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THE 2-DOMINATION NUMBER IN FUZZY GRAPHS

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Abstract:-
***I**n this paper we focus on 2- domination number of a fuzzy graph G by using effective edge and is denoted by $\gamma_2(G)$ and we obtain some results on this concept, the relationship between $\gamma_2(G)$ and some other concepts are obtained.*

Keywords:-*2-dominating set, 2-dominatic. Classification 2010: 03E72, 05C69, 05C72*

1 INTRODUCTION

Firstly L.A.Zadeh[15] in the year (1965) introduced the notion of fuzzy set firstly. Rosenfeld in(1974)[12] introduced the nation of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths,cycles and connectedness. The theory of dominating in graphs was begun by Ore and Berge [2,11]. Some important works in fuzzy graph theory can be found in[6,7,8]. Cockayne and Hedetniem studied the concept of domination num ber in graphs[3]. The domination number of fuzzy graph was introduced by A.Somasundarma and S.Somasundarma using effective edges [13,14]. Na goorgani and Chandrasekerem in the year (2006) discussed domination in fuzzy graph using strong arcs [9]. The concept of 2-domination in fuzzy graphs was also introduced by Nagoorgani using strong Arcs he considered $\mu(x) = 1, \forall x \in V(G)$ [10]. In this paper we introduce the concept of 2 domination number in fuzzy graphs using effective edges. we obtain some interesting results for this Parameter in fuzzy graphs.

2 Preliminaries

Definition 2.1 *A fuzzy graph $G = (\mu,\rho)$ is a pair of function $\mu : V \to [0,1]$ and $\rho : V \times V \to [0,1]$, where for all $\mu,\rho \in V$, we have $\rho(u,v) \leq \mu(u) \wedge \mu(v)$.*

Definition 2.2 *Let $G = (\mu,\rho)$ be any fuzzy graph and $v \in V(G)$. Then the vertex set $N(v) = \{u; u \in V(G) : \rho(u,v) = \mu(u) \wedge \mu(v)\}$ is called the open neighborhood set of v and the set $N[v] = N(v) \cup [v]$ is called the closed neighborhood set of v.An edge $e = (u,v)$ is called an effective edge if $\rho(u,v) = \mu(u) \wedge \mu(v)$.*

Definition 2.3 *Let $G = (\mu,\rho)$ be a fuzzy graph and $u \in V(G)$. The effective degree of a vertex u is defined as $d_E(u) = {}^{\mathrm{P}}\rho(u,v) : v \in N(u)$. and $d_N(u) = {}^{\mathrm{P}}\mu(v) : v \in N(u)$ is called the neighborhood degree of a vertex u. The minimum effective degree of a fuzzy graph is denoted by $\delta_E(G)$ and is defined as $d_E(G) = min\{d_E(u) : u \in V(G)\}$ and the maximum effective degree of a fuzzy graphs G is denoted by $\Delta_E(G)$ and defined as $\Delta_E(G) = \max\{d_E(u) : u \in V(G)\}$. In a similar way the minimum neigh borhood degree of G is defined as $\delta_N(G) = \min\{d_N(u) : u \in V(G)\}$ and the maximum neighborhood degree of G is defined as $\Delta_N(G) = \max\{d_N(u) :$*

$u \in V(G)\}$.

Definition 2.4 *A fuzzy graph $G = (\mu,\rho)$ is called complete fuzzy graph if $\rho(u,v) = \mu(u) \wedge \mu(v)$, $\forall u,v \in V(G)$ and is denoted by K_μ.*

Definition 2.5 *A fuzzy graph $G = (\mu,\rho)$ is said to be bipartite if the vertex set V can be partitioned into two nonempty subsets V_1 and V_2 such that $\rho(v_1,v_2) = 0$ if $v_1,v_2 \in V_1$ or $v_1,v_2 \in V_2$ further if $\rho(u,v) = \mu(u) \wedge \mu(v)$ for all $u \in V_1$ and $v \in V_2$ then G is called a complete bipartite fuzzy graph and is denoted by $K_{\mu 1,\mu 2}$.*

