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THE GLOBAL DOMINATION NUMBER IN PRODUCT FUZZY GRAPHS

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Abstract:-

In this paper we introduced the concepts of global domination number and global domatic number in product fuzzy graph and is denoted by $\gamma_g(G)$ and $d_g(G)$, respectively we determine the global domination number $\gamma_g(G)$ for several classes of product fuzzy graph and obtain Nordhaus-Gaddum type results for this parameter. Further, some bounds of $\gamma_g(G)$ and $d_g(G)$ are investigated. Also the relations between $\gamma_g(G)(d_g(G))$ and other known parameter in Product fuzzy graphs are investigated. Finally we introduce the concept of global full number and some results about this concept in product fuzzy graph are done.

Keyword:- *Product fuzzy graphs, global domination number, global domatic number and global full number.*

1 INTRODUCTION

In (1965)[15], L.A. Zadeh published his seminal paper on, "Fuzzy sets" which described fuzzy set theory and, consequently, fuzzy logic.

This theory essentially proposes graded membership for every element in a subset of a universal set by assigning a particular value for every element in the closed interval [0,1] this value is called the membership degree of such element. Zadeh's ideas have been applied to a wide range of scientific areas such as computer science, artificial intelligence, decision analysis, information science, system science, control engineering, expert systems, pattern recognition, management science, operation research. So also many areas of mathematics have also been touched by fuzzy set theory. The ideas of fuzzy set theory have been introduce into topology, abstract algebra, geometry, graph theory and analysis.

The fuzzy relation were also introduced by L.A. zadeh (1987)[16]. Rosenfeld (1975)[14], introduced the notion of fuzzy graph and several fuzzy analogues of graph theoretic concepts such as path, cycles and connectedness.

The mathematical formal definition of domination in graph was given by Ore(1962)[9].

Cockayne and Hedetnieme (1977)[4] published a survey paper on this topic.

R.B. Allan and R.Laskar(1978)[1] introduced the concept of an independent domination of graph. Mordeson and Nair(1996)[7] introduced the concept of cycles and cocycles of fuzzy graphs. Fuzzy cycle and Fuzzy trees was investigated by Mordeson and Y. Y. Yao(2002)[8]. The concepts of domination in fuzz graphs was investigated by A. Somasundaram, S. Somasundaram [13, 14]. The first definition of global dominating sets in graphs was introduced by Sampathkumar in (1989)[12].

The concepts of global domination number, domatic number and global domatic number in fuzzy graph was introduced by Mahioub shubatah(2009)[6].

The concept of product of fuzz graphs given by V. Ramaswamy (2009)[10]. Mahioub shubatah (2012)[5] introduced the concept of domination in product of fuzzy graph.

In this paper we introduce the concepts of global domination number and global domatic number in product fuzzy graph. we also discuss some theorems and bounds of global domination number in product fuzzy graph.

2 Definitions

In this section, we review briefly some definitions in Graphs, fuzzy graphs, product fuzzy graphs, and domination number in a product fuzzy graph.

A crisp graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex sets and the edges set of G are denoted by $V(G)$ and $E(G)$, respectively.

A fuzzy graph $G = (\mu,\rho)$ is a set V with two function $\mu : V \rightarrow [0,1]$ and $\rho : E \rightarrow [0,1]$ such that $\rho(\{u,v\}) \leq \mu(u) \wedge \mu(v)$ for all $u,v \in V$. We write $\rho(\{u,v\})$ for $\rho(u,v)$. The order p and size q of a fuzzy graph $G = (\mu,\rho)$ are defined to be $p = {}^{\mathrm{P}}{}_{u\in V}\ \mu(u)$ and $q = {}^{\mathrm{P}}{}_{(u,v)\in E}\, \rho(u,v)$.

A subset D of V in a fuzzy graph G is called a dominating set in G if for every $\upsilon \in V - D$, there exists $u \in D$ such that u dominates υ. A dominating set D of a fuzzy graph G is said to be a minimal dominating set $D-\{v\}$ is not dominating set of G for all $v \in D$.

The minimum fuzzy cardinality among all minimal dominating sets is called the domination number of G and is denoted by $\gamma(G)$.

The maximal fuzzy cardinality among all minimal dominating sets is called the upper domination number of G and is denoted by $\Gamma(G)$.

A dominating set D of a fuzzy graph G with $|D| = \gamma(G)$ is denoted by γ *−set* of G. A subset D of V in a fuzzy graph G is said to be global dominating set if D is a dominating set in bath G and complement of G.

The global dominating set D of a fuzzy graph G is said to be minimal global dominating set if $D - \{v\}$ is not global dominating set of G for all $v \in D$. The minimum fuzzy cardinality taken over all minimal global dominating sets in a fuzzy graph is called the global domination number and is denoted by γ_g(G). A global dominating set D of fuzzy cardinality $|D|=\{{}^{\mathrm{P}}\mu(u)$ for all $u \in D\} = \gamma_g(G)$ is denoted by $\gamma_g -$ *set*.

Let D is a $\gamma_g -$ *set* then D is connected if the fuzzy subgraph $< D >$ induced by D is connected.

The connected global domination number of a fuzzy graph G is the minimum cardinality taken over all connected global dominating set of G and is denoted by $\gamma_{cg}(G)$.

