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INDEPENDENT DOMINATIO N IN BIPOLAR FUZZY GRAPHS

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Abstract:-

In this paper we introduced and studies the concepts of independent domination of bipolar fuzzy graph G, ($\gamma_i(G)$). And chromatic number χ (G) of bipolar fuzzy graph .We investigated the relationship of γ_i and χ with the other known parameters. Finally we give γ_i for same standard bipolar fuzzy graph.

Keywords:- *Bipolar fuzzy graph independent domination number, chromatic number.*

Classification 2010: 03E72, 05C69, 05C72

1 INTRODUCTION

Zhang In(1994)[11,12] initiated the concept of bipolar fuzzy sets as a general ization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1, 1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0, 1] of an element indicates that the element somewhat satisfies the property and the membership degree [-1, 0) of an element indicates that the element somewhat satisfies the implicit counter property. Akram [1,2,3,4] introduced and studied the notations of bipolar fuzzy graph, bipolar fuzzy graphs with applications, regular bipolar fuzzy graph and metric in bipolar fuzzy graphs. A. Somasundaram and S. Somasundaram [10] introduced and discussed the concept of domination in fuzzy graphs. The inde pendent domination number and irredundance number in graphs are introduced by Cockayne [6] and Hedetniemi [7]. Nagoorgani and Vadivel [8] introduced and discussed the concepts of domination, independent domination and irredundance in fuzzy graphs using strong edges. The concept of domination in Intuitionistic fuzzy graphs was investigated by Parvathi and Thamizhendhi [9]. The concepts of domination, independence and irredundance number in bipolar fuzzy graph by Akarm and at al (2013)[5].

The aim of this paper is to introduce the concepts of independent dominating and chromatic number in bipolar fuzzy graph and we investigate the relationship between this concept and the others in bipolar fuzzy graphs.

2 Preliminaries

Definition 2.1: [1] A bipolar fuzzy graph (BFG) is of the form $G = (V,E)$ where

(i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1^+ : X \to [0,1]$ and $\mu_1^- : X \to [-1, 0]$

(ii) $E \subset V \times V$ where $\mu_2^+ : V \times V \longrightarrow [0,1]$ and $\mu_2^- : V \times V \longrightarrow [-1,0]$ such that

$\mu_{2ij}^+ = \mu_2^+(v_i, v_j) \leq \min(\mu_1^+(v_i), \mu_1^+(v_j))$

and $\mu_{2ij}^- = \mu_2^-(v_i, v_j) \geq \max\ (\mu_1^-(v_i), \mu_1^-(v_j)),\ \forall (v_i, v_j) \in E$

Definition 2.2:[1] A *bipolar fuzzy graph* $G = (V,E)$is called strong if

$\mu_2^+(v_i, v_j) = \min(\mu_1^-(v_i), \mu_1^-(v_j))$

and $\mu_2^-(v_i, v_j) = \max(\mu_1^+(v_i), \mu_1^+(v_j)),\ \forall (v_i, v_j)$

Definition 2.3:[5] Let $G = (V,E)$, be a BFG on V. Let $u,v \in V$, we say that u dominates v in G if there exists a strong edge between them.

Definition 2.4:[5] A subset S of V is called a dominating set in G if for every $v \in V - S$, there exists $u \in S$ such that u dominates v.

Definition 2.5:[5] A dominating set S of a BFG is said to be minimal dominating set if no proper subset of S is a dominating set.

Definition 2.6:[5] Two vertices u and v in a BFG ,$G = (V,E)$, are said to be independent if there is no strong edge between them.

Definition 2.7:[5]A subset S of V is said to be independent set if

$$(\mu_2^+)(u,v) < (\mu_2^+)^\infty(u,v) \text{ and } (\mu_2^-)(u,v) > (\mu_2^-)^\infty(u,v)\ \forall u,v \in S$$

Definition 2.8:[5]An independent set S of BFG, $G(V,E)$, is said to be maximal independent, if for every vertex $v \in V - S$, the set $S \cup \{v\}$ is not independent.

3 independent dominating in bipolar fuzzy graphs

Definition 3.1: A dominating set D in BFG is said to be an independent dominating set if D is an independent.

Definition 3.2: An independent dominating set D of a BFG, D is called minimal independent dominating set if D-{u} is not dominating $\forall u \in D$

Definition 3.3: The Order of bipolar fuzzy graph is denoted by P and is defined as

$$P(G) = \sum_{i=1}^{n}\left(\frac{1+\mu_1^+(v_i)+\mu_1^-(v_i)}{2}\right),\ \text{n is number of vertices in G}$$

Definition 3.4: The minimum fuzzy cardinality taken ever all independent dominating set in bipolar fuzzy graph G is called the independent domination and is denoted by $\gamma_i(G)$.