Definition 2.6 *Let $G = (\mu,\rho)$ be any fuzzy graph and $u,v \in V(G)$, then we say that u dominates v if $\rho(u,v) = \mu(u) \wedge \mu(v)$.*

Definition 2.7 *A vertex subset D of V in a fuzzy graph $G = (\mu,\rho)$ is called dominating set if $\forall v \in V - D$, there exists $u \in D$ such that u dominates v.*

Definition 2.8 *A dominating set D in a fuzzy graph $G = (\mu,\rho)$ is called minimal dominating set if $D - \{v\}$, $\forall v \in D$ is not dominating set of G.*

Definition 2.9 *The minimum fuzzy cardinality taken over all minimal dom inating set is called the domination number of a fuzzy graph G and is denoted*

by $\gamma(G)$.

Observation 2.1 The minimal dominating set D with $|D| = \gamma(G)$ is denoted by $\gamma - set$.

3 Main results

Definition 3.1 A vertex subset D of V in a fuzzy graph $G = (V,\mu,\rho)$is said to be 2-dominating set in G if for every $v \in V - D$, there is at least two vertices u and $w \in D$ such that $\rho(u,v) = \mu(u) \wedge \mu(v)$ and

$$\rho(w,v) = \mu(w) \wedge \mu(v).$$

Definition 3.2 A 2-dominating set D of a *fuzzy graph* $G = (V,\mu,\rho)$ is said to be minimal 2-dominating set of G if $D - \{u\}$ is not 2-dominating set of G for all $u \in D$.

Definition 3.3 The minimum fuzzy cardinality taken over all 2dominating set in a *fuzzy graph* $G = (V,\mu,\rho)$ is called the 2-domination number of G and is denoted by $\gamma_2(G)$ or simply γ_2. The maximum fuzzy cardinality taken over all minimal 2-dominating set in G is called the upper 2-domination number of G and is denoted by $\Gamma_2(G)$ or simply Γ_2. A minimal 2-dominating set D of a *fuzzy graph* G with minimal fuzzy cardinality dented by γ_2 - set.

Example 3.1 Consider a *fuzzy graph* $G = (V,\mu,\rho)$ which given in Figure 3.1.

G :

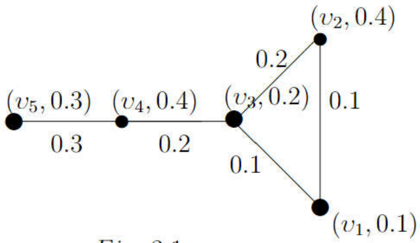

***Fig.* 3.1.**

We see that a vertices subset $D_1 = \{v_1, v_3, v_5\}$ is 2-dominating set further more D is minimal ,and $D_2 = \{v_2, v_3, v_5\}$ is minimal 2-dominating set, so $\gamma_2(G) = \min\{|D_1|, |D_2|\} = \min\{0.6, 0.9\} = 0.6$.
Now we begin with the trivial results.

Theorem 3.1 Every 2-dominating set of *fuzzy graph* $G = (V, \mu, \rho)$ is a dom-inating set of G.

Theorem 3.2 Let $G = (V, \mu, \rho)$ be any *fuzzy graph* and $v \in V(G)$, if v has only one neighbour, Then v belong to every 2-dominating set D of G.
Proof: Let G be a *fuzzy graph* and $v \in V(G)$ has at most one neighbour in G. Let D a 2-dominating set in G .
Cace(i):suppose that v has no neighbours in G. (i.e $N_v = \varphi$).
Then any vertex in D; $|N_v| = 0$ (i.e v is an isolated vertex dose not dominates v).
Cace(ii):suppose that v has only one neighbour in G and suppose that

$$v \notin D. \text{ Then } v \in V - D.$$

Since D is 2- dominating set in G, therefore there are at least two vertices in D, which dominate v. (i.e v has two neighbours u and w) but v has only one neighbor. Thus D is no 2- dominating set. Which is contradiction to assumption. Hence $v \in D$.

Corollary 3.1 if v is an end vertex in a *fuzzy graph* G. Then v must be in every 2- dominating set.