A global dominating set D of a fuzzy graph G is called an independent global dominating set if it is also independent.

The minimum fuzzy cardinality taken over all an independent global dominating sets is called an independence global domination number and is denoted by γ_{ig}(G).

A nonempty subset D of V in a fuzzy graph is said to be full set if $N(v)^{\mathrm{T}}(V - \quad D)$ 6= φ for all $v \in D$.

The full number of a fuzzy graph G is the maximum fuzzy cardinality taken over all full sets of G and is denoted by $f(G)$.

A nonempty subset D of V in a fuzzy graph G is said to be global full set if $N(v)^{\mathrm{T}}(V - D)$ 6= φ bath in G and complement of G for all $v \in D$.

The maximum fuzzy cardinality taken over all global full sets of a fuzzy graph G is called The global full number and is denoted by $f_g(G)$.

We say that a nonempty subset D of V in a fuzzy graph is called an irredundant set of G if for each vertex $v \in D$, $N[v] - N[D - \{v\}]$ =6 φ for all $v \in D$.

The minimum fuzzy cardinality taken over all an irredundant sets is called an irredundanc number and is denoted by $iR(G)$.

The maximum fuzzy cardinality taken over all an irredundant sets is called irredundanc number and is denoted by $IR(G)$.

We say that a nonempty subset D of V in a fuzzy graph is called global irredundant set of G if for each vertex $v \in D, N[v]-N[D-\{v\}] =6\ \varphi$ bath in G and complement of G for all $v \in D$.
The minimum fuzzy cardinality taken over all global irredundant sets in fuzzy graph is called global irredundanc number and is denoted by $iR_g(G)$.
The maximum fuzzy cardinality taken over all global irredundant sets in fuzzy graph is called global irredundanc number and is denoted by $IR_g(G)$.

An arc uv of a fuzzy graph G is called strong arc if $COND_{-uv}(u,v) \geq \rho^{\propto}(u,v)$, where the $COND_{-uv}(u,v)$ is the strength of connectedness between u and v.
Let G be a graph whose vertex set is V,μ be a fuzzy subset of V and ρ be a fuzzy subset of $V \times V$, we call (μ,ρ) a product partial fuzzy subgraph of G (in short, a product fuzzy graph) $if \rho(u,v) \leq \mu(u) \times \mu(v)$ for all $u,v \in V$.
A product fuzzy graph $G = (\mu,\rho)$ is called complete product fuzzy graph if $\rho(u,v) = \mu(u) \times \mu(v)$ for all $u,v \in V$.
A product fuzzy graph G is said to be bipartite product fuzzy graph if the vertex set V can be partitioned in to two nonempty sets $V_1 and V_2$ such that $\rho(u,v) = 0$ if $u,v \in V_1$ or $u,v \in V_2$.
We say that a bipartite product fuzzy graph is complete bipartite product fuzzy graph if $\rho(\{u,v\}) = \mu(u) \times \mu(v)$ for all $u \in V_1$, $v \in V_2$.
The complement of a product fuzzy graph $G = (V,\mu,\rho)$ is denoted by $\overline{G} = (V, \overline{\mu}, \overline{\rho})$ where
$\mu = \overline{\mu}$ and $\overline{\rho}(u,v) = \mu(u) \times \mu(v) - \rho(u,v).$
Let $G = (V,\mu,\rho)$ be a product fuzzy graph and $u,v \in V(G)$ then we say u dominates $v if \rho(u,v) = \mu(u) \times \mu(v)$ for all $u,v \in V$
.
Let $G = (V,\mu,\rho)$ be a product fuzzy graph then a vertex subset D of $V(G)$ is said to be dominating set of G if for every vertex $v \in (V-D)$ there exists a vertex $u \in D$ such that $\rho(u,v) = \mu(u) \times \mu(v)$.

The dominating set D of a product fuzzy graph is called minimal dominating set if $D - \{v\}$ is not dominating set of G, for all vertices in D.
The minimum fuzzy cardinality taken over all minimal dominating sets in a product fuzzy graph G is called the domination number of G and is denoted by $\gamma(G)$. A vertex subset D of $V(G)$ in a product fuzzy graph G is called an independent set if $(u,v) nin E(G)$ for all $u,v \in D$. An independent set D in a product fuzzy graph G is called maximal independent if $D \cup \{v\}$ is not independent for all $v \in V(G)$. The maximum fuzzy cardinality taken over all maximal independent sets of a product fuzzy graph G is called an independence number of G and is denoted by $\beta_0(G)$. If $e = (u,v)$ is an edge in a product fuzzy graph G then we say that u and v cover the edge e. A subset D of V is called a vertex cover set of a product fuzzy graph G if all edge e in G their is a vertex v in D such that v cover e.
The minimum fuzzy cardinality taken over all vertex cover sets of a product fuzzy graph G is called a vertex covering number of G and is denoted by $\alpha_0(G)$
A dominating set D of a product fuzzy graph $G = (V,\mu,\rho)$ is called connected dominating set of G if the fuzzy subgraph $< D >$ induced by D is connected. The connected domination number of a product fuzzy graph G is the minimum cardinality taken over all connected dominating sets in G and is denoted by $\gamma_c(G)$. A dominating set D of a product fuzzy graph G is called an independent dominating set if D is an independent.
The independence domination number of a fuzzy graph G is the minimum fuzzy cardinality taken over all an independent dominating sets in G and is denoted by
$\gamma_i(G)$.
We say that a partition of $V(G)$ is a domatic (global domatic) partition of a fuzzy graph G) if D_i is a dominating sets (global dominating sets) of a fuzzy graph G for all i.
The maximum fuzzy cardinality with $||P|| = \sum \frac{\mu(D_i)}{|D_i|}$ taken over all domatic sets (global domatic sets) of a fuzzy graph is called domatic (global) domatic number and is denoted by $d(G)(d_g(G))$.

