EPH - International Journal of Mathematics and Statistics
ISSN (Online): 2208-2212
Volume 1 Issue 2 July 2015

DOI:https://doi.org/10.53555/eijms.v5i1.36

DARCY CONVECTION ANISOTROPIC PROBLEM WITH MULTISTABILITY OF STATIONARY MOTIONS FOR RECTANGLE.

Abdelhafez M.A.[1*]**, Tsybulin V.G.**[2]
*[1,2]*Southern Federal University, Rostov-on-Don, 344006, Russia Sohag University,*
[1]*Sohag, 82524, the Arab Republic of Egypt*
[2]*E-mail: vtsybulin04@gmail.com*

Corresponding Author:-
Email: - mostafa.abdallah@yahoo.com

Abstract:-
Based on the Darcy model, uid convection in a porous rectangle is analyzed taking into account anisotropy of thermal characteristics and permeability. Relations between parameters for which the problem belongs to the class of cosymmetric systems are derived. For this case explicit formulas for the critical numbers of the loss of stability of mechanical equilibrium are found. Using a nite-di erence method that preserves the cosymmetry of the problem, family of stationary convective regimes is computed. Through the computational experiment the destruction of families is demonstrated in the case of violation of the conditions of cosymmetry. As result the appearance of a nite number of stationary regimes are obtained.

Convection equations of heat-conducting uid in a porous anisotropic medium based on Darcy's law. The plane problem of heating a rectangular container is considered $\Omega = [0,a] \times [0,b]$, on the boundary of which the conditions of impermeability and temperature pro le linear in height are given $T_*(y) = T_2 - y(T_2 - T_1)/b$, Where T_1 Ł T_2 temperature at the top ($y = b$) and bottom ($y = 0$) boundaries, respectively, the force of gravity acts in the direction opposite to the coordinate y. Next, a perturbation of the temperature eld is introduced $T(x,y,t) = T_*(y) + \theta(x,y,t)$ and a transition is made to dimensionless quantities [1]. For stream function ψ and temperature deviations θ the following initial-boundary problem is obtained with respect to the linear pro le:

$$0 = M\psi + \lambda\theta_x = f_1, \qquad \psi|_{\partial\Omega} = \theta|_{\partial\Omega} = 0. \qquad (1)$$

$$\dot{\theta} = L_D\theta - \psi_x - J(\psi,\theta) = f_2, \qquad J = \theta_x\psi_y - \theta_y\psi_x \qquad (2)$$

$$M = \partial y(\mu 11\partial y - \mu 12\partial x) + \partial x(-\mu 21\partial y + \mu 22\partial x), \qquad (3)$$

$$LD = \partial x\,(d11\partial x + d12\partial y) + \partial y\,(d21\partial x + d22\partial y). \qquad (4)$$

Here t time, μ_{ij} the components of the tensor of dimensionless coe cients of reverse permeability, d_{ij} coe cients thermal conductivity, λ Rayleigh ltration number.

Equations (1) (4) supplemented with initial conditions $\theta(x,y,0) = \theta_0(0,y)$.
With $\mu 11 = \mu 22 = d11 = d22 = 1$ Ł $\mu 12 = \mu 21 = d12 = d21 = 0$ from (1) (2) equations are obtained that correspond to the isotropic problem. In this case, the system of equations is kosymmetric according to [2], those there is a vector eld L, which is orthogonal to the vector eld of the problem and does not vanish on a nontrivial stationary solution. IN [1] set the conditions under which the task (1) (4) is cosymmetric, these conditions are clari ed by the following lemma

Lemma. Under the conditions

$$\mu_{11}\, d_{12} = -\mu_{12}\, d_{22}\,, \quad \mu_{11}\, d_{21} = -\mu_{21} d_{22}\,, \quad \mu_{11} d_{11} = \mu_{22}\, d_{22}$$

cosymmetry of the system (1) (4) is a vector function $L = (d_{22}\theta, -\mu_{11}\psi)$.

Mechanical Equilibrium Stability Analysis. Equations (1) (4) satis es zero solution $\theta = \psi = 0$, corresponding to mechanical equilibrium. In the case of $\mu_{12} = \mu_{21} = d_{12} = d_{21} = 0$ from (1) (4) for perturbations, a linear system is obtained
$0 = \mu 11\psi yy + \mu 22\psi xx + \lambda\theta x,\ \psi|\partial\Omega = 0,$ (6) $\dot{\theta} = d11\theta xx + d22\theta yy - \psi x,\ \theta|\partial\Omega = 0.$ (7)
It turns out that critical Rayleigh numbers λ, corresponding to the monotonic instability of mechanical equilibrium, are given by the formula

$$\lambda_{kj} = 4\pi^2\mu_{22}\left(\frac{d_{11}}{a^2}k^2 + \frac{d_{22}}{b^2}j^2\right), \quad k, j = 1, 2, ... \qquad (8)$$