Theorem 3.3 Let $G = (V, \mu, \rho)$ be any *fuzzy graph* and D be minimal 2 dominating set of G if G 6= K_P and G 6= $K_{n,m}$. Then $V - D$ need not be a 2-dominating set of G.
Proof: Let D be 2-dominating set and let $v \in V(G)$, Suppose that v has only one neighbour in G. Then v dominated only by one vertex in $V - D$. Therefore v must be in every minimal 2-dominating set of G. Then $V - D$ has either one neighbour of v or has no any neighbour of v. Thus $V - D$ is not 2-dominating set of G .

Now, suppose that every vertex in D has at least two neighbours in $V - D$ in this case every vertex in D is dominated by at least two vertices in $V - D$. Thus $V - D$ is 2-dominating set of G. Therefore $v \in V - D$ and thus $D - \{v\}$ is not minimal 2-dominating set which is contradict. Hence $V - D$ is not 2-dominating set.To show that the conditions G 6= K_P and G 6= $K_{n,m}$, we give the following examples.

Example 3.2 Let G be a complete fuzzy graph given in the Figure 3.2., where $\mu(v_1) = 0.1, \mu(v_2) = 0.2, \mu(v_3) = 0.3, \mu(v_4) = 0.4$.
We observe that $D_1 = \{v_1, v_2\}$ and $D_2 = \{v_3, v_4\}$
Hence D_1 is 2-dominating set of G. Therefore $V - D_1$ also is 2-dominating set of G. ($v$bu1$_b$,0.1) 0.1 ''u(v_2,0.2)

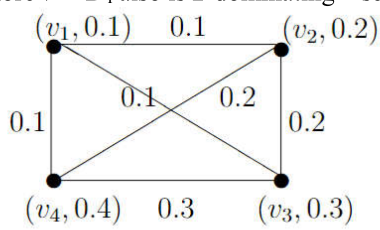

***Fig* 3.2**

Example 3.3 Let $G = K_{\mu 1, \mu 2}$ given in the Figure 3.3, where $\mu(v_1) = 0.1, \mu(v_2) = 0.2, \mu(v_3) = 0.3, \mu(v_4) = 0.4, \mu(v_5) = 0.1$.
We observe that $D_1 = \{v_4, v_5\}$ and $D_2 = \{v_1, v_2, v_3\}$
Hence D_1 is 2-dominating set of G, therefore $V - D_1$ also is 2-dominating set of G.

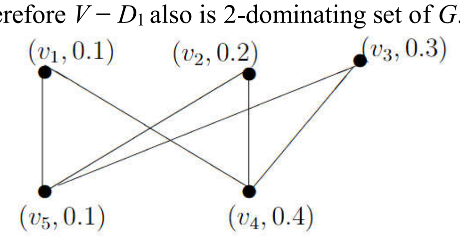

***Fig.* 3.3**

Example 3.4 Let G be a fuzzy graph defined in the Figure3.4, where, $\mu(v_1) = \mu(v_4) = 0.1$, $\mu(v_2) = 0.3$, $\mu(v_3) = 0.2$ and $\rho(u,v) = \mu(u)\wedge\mu(v)\forall u,v \in V$. We see that $D = \{v_1,v_3,v_4\}$, $\gamma_2(G) = |D| = 0.4$ and we see that $V - D$is not 2-dominating set.

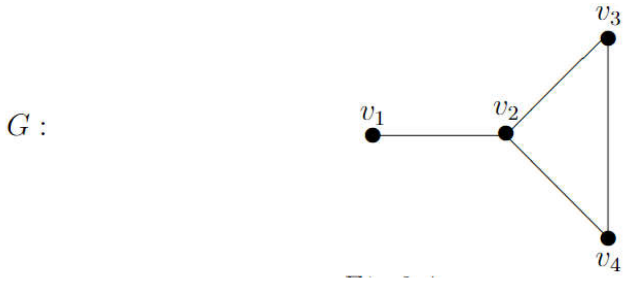

***Fig* 3.4**

In the following results we give $\gamma_2(G)$ for some standard fuzzy graph.