3 Global domination in product fuzzy graph

The aim of this section is to introduce and study the concepts of global domination, global domatic and global full in product fuzzy graphs.

Definition 1. *Let $G = (V,\mu,\rho)$ be any Product fuzzy graph a vertex subset D of $V(G)$ is called global dominating set of G if D is also a dominating set of the complement of G.*

Definition 2. *A global dominating set D of a product fuzzy graph G is called minimal global dominating set if $D - \{v\}$ is not global dominating set of G for all $v \in D$.*

Definition 3. *The minimum fuzzy cardinality taken over all minimal global dominating sets in a product fuzzy graph G is called the global domination number and is denoted by $\gamma_g(G)$.*

Example 4. *Let $G = (V,\mu,\rho)$ be a Product fuzzy graph given in Figure (3.1).*

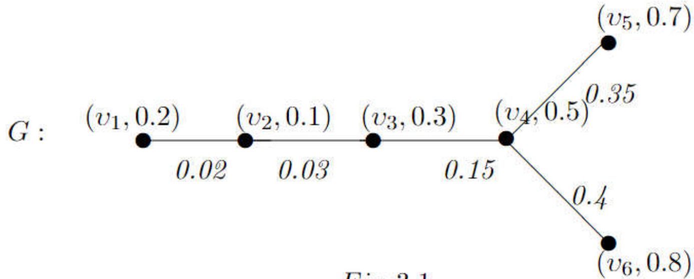

Fig 3.1

We see that the all minimal global dominating sets in the above product fuzzy graph are: $D_1 = \{v_2,v_4\}$, $D_2 = \{v_1,v_3,v_4\}$, $D_3 = \{v_1,v_3,v_5,v_6\}$, $D_4 = \{v_1,v_2,v_3,v_4\}$, $D_5 = \{v_2,v_3,v_4\}$, and $D_6 = \{v_1,v_2,v_3,v_4,v_5,v_6\}$ then the global domination number of G is $\gamma_g(G) = min\{D_1,D_2,D_3,D_4,D_5,D_6\} = min\{0.6,1,2,1.1,0.9,2.6,\} = 0.6$.
In the following results we give γ_g for some standard product fuzzy graphs we begin with the complete product fuzzy graph K_μ.

Theorem 5. *If* $G = (\mu,\rho)$ *is a complete product fuzzy graph then,*

$$\gamma_g(G) = p.$$

Proof. let $G = K_\mu$ be a complete product fuzzy graph then every vertex of G has $(n - 1)$ neighbors. Since the complement of G is the null graph then V is only the global dominating set of G and $\overline{G}$. Hence $\gamma_g(G) = p$. □

The following theorem gives γ_g of the complete product bipartite fuzzy graphs Kn,m.

Theorem 6. *If* $G = K_{n,m}$ *is complete product bipartite fuzzy graph, where* $n = |V_1|$ *and* $m = |V_2|$, *then* $\gamma_g(G) = min\{\mu(u), v \in V_1\} + min\{\mu(v), v \in V_2\}$.
Proof. let G is a complete product bipartite fuzzy graph and let D is minimal global dominating set of G then, every vertex in V_1 is dominated by at least a vertex in V_2 and the vice versa, so that V_1 and V_2 are independent in $\overline{G}$ and every vertex $v \in V_1$ is dominates the other vertices in V_1. Similarly, every vertex $u \in V_2$ dominates the other vertices in V_2. Therefore, $D = \{\{v,u\}; v \in V_1$ and $u \in V_2\}$ is a global dominating set of G. Hence,

$$\gamma_g(G) = min\{\mu(u), v \in V_1\} + min\{\mu(v), v \in V_2\}. \square$$

Theorem 7. *For any product fuzzy graph G,*

$$\gamma_g(G) = \gamma_g(\overline{G})$$

Proof. Let $G = (V,\mu,\rho)$ be any product fuzzy graph and $D$is a minimal global dominating set then, D is a dominating set of G and $\overline{G}$., clearly $\gamma_g(G) = \gamma_g(\overline{G})$.□

Corollary 8. *For any product fuzzy graph G,*

$$(i)\ \gamma(G) \leq \gamma_g(G).$$

$$(ii)\ \gamma(\overline{G}) \leq \gamma_g(\overline{G})$$

Proof. Let G be any product fuzzy graph and D is a minimal global dominating set of G. Therefore D is a dominating set of G. Hence $\gamma(G) \leq |D| = \gamma_g(G)$. Similarly, $\gamma(\overline{G}) \leq \gamma_g(\overline{G})$ □

Theorem 9. *For any product fuzzy graph* $G = (V,\mu,\rho)$ *we have,*

$$\gamma_g(G) \leq p.$$

Further, the equality hold if $\rho(u,v) < \mu(u) \times \mu(v)$ *for all* $u,v \in V(\overline{G})$.

