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NOTE ON THE CORE THEOREM OF THE L-TRANLATIVITY OF EXTENDED ABEL TRANSFORMATIONS

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Abstract: -

In this paper, we investigate the l-l Tranlativity Extended Abel Matrix. 2010 Mathematical Subject Classification: Primary 40A05, 40D99; Secondary 40C05

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1. BASIC NOTATION AND DEFINITIONS. Let $A=(a_{nk})$ be an infinite matrix defining a sequence to a sequence summability transformation given by

$$(Ax)_n = \sum_{k=0}^{\infty} a_{nk} x_k \qquad (2.1)$$

where $(Ax)_n$ denotes the nth term of the image sequence Ax. Let y be a complex number sequence. Throughout this paper, we use the following basic notations and definitions:

i. c={the set of all convergent complex number sequences}

ii. $l = \{y : \sum_{k=0}^{\infty} |y_k| converges\}$

iii. $l(A) = \{y : Ay \in \ell\}$

iv. $c(A)$ ={y: $y$is summable by A}

Definition 1. If X and Y are sets of complex number sequences, then the matrix A is called an X□Y matrix if the image Au of u under the transformation A is in Y whenever u is in X.

Definition 2. The summability matrix A is said to be ltranslative for the sequence u in provided that each of the sequences Tu and Su is in $\ell(A)$, where $T_u = \{u_1, u_2, u_3, ...\}$ and $S_u = \{0, u_0, u_1, ...\}$.

*The **Extended Abel** matrix, denoted by* $A_{\alpha,t}$, *the matrix is defined by*

$$a_{nk} = \binom{k+\alpha}{k} t_n^k (1-t_n)^{\alpha+1}, \quad \alpha > -1 \text{ and } 0 < t_n < 1.$$

Theorem 1. Every $\ell - \ell\ A_{\alpha,t}$ matrix is ℓ - *translative* for those sequences $x \in \ell(A_{\alpha,t})$ for which $\{x_k / k\} \in \ell, k = 1,2,3,....$

Proof. Suppose that x is a sequence in $\ell(A_{\alpha,t})$ for which $\{x_k / k\} \in \ell$. We show that

(1) $T_x \in \ell(A_{\alpha,t})$, and

(2) $S_x \in \ell(A_{\alpha,t})$, where T_x and S_x are as defined in Definition 2. Let us first show that (1) holds.

Note that

$$\left|(A_{\alpha,t}T_x)_n\right| = (1-t_n)^{\alpha+1}\left|\sum_{k=0}^{\infty}\binom{k+\alpha}{k}x_{k+1}t_n^k\right|$$

$$= \frac{(1-t_n)^{\alpha+1}}{t_n}\left|\sum_{k=0}^{\infty}\binom{k+\alpha}{k}x_{k+1}t_n^{k+1}\right|$$

$$= \frac{(1-t_n)^{\alpha+1}}{t_n}\left|\sum_{k=1}^{\infty}\binom{k-1+\alpha}{k-1}x_k t_n^k\right| \qquad (3.1)$$

$$= \frac{(1-t_n)^{\alpha+1}}{t_n}\left|\sum_{k=1}^{\infty}\binom{k+\alpha}{k}x_k t_n^k \frac{k}{k+\alpha}\right|$$

$$= \frac{(1-t_n)^{\alpha+1}}{t_n}\left|\sum_{k=1}^{\infty}\binom{k+\alpha}{k}x_k t_n^k\left(1-\frac{\alpha}{k+\alpha}\right)\right|$$

$$\le A_n + B_n,$$

where

$$A_n = \frac{(1-t_n)^{\alpha+1}}{t_n}\left|\sum_{k=1}^{\infty}\binom{k+\alpha}{k}x_k t_n^k\right| \qquad (3.2)$$

and

$$B_n = \frac{|\alpha|(1-t_n)^{\alpha+1}}{t_n}\left|\sum_{k=1}^{\infty}\binom{k+\alpha}{k}\frac{x_k}{k+\alpha}t_n^k\right| \qquad (3.3)$$

The use of the triangle inequality in equation (3.1) is legitimate as the radii of convergence of the power series are at least 1. Now if we show that both A and B are in ℓ, then (1) holds. But the condition that $A \in \ell$ and $B \in \ell$ follow easily from the hypotheses that $x \in \ell\left(A_{\alpha,t}\right)$ and $\{x_k / k\} \in \ell$, respectively. Next, we show that (2) holds as follows. We have

$$\begin{aligned}
\left|\left(A_{\alpha,t}S_x\right)_n\right| &= \left(1-t_n\right)^{\alpha+1}\left|\sum_{k=1}^{\infty}\binom{k+\alpha}{k}x_{k-1}t_n^k\right| \\
&= \left(1-t_n\right)^{\alpha+1}\left|\sum_{k=0}^{\infty}\binom{k+\alpha+1}{k+1}x_k t_n^{k+1}\right| \\
&= \left(1-t_n\right)^{\alpha+1}\left|\sum_{k=0}^{\infty}\binom{k+\alpha}{k}x_k t_n^{k+1}\left(\frac{k+\alpha+1}{k+1}\right)\right| \qquad (3.4) \\
&= \left(1-t_n\right)^{\alpha+1}\left|\sum_{k=0}^{\infty}\binom{k+\alpha}{k}x_k t_n^{k+1}\left(1+\frac{\alpha}{k=1}\right)\right| \\
&\le E_n + F_n
\end{aligned}$$

where

$$E_n = \left(1-t_n\right)^{\alpha+1}\left|\sum_{k=1}^{\infty}\binom{k+\alpha}{k}x_k t_n^k\right| \qquad (3.5)$$

and

$$F_n = \left(1-t_n\right)^{\alpha+1}|\alpha|\left|\sum_{k=0}^{\infty}\binom{k+\alpha}{k}\frac{x_k}{k+1}t_n^{k+1}\right|. \qquad (3.6)$$

The use of the triangle inequality in (3.4) is justified as above. If we show that E and F are in ℓ, then (2) holds. But the hypothesis that $x \in \ell\left(A_{\alpha,t}\right)$ and $\{x_k / k\} \in \ell$ implies that both E and F are in ℓ, resprectively, and hence the theorem follows.

Here we remark that a sequence x defined by $x_k = (-1)^k / k$ is one of the sequences which satisfies the condition of Theorem follows.

References

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