Theorem 3.4 Let G be a star fuzzy graph then $\gamma_2(G) = P - |u|$ such that $d_N(u) = \Delta_N(G)$.
Proof:Let (G) be a star fuzzy graph and $u \in V(G)$ when $d_N(u) = \Delta_N(G)$ then all vertices in graph G have only one neighbor except the vertex u.
Thus $V - \{u\}$ is 2-domination set of G and hence,$\gamma_2(G) = |V - \{u\}| = P - |u|$

Theorem 3.5 Let G is a complete bipartite fuzzy graph then $\gamma_2(G) = \min\{|V_1|,|V_2|\}$
Proof: Let $(G) = K_{\mu 1,\mu 2}$be a complete bipartite fuzzy graph.
Then $\rho(u,v) = 0$,$\forall\ u,v \in V_1$, $\rho(u,v) = \mu(u) \wedge \mu(v)\ \forall\ u \in V_1$ and $v \in V_2$.
We say that every vertex in V_1 dominates all vertices in V_2 and the vice versa. Then V_1 and V_2 are 2-dominating sets of G. Therefor $V_1 and V_2$ are minimal 2-dominating sets of G.
Hence $\gamma_2(G) = \min\{|V_1|,|V_2|\}$, where
$|V_1| = {}^{\mathrm{P}}\mu(u_i);\forall\ u \in V_1$
And$|V_2| = {}^{\mathrm{P}}\mu(v_i);\forall v \in V_2$.

Theorem 3.6 If $G = K_\mu$ is a complete *fuzzy graph*, then $\gamma_2(K_\mu) = \min\{|u| + |v|\}$, $\forall u,v \in V(G)$
Proof:- LetK_μ is a complete *fuzzy graph* then every vertex in K_μ is a neigh bour of all other vertices in G .
Thus any set of two vertices in K_μ will be a 2-dominating set of K_μ. Thus we conclude.

Example 3.5 In Figure 3.5 if G is a complete *fuzzy graph* we see that the set of vertices $D \subseteq V(G)$ where $D_1 = \{v_1,v_2\}$, $D_2 = \{v_1,v_3\}$, $D_3 = \{v_1,v_4\}$, $D_4 = \{v_2,v_4\}$, $D_5 = \{v_2,v_3\}$ and $D_6 = \{v_3,v_4\}$ are minimal dominating set of G. Hence $|D_1| = 0.1+0.4 = 0.5$,$|D_2| = 0.1+0.2 = 0.3$,$|D_3| = 0.1+0.5 =0.6$,$|D_4| = 0.4+0.5 = 0.9$,$|D_5| = 0.4+0.2 = 0.6$ and $|D_6| = 0.2+ 0.5 = 0.7$
Hence $\gamma_2(K_\mu) = \min\{|D_1|,|D_2|,|D_3|,|D_4|,|D_5|,|D_6|\}$.
Then $\gamma_2(K_\mu) = |D_2| = 0.3$
In above we see that $V - D$ is also 2-dominating set in G.

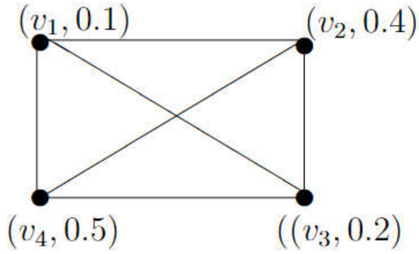

***Fig* 3.5**

Theorem 3.7 For any *fuzzy graph*, if every vertex of G has unique neighbor then $\gamma_2(G) = p$.
Proof: Let G any *fuzzy graph*, since all vertices of G has unique neighbor, then V is only 2-dominating set in G. Hence we conclude.

Corollary 3.2 If $\forall v,u \in V(G)$, $\rho(u,v) < \mu(u) \wedge \mu(v)$ For any *fuzzy graph* then $\gamma_2(G) = p$.

Theorem 3.8 For any *fuzzy graph* $\gamma_2(G)+\gamma(G) \leq 2p$ and equality hold if $\rho(u,v) < \mu(u) \wedge \mu(v)\ \forall v,u \in V(G)$.