Proof. Let G be any product fuzzy graph. Since V is a global dominating set of G.

Then $\gamma_g(G) \leq |V| = p$. Now if $\overline{G}$ is an independent. Then, V is only the dominating set of G and $\overline{G}$.
Hence, $\gamma_g(G) = p$. □
The following theorem follows directly from the definitions and corollary (8).

Theorem 10. *: For any product fuzzy graph G,*

$$\frac{\gamma + \overline{\gamma}}{2} \leq \gamma_g(G) \leq \gamma + \overline{\gamma}$$

Theorem 11. *: If G is a product fuzzy graph such that G or* $\overline{G}$ *is an independent, then,* $\gamma(G) = \gamma_g(G) = \Gamma(G) = p$.
Now we characterize some product fuzzy graphs with the above property.

(i) $\gamma(P_n) = \gamma_g(P_n), n \geq 6$.

(ii) $\gamma_g(K_\mu) = \Gamma_g(K_\mu)$.

(iii) $\gamma(K_{n,m}) = \gamma_g(K_{n,m})$.

Theorem 12. *For any product fuzzy graph G, if $\overline{G}$ is an independent, then*

$$\gamma_g(G) \geq \beta_0(G).$$

Further, equality holds if G is independent.

Proof. Let D be a global dominating set of G. Let S be an independent set of G, if

$\overline{G}$ is an independent, then $\gamma_g = p$. Hence $\gamma_g \geq \beta_0(G)$. And if G is also independent, then $\gamma_g = \beta_0(G) = p$ □

Corollary 13. *For any fuzzy graph G, if $\overline{G}$ is independent, then $\gamma_g(G) \geq p - \alpha_0(G)$. Further, equality holds if G is independent.*

Theorem 14. *Let G be any product fuzzy graph such that, G 6= K_μ without isolated vertices then,*

$$\gamma_g(G) \leq \beta_0(G).$$

Further, equality holds if $\rho(u,v) < \mu(u) \times \mu(v)$ for all $u,v \in V$.

Proof. Let G be any product fuzzy graph such that, G 6= K_μ. Since G =6 K_μ then by theorem 11 $\gamma_g(G) \ngeq \beta_0(G)$. Hence, $\gamma_g(G) \leq \beta_0(G)$. Now let $\rho(u,v) < \mu(u) \times \mu(v)$ for all $u,v \in V$ then G is an independent. Hence by theorem 11 $\gamma_g(G) \leq \beta_0(G)$. □

Corollary 15. *For any product fuzzy graph G 6= K_μ without isolated vertices,*

$$\gamma_g(G) \leq p - \alpha_0(G).$$

Definition 16. *A global dominating set D of a product fuzzy graphs G is connected if < D > is a connected product fuzzy subgraph of G.*

Definition 17. *A connected global dominating set D of a product fuzzy graphs G is minimal connected global dominating set of G if $D - \{v\}$ is not connected global dominating set for all $v \in D$.*

Definition 18. *The minimum fuzzy cardinality among all minimal connected global dominating sets in product fuzzy graph G is called the connected global domination number and is denoted by $\gamma_{cg}(G)$.*
A connected global dominating set D with $\gamma_{cg}(G) = |D|$ is denoted by $\gamma_{cg} -$ set.

Proposition 19. *For any product fuzzy graph G at lest one of the following holds*

$$(ii)\ \gamma_g(G) \leq \gamma_{cg}(G).$$
$$(ii)\gamma_g(\overline{G}) \leq \gamma_{cg}(G)$$

Proof. Let G be a product fuzzy graph and let D be a $\gamma_{cg} -$ *set* of G then D is a global dominating set of G. Hence, $\gamma_g \leq |D| = \gamma_{cg}$. Similarly, (ii) holds. □

Proposition 20. *For any product fuzzy graph G,*

$$\gamma(G) \leq \gamma_i(G) \leq \gamma_g(G) \leq \beta_0(G).$$

Further, the equality holds if $\rho(u,v) < \mu(u) \times \mu(v)$ for all $u,v \in G$.

Proof. Let G be a product fuzzy graph and D be an independent dominating set of G then, D is a dominating set of G. Hence, $\gamma(G) \leq \gamma_i(G)$. Since, D is an independent. Hence, $\gamma_i(G) \leq \beta_0(G)$. From theorem 14 we get, $\gamma_g(G) \leq \beta_0(G)$. Thus, $\gamma(G) \leq \gamma_i(G) \leq \gamma_g(G) \leq \beta_0(G)$. If $\rho(u,v) < \mu(u) \times \mu(v)$ for all $u,v \in V$ then, $\gamma(G) = \gamma_i(G) = \beta_0(G) = \gamma_g(G)$. □
For the global domination number $\gamma_g(G)$ the following theorem gives a Nordhaus- Gaddum type result