Definition 3.5: Let G be a BFG, and $u,v \in V(G)$, Then u,v are said to be adjacent there is strong edge between them

Definition 3.6: In a BFG G, a vertex and a edge are said to be incident if a vertex is the end vertex of a edge and if they are incident, then they are said to cover each other.

Definition 3.7: In a BFG G, a set of vertex which covers all the edge is called a vertex cover of G. In other words, a vertex covering set of a BFG ,G is a subset K of V such that every edge of G is incident with a vertex in K. A vertex covering set S of bipolar fuzzy graph G is called minimal covering set if $S-\{v\}$ is not covering set $\forall v \in S$.
The minimum fuzzy cardinality among all minimal vertex covering sets in bipolar fuzzy graph G,is called the vertex covering number and is denoted by $\alpha_0(G)$.

Exampe 3.1: Consider a bipolar fuzzy graph G, given figure 3.1

G :

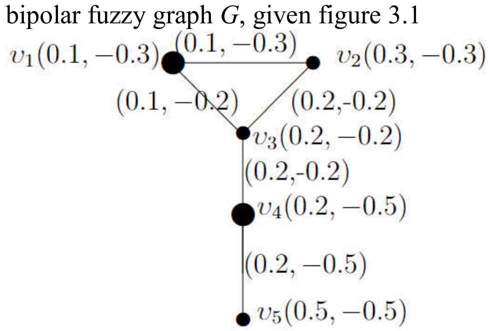

Fig 3.1.

We see that, $D_1 = \{v_1, v_4\}$, $D_2 = \{v_2, v_4\}$, and $D_3 = \{v_3, v_5\}$.

Hence $\gamma_i(G) = min\{|D_1|, |D_2|, |D_3|\} = 0.75$

Theorem 3.1: Let G be a BFG then $\gamma_i(G) \leq p$

Theorem 3.2: Let G be a BFG then $\gamma_i(G) \leq p - \Delta_N$
Proof: Let $v \in V$ such that $d_N(v) = \Delta_N$.
Then $V - N(v)$ is an independent dominating set of G so that $\gamma_i(G) \leq |V - N(v)| = p - \Delta_N$

Theorem 3.3: An independent dominating set D of a BFG,$G = (V,E)$ is a minimal independent dominating set if and only if for each $d \in D$ one of the following conditions holds.
(i) d is not a strong neighbor of any vertex in D
(ii) There is a vertex $v \in V - D$ such that $N(v)^{T}D = d$
Proof: Assume that D is a minimal independent dominating set of D. Then for every vertex $d \in D$,D-d is not an independent dominating set and hence there exists $v \in V - (D - \{d\})$ which is not independent dominated by any vertex in $D - \{d\}$. If $v = d$, we get, v is not a strong neighbor of any vertex in
D. If .v 6= d,v is not dominated by $D - \{v\}$, but is independent dominated by D, then the vertex v is a strong neighbor only to d in D. That is, $N_v{}^{T}D = d$. Conversely, assume that D is an independent dominating set and for each vertex $d \in D$, one of the two conditions holds. Suppose D is not a minimal indepen dent dominating set, then there exists a vertex $d \in D$,$D-\{d\}$ is an independent dominating set. Hence d is a strong neighbor to at least one vertex in $D - \{d\}$, the condition one does not hold. If $D - \{d\}$ is an independent dominating set then every vertex in $V - D$ is a strong neighbor to at least one vertex in $D-\{d\}$, the second condition does not hold which contradicts our assumption that at least one of the conditions hold. So D is a minimal independent dominating set.

Theorem 3.4: An independent set of BFG, G = (V,E), is a maximal indepen dent if and only if it is independent dominating.
Proof: Let D be a maximal independent set in a BFG. Hence for every vertex $v \in V - D$, the set $D \cup \{v\}$ is not independent. For every vertex $v \in V - D$, there is vertex $u \in D$ such that u is strong neighbor to v. Thus D is a dominating set. Hence D is dominating independent set.
Conversely, assume D is independent dominating. Suppose D is not a maximal independent, then there exists a vertex $v \in V - D$, the set $D \cup \{v\}$ is indepen dent. If $D \cup \{v\}$ is independent then no vertex in D is a strong neighbor to v. Hence D cannot be a dominating set, which is a contradiction. Hence D is a maximal independent set.

Theorem 3.5: In BFG G, with out isolated vertices a subset $D \subseteq V$ is an independent set of G if V - D is a vertex covering set of G.
Proof: By definition , D is an independent set of G if and only if no two vertices of D are adjacent ,if and only if every edge of D is incident with at least one vertex of V - D if and only if V - D is a vertex covering set of G.