Theorem 3.9 Let $G = (V,\mu,\rho)$ be any *fuzzy graph*. If $G^* = nK_2$, then $\gamma_2(G) = np$.
Let G any *fuzzy graph*, such that $G^* = nK_2$,
since K_2 is a complete fuzzy graph with two vertices. Then every vertex in K_2 is an end vertex. Thus every vertex in K_2 is an every 2-dominating set of G.
Therefore every vertex in a *fuzzy graph* G is in every 2-dominating set of G.

And if will be in the minimal 2-dominating set of G. Hence $\gamma_2(G) = np$.

Theorem 3.10 For any *fuzzy graph* $G = (V,\mu,\rho)$, $\gamma_2(G) + \gamma_2(\overline{G}) \leq 2p$, farther equality hold if $0 < \rho(u,v) < \mu(u) \wedge \mu(v)\ \forall u,v \in V(G)$.

Proof: The inequality is trivial since $\gamma_2(G) \leq p$ and $\gamma_2(\overline{G}) \leq p$.
Nwo, since $0 < \rho(u,v) < \mu(u) \wedge \mu(v)\ \forall u,v \in V(G)$.
$0 < \rho(\overline{u},v) = 1 - \rho(u,v) \leq \mu(u) \wedge \mu(v)\ \forall u,v \in V(G)$.
Thus $\gamma_2(G) = p$ and $\gamma_2(G) = p$.
Hence $\gamma_2(G) + \gamma_2(G) = p + p = 2p$.

Theorem 3.11 For any *fuzzy graph* $G = (V,\mu,\rho)$ without isolated vertices
$,\gamma_2(G) \leq \frac{p}{2}.$

Corollary 3.3 Let G be a *fuzzy graph* such that both G and $\overline{G}$ have no

isolated vertices. $\gamma_2(G) + \gamma_2(\overline{G}) \leq p$, farther equality holds if
$\gamma_2(G) = \gamma_2(\overline{G}) = \frac{p}{2}.$

Example 3.6 Consider the *fuzzy graph* $G = (\mu,\rho)$given in Figure3.6
$D = \{v_1,v_3\}$ is 2-dominating set of G, and $\gamma_2(G) = 0.5 + 0.5 = 1 = \frac{p}{2}$u

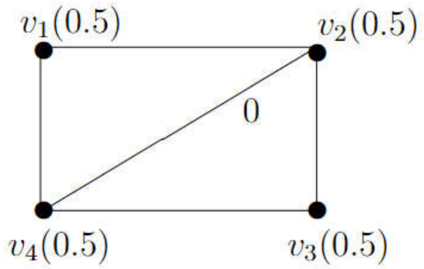

Fig.3.6

Theoram 3.12 For any fuzzy graph G = (V,μ,ρ) if $G \neq K_P,\ \gamma_2(G) \leq p - \delta_N.$
Proof: Suppose that G be any *fuzzy graph* and G 6= K_p.
Let $v \in V(G)$ such that $d_N(v) = \delta_N(G)$ and let D be minimal 2- dominating set of G. Then $V - N(v)$ is 2-dominating set of G.
Hence, $\gamma_2(G) \leq |V - N(v)| = p - \delta_N(G)$.

Corollary 3.4 For any *fuzzy graph* $G = (V,\mu,\rho)$ such that G 6= K_p, $\gamma_2(G) \leq p - \delta_E(G)$.
Proof: Since $\Delta_E \leq \Delta_N$ and $\delta_E \leq \delta_N$.
Then $p - \delta_N \leq p - \delta_E$. Hence by theorem 3.12 $\gamma_2(G) \leq p - \delta_E$.

Corollary 3.5 If $G = K_{n,m}$ complete bipartite *fuzzy graph*, then $\gamma_2(G) = p - \delta_N$.

Definition 3.4 A partition $p = \{D_1,D_2,...,D_m\}$ of $V(G)$ is 2-domatic partition of a *fuzzy graph G* if D_i is 2-dominating sets of $G\ \forall i$.

