than half the test items correctly; 25 pre-service teachers (49%) therefore mentally calculated more than half the test items correctly. The mark that was common among the scores is 52%; the average score was 49%. The lowest mark was 14% and the highest mark was 86%, indicating a difference of 72%.

5.2 Class performance per test item

Diagnostic test					
Pre-service teachers' mental computation skills					
Test items		Total number of pre-service teachers who got the answer right or wrong (51 wrote the test).		Percentage	
		Right	Wrong	% right	% wrong
1	5 + 7	49	2	96	4
2	19 + 15	46	5	90	10
3	23 + 18 + 37	24	27	47	53
4	23 - 16	33	18	65	35
5	151 - 98	20	31	39	61
6	563 - 292	9	42	18	82
7	4 × 50	46	5	90	10
8	32 × 15	4	47	8	92
9	16 × 25	9	42	18	82
10	16 ÷ 4	42	9	82	18
11	120 ÷ 6	23	28	45	55
12	14 × 8	13	38	25	75
13	59 + □ = 82	20	31	39	61
14	85 - □ = 67	18	33	35	65
15	□ - 38 = 89	8	43	16	84
16	192 ÷ 8	4	47	8	92
17	□ ÷ 15 = 20	19	32	37	63
18	15 × □ = 45	41	10	80	20
19	250 × 2	45	6	88	12
20	2 024 - 1 999	23	28	45	55
21	799 - 51	29	22	57	43

Table 3: Class performance per test item

With regard to specific items, as presented in Table 3, more than half the participants demonstrated an ability to calculate the following: 5 + 7, 19 + 15, 23 - 16, 4 × 50, 16 ÷ 4, 15 × □ = 45, 250 × 2 and 799 - 51. More than half the participants were unable to calculate 23 + 18 +
37, 151 - 98, 563 - 292, 32 × 15, 16 × 25, 120 ÷ 6, 14 × 8, 59 + □ = 82, 85 - □ = 67, □ - 38 = 89, 192 ÷ 8, □ ÷ 15 = 20 and 2 024 – 1 999.

6. Discussion and recommendations

The findings of this study indicate that few pre-service teachers (49%) have adequate knowledge to compute algorithms mentally upon entry to university: the majority of pre-service teachers (51%) were unable to calculate more than half the test items correctly, as reflected in Table 2. The gap between the highest and low score is also very wide (72%), which signifies that, while 49% of pre-service teachers were found to be well below average, 31% scored above average, while 20% had a highly significant knowledge base of flexible mental computation well above average. Performance per test item indicates that the majority of pre-service teachers are able to add, subtract, multiply and divide algorithms involving two one-digit numbers and two two-digit numbers below 50 and a one-digit number and two-digit number, a three-digit number and onedigit number and a three-digit number and two-digit number close to a multiple of 10. This means that the majority of the pre-service teachers involved in the study cannot compute algorithms involving two two-digit numbers beyond 30 or algorithms with three and four-digit numbers, as reflected in Table 3.

The findings also indicate that the majority of pre-service teachers lack number sense and a basic knowledge of calculation strategies involving, for example, decomposition, making a friendly number, derived facts, reversibility, commutative property, number and basic operation relationships, and factors.

The results of this study therefore confirm Kajander's (2010) findings that the majority of preservice elementary mathematics teachers, in most cases, have a weak knowledge base of mathematics upon entry to university. If the existing knowledge of pre-service teachers is not strengthened, pre-service teachers will continue to demonstrate a weak ability to perform calculations mentally, even in their final year of study (Courtney-Clarke & Wessels, 2014), and will teach with a weak knowledge base in their schools (Nambira, Kapenda, Tjipueja & Sichombe, 2009).

Pre-service teachers in this study illuminate what teacher educators could expect from their preservice teachers upon entry to university. Teacher educators could therefore either adopt or adapt the diagnostic test used in this study to determine and improve the entry knowledge to university of their pre-service elementary mathematics teachers. With regard to the items that the preservice teachers could not calculate mentally, as reflected in Table 2, teacher educators could be provided a guide for the planning of initial instruction for flexible mental computation.

Therefore, this study recommends that teacher educators make an effort to establish the knowledge of pre-service teachers before instruction for the sake of the relevance of instruction in mathematics teacher education.

7. Conclusion

Studies on pre-service elementary mathematics teachers both in Namibia and internationally have found that, in many cases, pre-service teachers come with a weak mathematical knowledge base to university. The findings of this study also indicate inadequate knowledge to compute calculations mentally among pre-service elementary mathematics teachers. As a result, the study offers an instrument that teacher educators could use to determine the knowledge of the flexible mental computation of their pre-service teachers. Furthermore, the test items found difficult by the pre-service teachers in this study could also be incorporated into lessons in the form of activities to improve the knowledge of pre-service teachers. The study recommends further research into establishing the degree to which the university curriculum for elementary mathematics teachers responds to the level of preparation that pre-service teachers need to teach elementary school mathematics. Finally, unless teacher educators align instruction with the cognitive needs of pre-service teachers, these teachers will continue to graduate and practice in classrooms with a weak knowledge base.

References

[1]. Bolden, D., Barmby, P., & Harries, T. (2013). A representational approach to developing primary ITT students' confidence in their mathematics. *International Journal of Mathematical Education in Science and Technology, 44*(1), 70–83.

[2]. Bruner, J. (1977). *The process of education: A landmark in educational theory.* Cambridge: Harvard University Press.