Theorem 21. *For any product fuzzy graph G,*

$$\gamma_g(G) + \gamma_g(\overline{G}) \leq 2p$$

Further, equality holds if $\rho(u,v) < \mu(u) \times \mu(v)$ for all $u,v \in V$.
Proof. Let G be a product fuzzy graph. Since V it self a global dominating set of G, then $\gamma_g(G) \leq |V| = p$ and $\gamma_g(\overline{G}) \leq |V| = p$. Hence, $\gamma_g(G) + \gamma_g(\overline{G}) \leq 2p$. If $\rho(u,v) < \mu(u) \times \mu(v)$ for all $u,v \in V$, then $\gamma_g(G) = \gamma_g(\overline{G}) = p$. Hence $\gamma_g(G) + \gamma_g(\overline{G}) = 2p$
□

Proposition 22. *Let D be a $\gamma -$ set of a product fuzzy graph G, If there exists a vertex v in $V - D$ adjacent to only vertices in D, then*

$$\gamma_g(G) \leq \gamma + \mu(v).$$

Proof. This follows, since $D \cup \{v\}$ is a global dominating set. □

Corollary 23. *If $V - D$ is independent, then the inequality in proposition* (22) *holds.*
Corollary 24. *If $\forall\ v \in V\ \exists\ u \in V$ such that $\rho(u,v) = \mu(u) \wedge \mu(v)$, then the inequality in proposition* (22) *holds.*
Corollary 25. *If G is a tree, then the inequality in proposition* (22) *holds.*
Proposition 26. *Let G be a product fuzzy, if G = T is a tree then,*

$$\gamma_g(T) \le \gamma(T) + \mu(u).$$

Where u is a vertex of maximum membership value.

Proof. Let D is a $\gamma - set$ it follows that, $D \cup \{u\}$ is a global dominating set such that u is of maximum membership of value. Hence $\gamma_g(T) \le \gamma(T) + \mu(t)$. □

Let α_0 and β_0 respectively, denote to the vertex covering and vertex independence numbers of a product fuzzy graph G.

Theorem 27. *Let G be a product fuzzy graph without isolated vertices if for each* $(u,v) \in p*$ *and,* $\rho(\{u,v\}) = \mu(u) \times \mu(v)$ *then,*

$$\gamma_g(G) \le p - \beta_0 + t,\ t = max\{\mu(v), v \in V\}.$$

Proof. Let I be an independent set with $|I| = \beta_0$. Since G has no isolated vertex, $V - I$ is a dominating set of G. Clearly for any vertex $v \in I, \{V - I\} \cup \{v\}$ is a global dominating set of G. Thus, $\gamma_g(G) \le |(V - I) \cup \{v\}| = p - \beta_0 + t, t = \mu(u)$. □

Proposition 28. *For any product fuzzy graph G if* $\rho(u,v) \le \mu(u) \times \mu(v)$ *for all* $u,v \in p*$ *then,*

$$\gamma_g(G) \le \alpha_0(G) + t,\ t = max\{\mu(u) \text{ for all } u \in V(G)\}.$$

Proof. This follows from theorem 27. □

As a consequence of theorem (27) and proposition (20), we have

Theorem 29. *For any product fuzzy graph G of order p without isolates.*

(i) $\gamma_i(G) + \gamma_g(G) \le p + t.$

(ii) $\gamma(G) + \gamma_g(G) \le p + t.$

Proof. Let D be a γ *−set* of a product fuzzy graph G and let $v \in D$ such that, $\mu(u) = \max\{\mu(u)$ for all $u \in V(G)\}$. Then, $V - D \cup \{v\}$ is a global domination set. Therefore, $\gamma_g(G) \le |V - D| + \mu(v) = p - \gamma(G) + t$. Hence, $\gamma_i(G) + \gamma_g(G) \le p + t$. holds Since every independent dominating set of G is a dominating set of G. Hence (ii) holds. □

Theorem 30. *Let T be a product fuzzy tree,* $n \ge 3$ *if T* 6= $K_{1,m}$, $m \ge 2$ *is an integer then* $\gamma_{cg}(T) = p - \omega(T)$, *where* $\omega(T) = |U|$, *U is the set of leaves of T.*

Proof. let T be any tree such that T 6= $K_{1,m}$ we note that T has at lest two leaves, let U be the set of all leaves in T. Since T 6= $K_{1,m}$, $T - U$ is non trivial and connected we note that the fuzzy sub graph induced by $< V(T) - U >$ is a tree so it is connected produced fuzzy subgraph of T as every leaf of T is joined to some vertex in $V(T) - U$ but not all, then $V(T) - U$ is a connected global dominating of T. Thus $\gamma_{cg}(T) \le |V - U| = p - \omega(T)$. □

Theorem 31. *Every global dominating set of a product fuzzy graph G is a global dominating set in crisp graph G* but the converse is not true.*

Proof. Let $G = (V,\mu,\rho)$ be a product fuzzy graph and D is a global dominating set of G then for each $v \in V - D$ their exists a vertex $u \in D$ such that $\rho(\{u,v\}) = \mu(u) \times \mu(v) > 0$ and $\bar{\rho}(u,v) = \bar{\mu}(u) \times \bar{\mu}(v)$ for all $(u,v) \in \bar{\rho}*$, then $(u,v) \in \rho*$. Therefor, v adjacent to u in $G*$ and $\bar{G}^*$ for all $u,v \in \mu^*$. Thus D is a global dominating in G^*. □

To show that the converse of the above theorem is not true we give the following example.