A fuzzy cardinality of a partition P is denoted by $||p||$ and is defined as $||p|| = \sum_{i=1}^{m} \frac{\mu(D_i)}{|D_i|}$, where $|D_i|$ is number of vertices in D_i, the 2-domatic number of *fuzzy graph G* is the maximum fuzzy cardinality of $||P||$. That is,

$$d_2(G) = max\{||p_i||\} = max\{\sum_{i=1}^{m} \frac{\mu(D_i)}{|D_i|}\}.$$

Example 3.7 Let $G = (V,\mu,\rho)$ be a *fuzzy graph* such that
$V = \{v_1,v_2,v_3,v_4,v_5\}$ and $\mu(v_1) = 0.3$,$\mu(v_2) = 0.4$,$\mu(v_3) = 0.2$, $\mu(v_4) = 0.5$,$\mu(v_5) = 0.1$,and $\rho(u,v) = \mu(u) \wedge \mu(v), \forall u,v \in V(G)$.
There are the following partitions of $V(G)$ into 2-dominating set .
$p_1 = \{v_1,v_3,v_5\},\{v_2,v_4,v_5\}$, $p_2 = \{v_2,v_4,v_5\},\{v_1,v_3,v_5\}$, $p_3 = \{v_1,v_3,v_4\},\{v_2,v_4,v_5\}$, $p_4 = \{v_1,v_2,v_3,v_5\}$ with,

$$||p_1|| = \frac{0.3+0.2+0.1+0.4+0.5+0.1}{3} = 0.2 + 0.33 = 0.53$$

$$||p_2|| = \frac{1+0.6}{3} = 0.8$$

$$||p_3|| = \frac{1+1}{3} = 0.67$$

$$||p_4|| = \frac{0.3+0.4+0.2+0.1}{4} = 0.25$$

Hence $d_2(G) = 0.67$

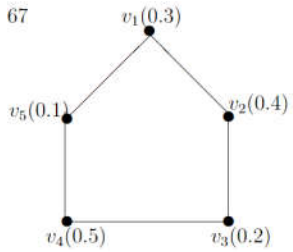

***Fig* 3.7**

In the following theorem we give the relationship between 2-domination number of a fuzzy graph G and crisp graph G^*.

Theorem 3.13[5] Every dominating set of a *fuzzy graph* $G = (V,\mu,\rho)$ is a dominating set of a crisp graph G^* but the converse is not true.

Theorem 3.14[5] Let $G = (V,\mu,\rho)$ be *fuzzy graph*. A dominating set D of $G^* = (\mu^*,\rho^*)$ is a dominating set of G if $\rho(u,v) = \mu(u) \wedge \mu(v)\ \forall u,v \in V(G)$. Consequently with the above results we get the following;

Theorem 3.15 Every 2-dominating set D of a *fuzzy graph* $G = (V,\mu,\rho)$ is a 2-dominating set of G^* but the converse is not true.
proof: Let $G = (V,\mu,\rho)$ a *fuzzy graph* and D is 2-dominating set of G, then for each $v \in V - D, v$ has at least two neighbours in D,$(\exists u,w \in D$ such that $\rho(u,v) = \mu(u) \wedge \mu(v) > 0, uv \in \rho^*$ and $\rho(v,w) = \mu(v) \wedge \mu(w) > 0, vw \in \rho^*$. Therefor v has two neighbours in D. Hence the theorem.
The following example shew that the converse of the above theorem is not true.

Example 3.8 Let $G = (V,\mu,\rho)$ be a *fuzzy graph* given in Figure3.8 and $G^* = (\mu^*,\rho^*)$ given in Figure 3.9

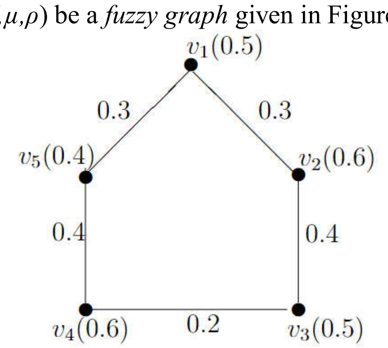

Fig 3.8

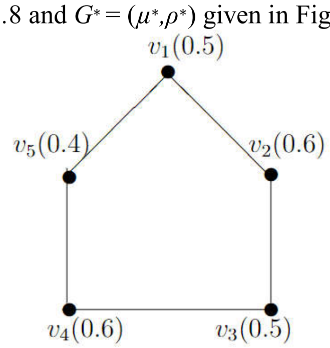

Fig. 3.9

We see that $D = \{v_1, v_3, v_5\}$ is 2-dominating set of G^* but is not 2-dominating set of G.