Theorem 3.6: Let G be a BFG without isolated vertices, then $\alpha_0 + \beta_0 = P$ **Proof:** Let D be an independent set of G with maximum cardinality and K is a vertex covering set with minimum cardinality. Then $|D| = \beta_0$ and $|K| = \alpha_0$. Now, $V - D$ is a vertex covering set of G by theorem (3.5) and K a vertex covering set with minimum fuzzy cardinality .

Therefor $|K| \leq | V - D | - D| \Longrightarrow \alpha_0 \leq P - \alpha_0$.

, Hence $\alpha_0 + \beta_0 \leq P \rightarrow (1)$.

Also V-K is an independent set and D is an independent set of G with maximum fuzzy cardinality. Hence $|D| \geq |V - K|$ $\Longrightarrow \alpha_0 + \beta_0 \geq P \rightarrow (2)$. From (1) and (2) we get $\alpha_0 + \beta_0 = P$.

Theorem 3.7: Let G is a BFG, then $\gamma(G) \leq \gamma_i(G)$ and equality holds if $V(G)$ is independent

Proof: Let D be an independent dominating set of BFG G, then D is domi nating set of G. Hence $\gamma(G) \leq \gamma_i(G)$
if $V(G)$ is independent then, V is the only dominating set of G.
Hence $\gamma(G) = \gamma_i(G)$.
In the following example we explain the above theorem.

Example 3.2: Consider a bipolar fuzzy graph G, given figure 3.2

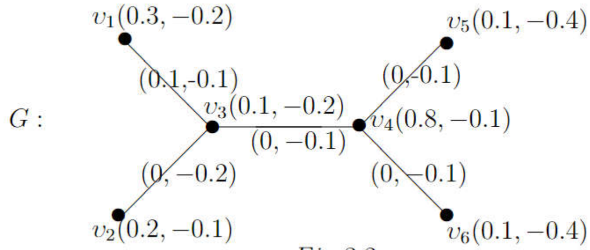

Fig 3.2.

We see that G is independent.

Hence $\gamma(G)$ =0.45+0.35+0.35=1.15=$\gamma_i(G)$

In the following examples we give γ_i for some standards bipolar fuzzy graph

Example 3.3: Consider a strong bipolar fuzzy graphs given in figures (3.3),(3.4), (3.5), (3.6). We have

1. If G is star bipolar fuzzy graph, then γ_i(G)= $\{p - t, t = |u|$, u is a root$\}$
2. If G is complete bipolar fuzzy graph, then γ_i(G)= $\min\{|v_i|, i = 1,2,...,n\}$
3. If G is wheel bipolar fuzzy graph, then γ_i(G) $\leq \min\{\sum_{k=0} |v_{2k+1}|, \sum_{k=1} |v_{2k}|\}$
4. If G is bipartite bipolar fuzzy graph, then γ_i(G)= $\min\{|V_1|, |V_2|\}$

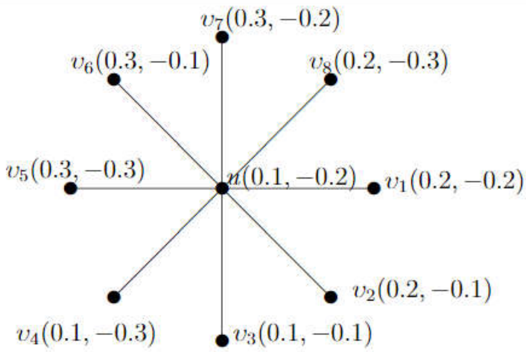

Fig 3.3.

γ_i(G)= $|v_1| = 0.45$

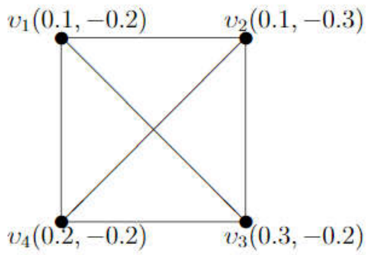

Fig 3.4.

$\gamma_i(G) = 0.4$

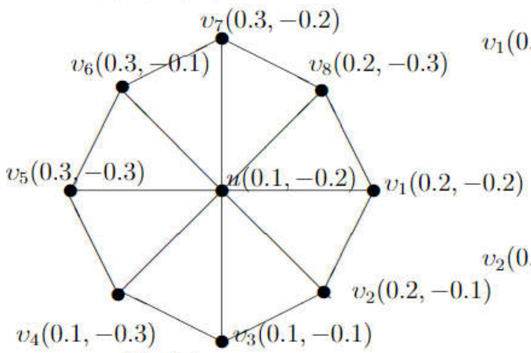

Fig 3.5.