Let $G = (V, \mu, \rho)$ be a product fuzzy graph given in the Figure 3.2

G :

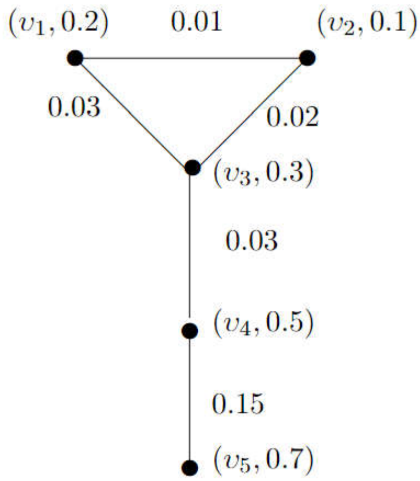

Fig.3.2

The crisp graph $G^* = (\mu^*, \rho^*)$ given in Figure 3.3 the subset D of $V = \{v_3, v_4, v_5\}$ is a dominating set of G^* but is not a global dominating set of G.

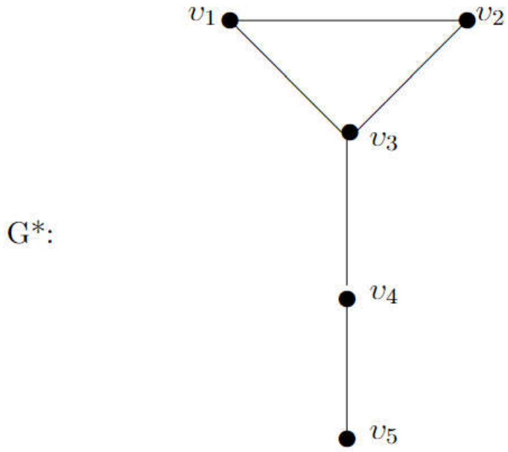

***Fig.* 3.3**

Theorem 32. *Let $G = (V, \mu, \rho)$ be a Product fuzzy graph then, $\gamma_g(G) \leq \gamma_g(G^*)$. Further, equality holds if and only if $\mu(v) = 1$, for all $v \in V(G)$.*

Proof. Let D is a minimal global dominating of a product fuzzy graph G, then by theorem 31, D is a minimal global dominating set of G^*. Hence, $\gamma_g(G) \leq \gamma_g(G^*)$. Furthermore, if D is a global dominating set of G and $\mu(v) = 1$ for all $v \in V(G)$ then, $\rho(u,v) = \mu(u) \times \mu(v)$ for all $(u,v) \in \rho^*$. Then, D is a global dominating set of crisp graph G^*. Hence, $\gamma_g(G) = \gamma(G^*)$. □

4 Full And Global Full Numbers

Definition 33. *Let G be a product fuzzy graph. A vertex subset S of V is called full set if $N(v) \cap (V - S)$ 6= φ $\forall$ $v \in S$. The full number of a product fuzzy graph G is the maximum fuzzy cardinality among of all full sets of G and denoted by $f(G)$.*

Definition 34. *Let G be a product fuzzy graph a vertex subset D of V is said to be global full set (g- full) if $N(v)^{\mathrm{T}}(V - D)$ =6 φ both in G and the complement of G for all $v \in D$.*

Definition 35. *The maximum fuzzy cardinality taken over all global full sets of a product fuzzy graph G is called The global full number and is denoted by $f_g(G)$.*

The following remark follows directly from the definitions

Remark 36. *For any product fuzzy graph $G, f_g(G) = f_g(\bar{G})$.*

Theorem 37. *If G is a product fuzzy graph of order p, then $\gamma(G) + f(G) = p$.*

Proof. Let D be a $\gamma - set$ of G and $v \in V - D$. Then $N(v) \cap D$ =6 φ in G. Hence $V - D$ is a full and $p - \gamma = |V - D| \leq f$. On the other hand. Suppose $S \subseteq V$ is a full with $|S| = {}^{\mathrm{P}}\mu(v)$, $\forall v \in S = f$. Then, for all $v \in S$, $N(v) \cap (V - S)$ 6= φ in G. This implies that $V - S$ is a dominating set. Hence $\gamma \leq |V - S| = p - f$ and the equality holds □

.

Corollary 38. *Let $G = (\mu, \rho)$ be a product fuzzy graph of order p, if V is independent, then $f(G) = 0$.*

Example 39. *Let $G = (\mu, \rho)$ be a product fuzzy graph on V given in the Figure(4.1)* "u

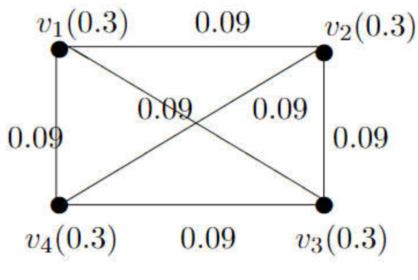

***Fig* 4.1**

We see that $f_g(G) = 0$.

Similarly we have the following result.