Theorem 3.16 Let $G = (V,\mu,\rho)$ be a *fuzzy graph*.
A 2-dominating set D of $G^* = (\mu^*,\rho^*)$ is 2-dominating set of a *fuzzy graphs* G if $\rho(u,v) = \mu(u) \wedge \mu(v) \forall u,v \in V(G)$. **proof:** Let D be $\gamma_2 -$ *set* of $G^* = (\mu^*,\rho^*)$, then $\forall v \in V - D$, there exists two vertices u and $v \in D$ such that $(u,v) \in \rho^*$ and $(v,w) \in \rho^*$. Therefor $\rho(u,v) > 0$ and $\rho(v,w) > 0$ and since $\rho(u,v) = \mu(u) \wedge \mu(v)$ and $\rho(v,w) = \mu(v) \wedge \mu(w)$. Thus v has two neighbours in D. Hence D is 2dominating set of G.
A consequence of the above theorem we have the following result.

Theorem 3.17 Let $G = (V,\mu,\rho)$ be a *fuzzy graph*, then $\gamma_2(G) \leq \gamma_2(G^*)$.
Furthermore equality hold if $\rho(x,y) = 1 = \mu(x) \wedge \mu(y) \forall (x,y) \in \rho^*$.
proof: Let D be 2-dominating set of a *fuzzy graph* G, then by theorem 3.15 D is 2-dominating of a crisp graph $G^* = (\mu^*,\rho^*)$. Hence $\gamma_2(G) \leq \gamma_2(G^*)$. If D is 2-dominating set of G^* and $\rho(x,y) = \mu(x) \wedge \mu(y) = 1$, then by theorem3.16 D is 2-dominating set of a fuzzy graphs G. Hence $\gamma_2(G) = \gamma_2(G^*)$.

Definition 3.5[4] A *subset* $S \subseteq V(G)$ *is* a *double dominating set of* G if S *dominates every vertex* of G at least *twice*. The double domination number of G is the minimum fuzzy cardinality of double dominating sets of G and is denoted by $\gamma_{dd}(G)$ or simply γ_{dd}.

Theorem 3.18 Every double dominating set D of *fuzzy graph* G is a 2- dominating set of G.
proof: Let $G = (V,\mu,\rho)$ be any *fuzzy graph* and D is double dominating set of G. Then $\forall v \in V(G)$ there exists at least tow vertices u and w in D such that $\rho(u,v) = \mu(u) \wedge \mu(v)$ and $\rho(v,w) = \mu(v) \wedge \mu(w)$. Then $\forall v \in V - D$ there exist at least tow vertices u and w in D such that u and w dominate v. Thus D is 2-dominating set of G.

Corollary 3.6 For any *fuzzy graph* $G, \gamma_2(G) \leq \gamma_{dd}(G)$.

Theorem 3.19[1] For any graph G, $\lceil \frac{p}{\Delta_E+1} \rceil \leq \gamma(G) \leq n - \Delta_E(G)$.

Theorem 3.20 For any *fuzzy graph* $\gamma_2(G) \geq \frac{p}{\Delta_E+1}$.

proof: Since $\lceil \frac{p}{\Delta_E+1} \rceil \leq \gamma(G)$ by theorem 3.19 and $\gamma_2(G) \geq \gamma(G)$, then

$$\gamma_2(G) + \gamma(G) \geq \gamma(G) + \frac{p}{\Delta_E+1} \Longrightarrow \gamma_2(G) \geq \frac{p}{\Delta+1}.$$

Corollary 3.7 For any fuzzy graph, $\gamma_2(G) \geq p - \Delta_E(G)$.