The emergence of the instability of mechanical equilibrium corresponds to the eigenvalue λ_{11}. Critical values of the Rayleigh ltration number λ for isotropic case follow from (8) at $\mu_{22} = d_{11} = d_{22} = 1$. In [2] for the isotropic Darcy problem, it is shown that the rst critical value λ_{11} twice for an arbitrary region, and when the transition parameter λ through λ_{11} from a state of mechanical equilibrium a family of stationary modes branches o (equilibria). The calculations performed in this paper showed that a similar scenario is realized for the Darcy anisotropic problem. The equilibrium family for the plane problem of ltration convection has a variable spectrum, this distinguishes the cosymmetric situation from the symmetric one. Every transition λ through subsequent critical values λ_{kj} corresponds to the bifurcation of the birth of a family of unstable stationary modes.

In case of violation of conditions (5) vector function $L = (d_{22}\theta, -\mu_{11}\psi)$ is not a problem cosymmetry. In this case, instead of a one-parameter family, a nite number of convective regimes are formed. (Stationary or non-stationary) [3 5].

Further numerical research is carried out for the case $\mu_{ij} = d_{ij} = 0$, (i 6= j). Following [3], it turns out selective (selection) function

$$S(\nu) = \int_{\Omega} \psi_x\theta_x(d'_{22}\mu'_{22} - \mu'_{11}d'_{11})dxdy \qquad (9)$$

Here ψ and θ are the solution to the problem (1) (4) on condition $\mu_{11}d_{11} = \mu_{22}d_{22}$ (case of cosymmetry) belonging to the family of stationary modes. Parameter $v$sets parametrization on the family, $v \in [0, 1]$. Symbols μ'_{ii} и d'_{ij} denoted parameter values that di er from those for which the family is calculated. At $\mu^0_{ii} = \mu_{ii}$, $d^0_{ii} = d_{ii}$, ($i = 1, 2$) selective function $S(v) = 0$. If a μ^0_{ii} 6= μ_{ii} and/or d^0_{ii} =6 d_{ii}, ($i = 1, 2$), then there are modes corresponding to the solutions of the equation $S(v) = 0$. If one parameter is disturbed $d'_{11} = d_{11} + \varepsilon$ selective equation (9) will take the following form

$$S(v) = -\varepsilon\mu_{11} \int_{\Omega} \psi_x\theta_x dxdy = 0 \qquad (10)$$

4. Numerical study. In the case of anisotropy, the Darcy problem with cosymmetry in the [1] an analysis of the occurrence of stationary convective regimes, branching o from the loss of stability of mechanical equilibrium, is given. The conditions on the coe cients of the system under which the problem has cosymmetry and the branching of the continuous family of stationary modes are analytically determined. In this paper, the results of calculations of the families themselves are presented on the basis of the scheme [6], and the study of their destruction under violation of the conditions. Table 1 presents the results of calculations of the critical Rayleigh numbers depending

Table 1 Critical Rayleigh numbers of various parameters μ_{ij} Ł d_{22} at d_{11} =1, a =2.5, b =1

μ_{11}	μ_{22}	$d22$	λ_{11}	λ_{21}	λ_1^h	λ_2^h	λ_3^h	λ_4^h	Œ æ
1	1	1	45.795	64.745	46.121	46.121	65.770	65.770	+
1.2	1	1.2	53.691	72.640	54.135	54.135	73.876	73.876	+
1.2	0.8	1.5	52.427	67.587	52.953	52.953	68.856	68.856	+
1.21	1.1	1.1	54.717	75.562	55.139	55.139	76.803	76.803	+
1.1	0.8	1.5			50.909	50.928	66.719	66.879	-
1.3	0.8	1.5			54.932	54.961	70.810	70.935	-

on combinations of parameters of reverse permeability μ_{ii} and thermal conductivity d_{22} Ł d_{11} = 1, a = 2.5, b = 1. The value λ_{11}, λ_{21} is calculated by the formula (8), and the quantities λ_1^h, λ_2^h, λ_3^h meet the rst three critical values on the grid 36×12. In the last column Œ æ¿ notes the ful llment of the conditions of cosymmetry (5).

The rst row of the table answers the isotropic problem [4], the next three lines are cosymmetries in the anisotropic case. The second line corresponds to the conditions given in [1], and the third and fourth conditions (5). Thus, the conditions for the existence of cosymmetry (5) allow you to expand the set of values of the coe cients for which you can apply the formula for calculating the critical Rayleigh numbers [1]. Critical numbers characterize the occurrence of convection as a result of monotonous loss of stability of mechanical equilibrium. The last two lines of the table 1 present the results of calculating the critical numbers when the condition for the existence of cosymmetry is violated. It can be seen that the duplicity of the eigenvalues of the corresponding spectral problem disappears.