Theorem 40. *If G is a product fuzzy graph of order p such that G has at least one effective edge, then $\gamma_g(G) + f_g(G) = p$.*

Proof. Let D be a global dominating set of G with a minimum fuzzy cardinality and $v \in V - D$. Then $N(v) \cap D$ 6= φ both in G and $\overline{G}$. Hence $V - D$ is a g-full and $p - \gamma_g = |V - D| \leq f_g$. On the other hand. Suppose $S \subseteq V$ is a $g - full$ with

$|S| = f_g$. Then, for all $v \in S$, $N(v) \cap (V - S)$ =6 φ both in G and G. This implies that $V - S$ is a global dominating set. Hence $\gamma_g \leq |V - S| = p - f_g$ and the equality holds □

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Corollary 41. *If $F \subseteq V$ is a $g-$full, then $\gamma_g(G) \leq p - |F|$*

.

Corollary 42. *If G has no isolates, and $F \subseteq V$ such that $N(v) \cap N(u) = \varphi$ for all $u,v \in F$, then F is $g-$full, and corollary (41) holds.*

Corollary 43. *If $G = K_\mu$ or $\overline{K_\mu}$, then $f_g = 0$.*

Corollary 44. *Let $G = (\mu,\rho)$ be a product fuzzy graph of order p, if G or $\overline{G}$ is independent, then $f_g(G) = 0$.*
Since $\gamma \leq \gamma_g$, $\gamma(G) + f(G) = p$ and $\gamma_g(G) + f_g(G) = p$ we have

Corollary 45. *for any product fuzzy graph G, $f_g \leq f$.*

Definition 46. *Let G be a product fuzzy graph. The private neighbor of a vertex $v \in V$ with respect to a set S, denoted by $PN[v,S]$, is the set $N[v] - N[S - \{v\}]$. (i.e. $PN[v,S] = N[v] - N[S - \{v\}]$).*
if $PN[v,S]$ 6= φ for some vertex v and some $S \subseteq V$, then every vertex of $PN[v,S]$ is called a private neighbor of v with respect to S.

Definition 47. *We say that a nonempty subset D of V in a product fuzzy graph is called global irredundant set of G if for each vertex $v \in D$, $N[v] - N[D - \{v\}]$ 6= φ bath in G and the complement of G for all $v \in D$.*

Definition 48. *The minimum fuzzy cardinality taken over all global irredundant sets in a product fuzzy graph is called global irredundanc number and is denoted by $iR_g(G)$.*

Definition 49. *The maximum fuzzy cardinality taken over all global irredundant sets in a product fuzzy graph is called global irredundanc number and is denoted by $IR_g(G)$.*

Example 50. *Let $G = (V,\mu,\rho)$ be a Product fuzzy graph given in Figure (4.2).*

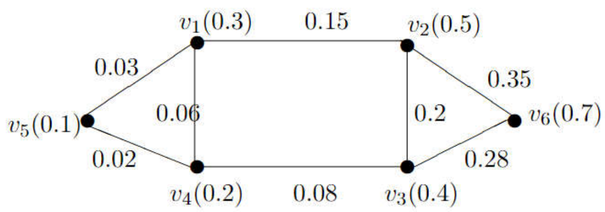

***Fig* 4.2**

We see that the global irredundant dominating sets in product fuzzy graph G are $D_1 = \{v_1,v_2\}$, $D_2 = \{v_2,v_3\}$, $D_3 = \{v_1,v_6\}$, $D_4 = \{v_1,v_4\}$, $D_5 = \{v_2,v_3\}$, and $D_6 = \{v_5,v_6\}$ then the global irredundanc number $iR_g(G) = 0.5$ and $IR_g(G) = 1$.
As a consequence of theorem (37) and theorem (40), we have

Proposition 51. *For any product fuzzy graph G of order p,*

(i) $\gamma(G) \leq P - IR(G)$;

(ii) $\gamma_g(G) \leq P - IR(G)$.

Proof. Let G is a product fuzzy graph and let D be an irredundant set such that $|D| = {}^{\mathrm{P}}\mu(v) forallv \in D = IR(G)$, then for all $v \in D$, $N[v] - N[D - \{v\}]$ 6= φ in G this implies that $V - D$ is a dominating set. Hence, $\gamma(G) \leq |V - D| = P - IR(G)$. (ii) Suppose $D \subseteq V$ is a global irredundant set with $|D| = IR(G)$, then for all

$v \in D$, $N[v] - N[D - \{v\}]$ 6= φ in G and G this implies that $V - D$ is a global dominating set. Hence $\gamma_g(G) \leq |V - D| = P - IR(G)$. □

5 Global domatic number

Definition 52. *We say that a partition $P = \{D_1,D_2,D_3,...,D_m\}$ of $V(G)$ is a domatic (global domatic)partition of a product fuzzy graph G if D_i is a dominating sets (global dominating sets) of a product fuzzy graph G for all i such that $||P|| = \sum \frac{\mu(D_i)}{|D_i|}$ D. is called fuzzy cardinality of P and $|D|$ is the number of vertices in*

Definition 53. *The maximum fuzzy cardinality taken over all global domatic partition of a product fuzzy graph G is called global domatic number and is denoted by $d_g(G)$.*
Let $G = (V,\mu,\rho)$ be a Product fuzzy graph defined as follows:

$V = \{v_1,v_2,v_3,v_4\}, \mu(v_1) = 0.2, \mu(v_2) = 0.3, \mu(v_3) = 0.3, \mu(v_4) = 0.4, \rho(v_1,v_2) = 0.06, \rho(v_2,v_3) = 0.09, \rho(v_3,v_4) = 0.12,$ their is three partition of $V(G)$ in to global dominating sets. The trivial partition $P_1 = \{v_1,v_2,v_3,v_4\}$, $P_2 = \{\{v_1,v_2\},\{v_3,v_4\}\}$,