Corollary 3.8 Let G a *fuzzy graph* then $\gamma_2(G) \leq \gamma(G) + \beta_\circ(G)$

Theorem 3.21 Let G a *fuzzy graph* then $\gamma_2(G) \leq \frac{\gamma(G)+3\beta_\circ(G)}{2}$.
proof: let G be any fuzzy graph, since $\gamma_2(G) \leq \gamma(G) + \beta_\circ(G)$ and $\gamma_2(G) \leq 2\beta_\circ(G)$.
Then $2\gamma_2(G) \leq \gamma(G) + 3\beta_\circ(G)$. Thus we conclude.

Theorem 3.22 Let G a *fuzzy graph* and G is a cycle with n vertex, then the 2-domination number of G given by:
$\gamma_2(G) = min\{|D_1|, |D_2|\}$, such that $|D_1| = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \mu(v_{1+2i})$ and

$$|D_2| = (\sum_{i=1}^{\frac{n}{2}} \mu(v_{2i})) \cup min\{v_1, v_n\}.$$

Example 3.9 Let $G = (V, \mu, \rho)$ be a *fuzzy graph* and G is a cycle, in Figure 3.10 we see that $D_1 = \{v_1, v_3, v_5\}$, then $|D_1| = 0.1 + 0.3 + 0.2 = 0.6$ and $D_2 = \{v_2, v_4\} \cup min\{v_1, v_5\}$, then $|D_2| = 0.3 + 0.1 + min\{0.1, 0.2\} = 0.5$

$\Longrightarrow \gamma_2(G) = min\{|D_1|, |D_2|\} = 0.5$.

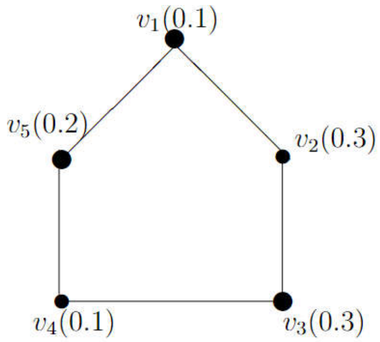

***Fig.* 3.10**

Theorem 3.23 If P_n is a fuzzy path with n vertices and $\mu(v) = t \ \forall v \in V, t \in [0,1]$ then

$$\gamma_2(G) = \begin{cases} \sum_{i=1}^{n} \frac{p+t}{2} & \text{if } n \text{ odd;} \\ \sum_{i=1}^{n} \frac{p}{2} + t & if\ n\ even; \end{cases}$$

Example3.10 G= (V,μ,ρ) be a fuzzy graph and G is a path, in Figure 3.11 we see that $D = \{v_1, v_3, v_5\}$ is minimal 2-dominating set with $\gamma_2(G) = |D| = 0.1 + 0.1 + 0.1 = 0.3$.
and by using theorem 3.23, we have $\gamma_2(G) = \sum_{i=1}^{n} \frac{p+t}{2} = \frac{0.5+0.1}{2} = 0.3$.

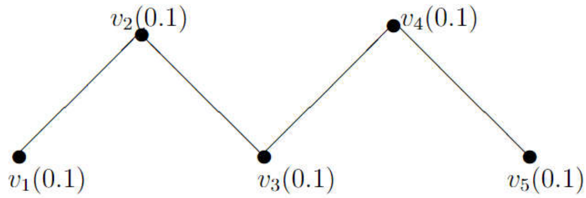

***Fig.* 3.11**

And in Figure 3.12 we see that $D = \{v_1, v_3, v_5, v_6\}$ is minimal 2-dominating set with $\gamma_2(G) = |D| = 0.1 + 0.1 + 0.1 + 0.1 = 0.4$, and by using theorem 3.23, we have $\gamma_2(G) = \sum_{i=1}^{n} \frac{p}{2} + t = \frac{0.6}{2} + 0.1 = 0.4$.

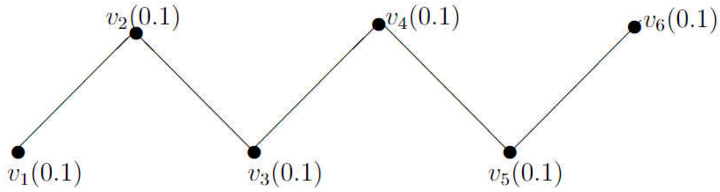

***Fig* 3.12**

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