

A REVIEW ON GO_{Δ} - GENERALIZED ORDERED TOPOLOGICAL SPACES

Pralahad Mahagaonkar^{1*}

¹Department of Mathematics, Ballari Institute of Technology and Management, Ballari. (India).

***Corresponding Author:-**

Abstract:-

In this paper we defined LOTS or GO spaces, further we established some results on GO_{δ} -spaces on generalized metric classes with help of G_{δ} -diagonal spaces in GO- spaces.

Keywords:-*GO-spaces, locally connected spaces, compactness, and para-compactness.*

INTRODUCTION:

LOTS or a GO space is topological space already equipped with a compatible ordering. Over a years, some effort has been devoted to giving a characterization of those topological spaces for which some compatible ordering can be constructed .Some results are called as orderability theorems. Characterization of the cantor set and space of irrationals might be viewed as orderability theorem (Eventually, any compact, separable connected, locally connected space with at most non-cut points is homeomorphic to $[0,1]$ and is therefore orderable) .However , the first results dealing directly with order ability seem to be the theorems of Eilenberg[1] who proved.

Corollary 1: Any GO space hereditary normal.

Every GO space is collection wise normal by corollary 1 so that, in the light of general theory, many well-know covering properties,

However, there are other properties, not generally equivalent to paracompactness in collection wise normal spaces, which reduce to paracompactness in a GO space.

Main results

Theorem 1.1: Every GO-space is hereditarily S_0 fully normal.

Theorem 1.2: A LOTS is metrizable iff it has a G_δ - diagonal.

Proof: Proving that a LOTS with G_δ -diagonal is Moore space by combing with above statement theorem, completes proof. Unfortunately, Theorem does not hold for an arbitrary GO-spaces. However, the existence of a G_δ -diagonal in a GO space does yield some special structure for the space, eg hereditary paracompactness, can be from one of the theorem and the fact that no stationary set in a regular uncountable cardinal has a G_δ -diagonal.

Alster [3] gave detailed study of the special properties and product theory of GO-spaces having G_δ -diagonal (which is so called as GO_δ -spaces).

Since any paracompact space with a G_δ -diagonal is sub-metrizable , every GO_δ -spaces has a weaker metrizable topology.

Theorem 1.3: If $(X,T, <)$ is a GO-space with a GO_δ -spaces , then there is a metrizable topology $S \subset T$ such that $(X,T <)$ is also a GO-space.

Another example of special structures of GO_δ -spaces is given by Alster's result that, module isolated points, each GO_δ -spaces is perfect, to more precise, Alster[3] showed that if X is a GO_δ -spaces then that X^d , the set of non-isolated points of X , is perfect. As Alster[3] used the special of GO_δ -spaces to obtain results in this theory.

Thus the GO_δ -spaces for LOTS turns out to be a special case for G_δ - diagonal for arbitrary GO spaces.

Theorem 1.4: A GO space is metrizable if and only if it is semi-stratifiable.

Proof : A space X is semistratifiable if corresponding to the each open set G it is possible to choose set $C(G,n)$ such that $G = \cup \{C(G,n): n \geq 1\}$ and such that if the open sets G,H then $C(G,n) \subset C(H,n)$ for every $n \geq 1$.

A related metrization theorem for LOTS was given by Nedev [4] and it can be easily proved the theorem.

Theorem 1.5: Any symmetrizable GO space is metrizable.

Proof: Any symmetrizable space satisfies the GO-spaces. But a first countable symmetrizable space is known to be semi-metrizable and hence semistratifiable . Now by using above theorem.

The Generalized metric space theory for p-spaces and M-spaces. The solution of problems which are not related with metrization theorem. and each class has since been widely used in metrization theory. In genral, the both classes are distinct -there are p-spaces which not Mspaces, and vice versa but VanWouwe [5] have shown that the properties are related in the family of LOTS.

References:

[1] **Eilenberg:** A synthetic molecular trefoil knot . Angew chem..Int,Ed .Engel 28,189192.

[2]**Gillman-Henriksen:** Rings of Continuous functions .Van Nostrad Princeton.

[3]**Alster .K :** Convergence inn topology .Duke Maths .Jol.17.277-283.

[4]**Nedev :** K-spaces, Doctoral dissertation , University of Rochester.

[5]**Van Wouwe :** Fixed and almost fixed points . Mathematical centre tract No.1Mathemaicsch Cntrum,Amsterdam.