$P_3 = \{\{v_2,v_3\},\{v_1,v_4\}\}$,

withe $||P_1|| = \sum_1^4 \frac{\mu(D_1)}{|D|} = \frac{1.1}{4} = 0.27, ||P_2|| = \frac{\mu(D_1)}{|D_1|} + \frac{\mu(D_2)}{|D_2|} = \frac{0.2+0.3}{2} + \frac{0.2+04}{2} =$ 0. $55, ||P_3|| = \frac{\mu(D_1)}{|D_1|} + \frac{\mu(D_2)}{|D_2|} = \frac{0.3+0.2}{2} + \frac{0.2+04}{2} = 0.$ 55. Thus $d_g(G) = 0.55$.

Theorem 54. *If G is a product fuzzy graph of order P then,*

$$d_g(G) \le P.$$

Proof. Suppose that $P = \{D_1,D_2,D_3,...,D_m\}$ be a maximum partition. We have $^P\mu(D_i) = P$. Since, any global dominating set is nonempty so $|D_i| > 1$. Hence theProof. □

Theorem 55. *For any complete product fuzzy graph $G = K_\mu$. Then,*

$$d_g(K_\mu) = d_g(\bar{K_\mu}) = \frac{P}{n},$$

n is number of vertices in K_μ.

Proof. Let $G = K_\mu$ is a complet product fuzzy graph. Since, V is the only global dominating set in G. Then the global domatic partition are $||P|| = V$. Therefor, the global domatic partition number $d_g(G) = \sum_{i=1}^{n} \frac{\mu(D_i)}{|D_i|}$. Similarly, $d_g(\bar{K_\mu}) = \frac{P}{n}$, Hence, $d_g(K_\mu) = d_g(\bar{K_\mu}) = \frac{P}{n}$. □

Example 56. *Let $G = (\mu,\rho)$ be a product fuzzy graph on V given in the Figure(5.1)* "u

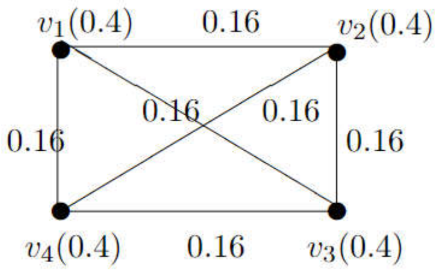

***Fig.* 5.1**

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We see that $d_g(G) = 0.1$.

Theorem 57. *If G is a product fuzzy graph of order P then,*

$$(i)\gamma(G) + d(G) \le P + t,\ t = min\{\mu(v) : v \in V\};$$

$$(ii)\gamma_g(G) + d_g(G) \le \frac{n+1}{n}p.$$

Further, equality hold if and only if $G = K_\mu$.

Proof. Let G be a product bipartite fuzzy graph and let D be a $\gamma - set$ of G, let $D \cup \{v\}$ is a domatic partition. Therefore, $d(G) \le |V - D| + \mu(v) = P - \gamma(G) + t$. Henc, $\gamma(G) + d(G) \le P + t, t = min\{\mu(v) : v \in V\}$. Since $\gamma_g(G) \le P$ and by the fact that $d_g(G) \le \frac{P}{n}$. Hence $\gamma_g(G) + d_g(G) \le \frac{n+1}{n}P$. Now $\gamma(G)+d(G) = p+t$ if and only if $\gamma(G) = min\{\mu(v) : v \in V\}$ and $d(G) = p$ if and only if $G = K_\mu$. Similarly, $\gamma(G) + d_g(G) = \frac{n+1}{n}P$ if and only if $\gamma_g(G) = p$ and $d_g(G) = \frac{P}{n}$ if and only if V is the unique global dominating set of G if and only if $G = K_\mu$. □

Corollary 58. *For any product fuzzy graph G of order p, $\gamma(G) + d(G) \le \frac{n+1}{n}P$, n is the number of vertices in G, and quality holds if and only if $\rho(u,v) < \mu(u)\times\mu(v)$ for all $u,v \in V$.*

Proof. The inequality is trivial. Now $\gamma(G) = p$ if and only if $\rho(u,v) < \mu(u) \times \mu(v)$ for all $u,v \in V$ if and only if V is an independent if and only if $d(G)$ = max $||P||$ $= \sum_{i=1}^{n} \frac{\mu(D_i)}{|D_i|}$ for all i = 1, 2, 3, ...n. Hence, $\gamma(G) + d(G) = p + \frac{P}{n} = (\frac{nP+P}{n}) =$ □

Lemma 59. *If G is a complete product fuzzy graph of order p. Then, $d_g(G)+d_g(\bar{G}) = \frac{2}{n}p$.*

Proof. The equality follows from theorem 55. □

6 Conclusion

In this paper, global domination number and global domatic number are defined on product fuzzy graphs and also applied for the various types of product fuzzy graphs and suitable examples are given. We have done some results with examples and Relations of global domination number and known parameters in product fuzzy graph were discussed with the suitable examples. Further, we introduced and investigated some result of global domatic number in product fuzzy graph and some suitable examples are given.

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