$\gamma_i(G)$=2

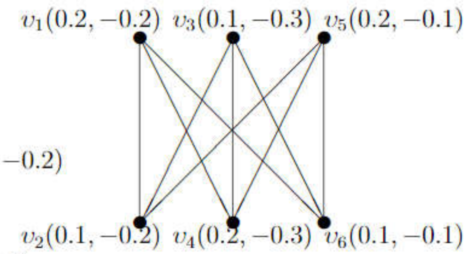

Fig 3.6.

$\gamma_i(G)$=1.4

Definition 3.8:
Let G be a BFG, the minimum number of colours required to colour all the vertices of G such that adjacent vertices do not receive the same colour is called the chromatic number and is denoted by $\chi(G)$.

Exampe 3.4: Consider a bipolar fuzzy tree given in figure 3.5

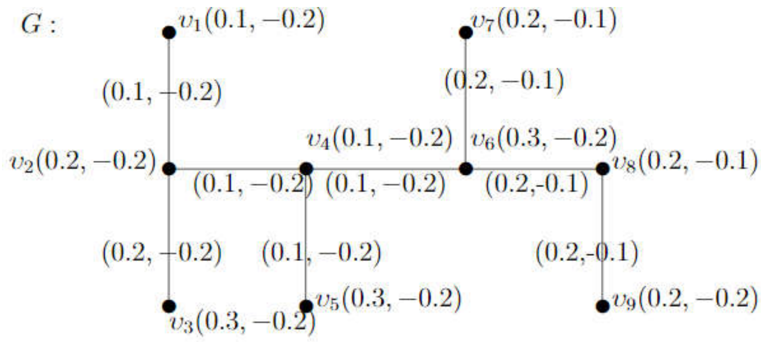

Fig 3.5.

The chromatic number $\chi(G) = 2$

Theorem 3.8: Let G be a BFG, then $\chi(G) \leq$ d$\Delta_N(G)$e.
Proof: Let $v \in V(G)$ such that $d(G) = \Delta_N(G)$, $N(v)$ is the set of colours then

$$\chi(G) \leq N(v) \leq \lceil \Delta_N(G) \rceil.$$

Theorem 3.9: For any connected strong BFG, G $\gamma_i(G) + \chi(G) <$ d2pe
Forther more, equality holds if $V(G)$ is independent
Proof: By theorem (3.1) $\gamma_i(G) \leq p$ and by theorem(3.8) $\chi(G) \leq$ d$\Delta_N(G)$e.
Then $\gamma_i(G)+\chi(G) \leq P$ +d$\Delta_N(G)$e $\leq P$ +dPe $\leq$ d2Pe If V is independent $\Longrightarrow$ V is the only dominating set in G thus V is $\gamma(G) = P$ $\gamma_i(G) + \chi(G) =$ d2Pe

Theorem 3.10: For any connected strong BFG, $G(V,E)$ then $\gamma_i(G) + \chi(G) = \delta_N + 2$ if $G \tilde{=} k_2$
Proof: Let G be a BFG, $G \tilde{=} k_2$
let $d(v_1) = \Delta_N, d(v_2) = \delta_N$
Then $\gamma_i(G) = \delta_N \rightarrow (1)$
$\chi(G) = 2 \rightarrow (2)$
from 1 and 2 we get $\gamma_i(G) + \chi(G) = \delta_N + 2$.

Theorem 3.11: For any connected strong BFG, $G(V,E)$ then $\gamma_i(G) + \chi(G) = P - \Delta_N + 3$ if $G \tilde{=} k_3$
Proof: Let $G \tilde{=} k_3$ suppose that $v \in V(G)$ such that $d_{(v)} = \Delta_N(G)$
Then v has minimum cardinality then $\gamma_i(G) = |v| = |V - d_n(v)|$
$\gamma_i(G) = p - \Delta_N \rightarrow (1)$
$\because \chi(G) = 3 \rightarrow (2)$
From (1) and (2) we get
$\gamma_i(G) + \chi(G) = P - \Delta_N + 3$

Theorem 3.12: For any connected strong BFG, $G(V, E)$ then $\gamma_i(G) + \chi(G) = P - \Delta_N + n$ if $G \tilde{=} k_n$
Proof: Let $G \tilde{=} k_n$ suppose that $v \in V(G)$ such that $d_{(v)} = \Delta_N(G)$
Then v has minimum cardinality then $\gamma_i(G) = |v| = |V - d_n(v)|$
$\gamma_i(G) = p - \Delta_N \rightarrow (1)$
$\because \chi(G) = n \rightarrow (2)$ From (1) and (2) we get $\gamma_i(G) + \chi(G) = P - \Delta_N + n$

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