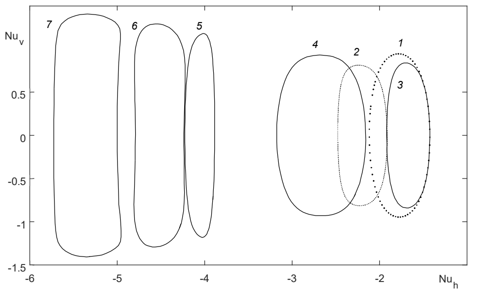

Figure:-1 Stationary mode families with cosymmetry: 1) isotropic case; 2) μ_{11} = d_{22} =1.2, μ_{22} =1; μ_{11} =1.2, μ_{22} =0.8, d_{22} =1.5, λ =90 (3), λ =120 (4), λ =180 (5), λ =210 (6), λ =270 (7); d_{11} =1; d_{11} =1

In fig. 1 calculated families of stationary modes are presented in coordinates Nu_h и Nu_v:

$$Nu_h = \int_0^b \theta_x\big|_{x=a/2} dy, \quad Nu_v = \int_0^a \theta_y\big|_{y=0} dx \qquad (11)$$

Isotropic case corresponds to the curve 1. With changing parameters μ_{11}, d_{22}, μ_{22}, according to the formula (5), cosymmetry is preserved, while the family shifts, see curves 2 Ł 3. With the growth of the Rayleigh number λ the family increases in size and shifts towards negative values Nu_h.

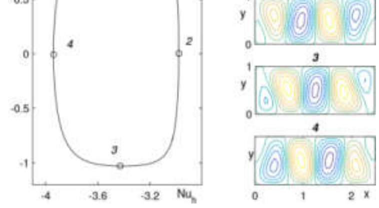

Figure:-2 Stationary families when λ =90 (curve 1), λ =120 (2), λ =150 (3), λ =180 (4), λ =270 (5); μ_{11} = d_{22} =1.2, μ_{22} = d_{11} =1.

In g. 2 on the left is a family curve calculated at $\lambda = 150$. The dots mark the stationary modes, the stream functions of which are given on the right. Depending on the position of the point on the family, the resulting mode consists of four shafts (points 2, 4) or three main and two angular shafts (points 1, 3).

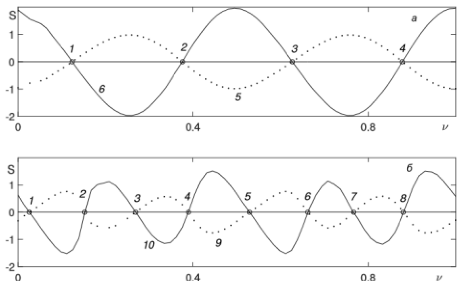

Figure:-3 Graph of the selective function for d_{11} =1.2 (dotted curve), d_{11} =1.5 (solid curve); λ =120, $\mu_{11} = d_{22}$ =1.2, μ_{22} =1.0

In g. 3 graphs of discrete analogue of the selective function are presented (10) for $\mu_{11} = d_{22} = 1.2$, $\mu_{22} = 1.0$ and various meanings d_{11}. When the perturbation of the parameter $\tilde{d}_{11} = d_{11} + \varepsilon$ grid analog of the selective equation (10) has the following form:

$$S(\nu) = \varepsilon \mu_{11} \sum_{i=0}^{n} \sum_{j=0}^{m} \left(\psi_{i+1}^{j}(\nu) - \psi_{i}^{j}(\nu) \right) \left(\theta_{i+1}^{j}(\nu) - \theta_{i}^{j}(\nu) \right) \tag{12}$$

Here through $\psi_i^j\ (\nu), \theta_i^j\ (\nu)$ marked temperature and current function corresponding to the point family of stationary modes with number ν.

REFERENCES

[1]. Abdelhafez M.A., Tsybulin V.G. Anisotropy e ect on the convection of a heat-conducting uid in a porous medium and cosymmetry of the darcy problem // Fluid Dynamics. 2017. . 52. 1. . 49-57.

[2]. Yudovich V. I. Cosymmetry, degeneration of solutions of operator equations, and onset of a ltration convection // Mathematical Notes of the Academy of Sciences of the USSR. 1991. Vol. 49, no. 5. P. 540 545.

[3]. Yudovich V. I. Bifurcations for Perturbations Violating Cosymmetry // Dokl. Ross. Akad. Nauk 398, 57-61 (2004).

[4]. Tsybulin V., Karasozen B. Destruction of the family of steady states in the planar problem of Darcy convection. // Phys. Lett. A. 2008. Vol. 372, N 35. P. 5639 5643.

[5]. Govorukhin V.N., Shevchenko I.V. Multiple equilibria, bifurcations and selection scenarios in cosymmetric problem of thermal convection in porous medium //Phys. D. 2017. Vol. 361. P. 42 58.

[6]. Abdelhafez M.A., Tsybulin V.G. umerical simulation of convective motion in an anisotropic porous medium and cosymmetry conservation // Computational Mathematics and Mathematical Physics. 2017. . 57. 10. . 1706-1719.