[3]. Can, T. (2006, April 07). *Constructivist education: What is constructivism?* [Blog post].

[4]. Retrieved from http://www.constructivist-education.blogspot.com/2006/04what-isconstructivism.html Carpenter, F. (1974). *The Skinner primer: Behind freedom and dignity.* London: Collier Macmillan Publishers.

[5]. Courtney-Clarke, M. & Wessels, H. (2014). Number sense of final year pre-service primary school teachers. *Pythagoras, 35*(1).

[6]. Creswell, J.W. (2013). *Qualitative inquiry and research design: Choosing among five approaches.* Los Angeles: SAGE.

[7]. Creswell, J.W. (2015). *A concise introduction to mixed methods research.* Los Angeles: SAGE.

[8]. Given, L.M. (2008). *The Sage encyclopedia of qualitative research methods.* Los Angeles: SAGE.

[9]. Gresham, G. (2007). A study of mathematics anxiety in pre-service teachers. *Early Childhood Education Journal, 35*(2), 181–188.

[10]. Hart, L.C., Oesterle, S., & Swars, S.L. (2013). The juxtaposition of instructor and student perspectives on mathematics courses for elementary teachers. *Edc Stud Math, 83*, 429– 451.

[11]. Hartnett, J. (2007). Categorisation of mental computation strategies to support teaching and to encourage classroom dialogue. In J. Watson, & K. Beswick (Ed.), *Proceedings 30th annual conference of the mathematics education research group of Australasia, Mathematics: Essential research, essential practice* (pp. 345-352). Hobart, Tasmania: MERGA Incl.
[12]. Hein, G.E. (1991). Constructivist learning theory: The museum and the needs of people. *CECA (International Committee of Museum Educators) Conference* (pp. 1–10). Jerusalem: Lesley College, Massachusetts USA.
[13]. Heirdsfield, A. (2002). The interview in mathematics education: The case of mental computation. *Annual Conference of the Australian Association for Research in Education* (pp. 1-17). Brisbane: Australian Association for Research in Education.
[14]. Jakimovik, S. (2014). *The mathematics education of pre-school teachers.* Macedonia: Faculty of Pedagogy "St. Kliment Ohridski".
[15]. Kasanda, C. (2005). Education in Africa: Post colonialism and globalisation in science and mathematics education: The case of Namibia and Zambia. In M. Mostert & A.C. Kasanda (Eds.), *Education in Namibia: A collection of essays* (pp. 106–123). Windhoek: University of Namibia Publishers.
[16]. Kajander, A. (2010). Elementary mathematics teacher preparation in an era of reform: The development and assessment of mathematics for teaching. *Canadian Journal of Education, 33* (1), 228-255.
[17]. Kesicioğlu, O.S. (2015). The effects of an undergraduate programme of preschool teaching on preservice teachers' attitudes towards early mathematics education in Turkey: A longitudinal study. *Early Child Development and Care, 185*(1), 84–99.
[18]. Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* New York: Routledge, Taylor and Francis Group.
[19]. McIntosh, A. (2004b). Developing computation. *APMC, 9*(4), 47–49.
[20]. Mcintosh, A., Reys, B.J., & Reys, R.E. (1992). A proposed framework for examining number sense. *For the Learning of Mathematics, 12*(3), 2–8, 44.
[21]. Nambira, G., Kapenda, L., Tjipueja, G., & Sichombe, B. (2009). Performance of learners in mathematics at upper primary phase in Okahandja district: Examining reasons for low performances. Okahandja: The National Institute for Educational Development.
[22]. National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.
[23]. Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57(1),* 1–21.
[24]. Star, J.R., & Stylianides, A.G. (2013). Procedural and conceptual knowledge: Exploring the gap between knowledge type and knowledge quality. *Canadian Journal of Science, Mathematics and Technology Education, 13(2)*, 169–181.
[25]. Tarasenkova, N.A. & Akulenko, I.A. (2013). Determination of students' beliefs is one of the aspects of competence oriented system of mathematics teachers' methodical preparation. *American Journal of Educational Research, 1*(11), 477–483.
[26]. Thorndike, E.L. (1921). *The psychology of learning.* New York: Teachers College Columbia.
[27]. Van den Heuvel-Panhuizen, M. & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 521–525). Utrecht: Springer Netherlands.
[28]. Vatilifa, N. (2014). Investigating the experiences of level 5-7 student teachers when teaching fractions: A case study. In M. Schäfer, D. Samson, & B. Brown (Eds.), *Namibia counts:*
[29]. *Stories of mathematics education research in Namibia* (pp. 123–137). Grahamstown: Education Department, Rhodes University.
[30]. Vygotsky, L. (1978). *Mind and society: The development of higher psychological processes.* Cambridge: Harvard University Press.
[31]. Vygotsky, L.S. (1987). Thinking and speech. In R.W. Rieber & A.S.Carton (Eds.), *the collected works of L.S.Vygotsky* (pp. 39–243). New York: Plenum Press.
[32]. Wang, F. & Hannafin, M.J. (2005). Design-based research and technology-enhanced learning systems. *Educational Technology Research and Development, 53*(4), 5–23.
[33]. Whitacre, I. & Nickerson, S.D. (2006). Pedagogy that makes (number) sense: A classroom teaching experiment around mental math. *Proceedings of the 28th annual meeting of the North American chapter of the International group for the psychology of mathematics education* (pp. 736–743). Mérida: Universidad Pedagógica Nacional.