EPH - International Journal of Mathematics and Statistics
ISSN (Online): 2208-2212
Volume 02 Issue 02 October.2016

DOI:https://doi.org/10.53555/eijms.v2i2.5

METHOD OF ESTIMATING JOINT USE OF GROUNDWATER IN TRANSBOUNDARY AQUIFERS ON MATHEMATICAL MODELS

Md. Irina Polshkova*
**Water Problem Institute of Russian Academy of Sciences, 119333, Moscow, Russia*

***Corresponding Author:-**
E-mail:- z_irpol1@mail.ru

Abstract:-

The methodological approach to solving the problems of optimal sharing of transboundary groundwater is determined as a result of mathematical modelling. Differential equation reproduces the confined-unconfined filtration of groundwater in the multi-stratified system as the basic approximation. Must be considered a single flow of underground waters, whose existence and properties depend on the areas of supply and discharge, which may be located in the territories of different countries. For regulation of transboundary water use the following criteria are suggested: quantitative estimation of ground and surface water depletion due to long time water discharge, calculation of extending time from contamination sources, identification of unknown location a possible source of contamination by groundwater flow velocity inversion. As an example, the results of modelling of cross-border flows underground in the territories Russia-Ukraine and Russia-Kazakhstan are given. The hydrodynamic math models have to be a subsystem of the total monitoring. Using such approach, both geological service and government authorities meet the real instrument for assessing and forecasting conditions of underground hydrosphere and efficient regulating of the anthropogenic load.

Key words:-*Transboundary groundwater flows, hydrodynamics math models, extending of contamination.*

INTRODUCTION

Research and forecasting of hydrogeological processes in transboundary aquifers of neighboring countries demands an exact quantitative estimation, especially in case of anthropogenic load increasing on groundwater. The solution to this problem is entirely the creation of mathematical models, taking into account that groundwater areas recharge and discharge may be present in the territories of different countries. The actual monitoring data on the territories of neighboring countries are required to assess the adequacy of the model, so you must have an agreement for their use at the inter-State. Information on allocation of existed or planned sources of possible groundwater pollution is of critical importance. Thorough analysis of the hydrogeological situation in the near-boundary zone is necessary, which have to work experts from neighboring countries. Only such analysis can show which of the bordering countries breaks the natural transboundary groundwater flow and inflicts damage to the neighbor.

II. STUDY METHODS

In general case the law of groundwater movement at confined-unconfined geofiltration, can be described by the system of nonlinear differential equations, the dimension of which is determined by the number of simulated aquifers and commonly for aquifer *n* can be written as:

$$\frac{\partial}{\partial x}\left(T_{xn}\frac{\partial H_n}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_{yn}\frac{\partial H_n}{\partial y}\right)+\frac{\bar{K}_{n-1}}{M_{n-1}}\left(H_{n-1}-H_n\right)+\frac{\bar{K}_n}{M_n}\left(H_{n+1}-H_n\right)+Q_2+Q_3+W_n=\mu_n\frac{\partial H_n}{\partial t}$$

The basic equation is reduced to a kind having simple physical sense - the sum of flow rates in each point ***i*** of aquifer ***n*** equals to 0 in natural conditions or difference in capacity in the broken:

$$\sum_i Q^n_{xi}+\sum_i Q^n_{yi}+\sum_i Q^{n-1}_{zi}+\sum_i Q^{n+1}_{zi}++\sum_i Q^n_{2i}+\sum_i Q^n_{3i}+\sum_i Q^n_{wi}=\sum_i Q^n_{ci}(1)$$

Where in each node point ***i*** of aquifer ***n*** are calculated specific flow rates, [m/day]: water conductivity along Y,
The water conductivity at calculated node point can be determined by top and bottom absolute marks of aquifer by the following relationship:

$$T^i_{xn}=\begin{cases} k^i_{xn(yn)}\left(z^i_{tn}-z^i_{bn}\right) \text{ if } H^i_n\ge z^i_{bn}; \\ k^i_{xn(yn)}\left(H^i_n-z^i_{bn}\right) \text{ if } z^i_{tn}\le H^i_n\le z^i_{bn}; \\ 0 \qquad \text{if} \quad H^i_n\le z^i_{bn} \end{cases}$$

The coefficient of lateral filtration $k_{xn(yn)}$ is determined as average characteristic typical to deposits of aquifer,

- $Q^{n-1}_{zi}=\frac{k^i_{zn-1}}{m^i_{zn-1}}\left(H^i_{n-1}-H^i_n\right);\ Q^{n+1}_{zi}=\frac{k^i_{zn}}{m^i_{zn}}\left(H^i_{n+1}-H^i_n\right)$ -

Vertical flows from adjacent aquifers,
$k_{zn}{}^i$ - vertical filtration coefficient of semi permeable aquifer *n* in point *i,*
$m_{zn}{}^i$ - thickness of semi permeable layer *n* in point *i,*
$H_n{}^i{}_{\square 1}$- groundwater level in under lying aquifer,
$H_n{}^i{}_{\square 1}$- groundwater level in above lying aquifer,
$Q_{wi}{}^n$ - infiltration,

- $Q_2{}^n{}_i$ - intensity of groundwater extraction, this function may be time dependent,
- $Q_3{}^n{}_i \square (H_{sn}{}^i \square H_n{}^i) G_{sn}{}^i$ - specific flow rate of groundwater/surface interaction: $H_{sn}{}^i$ - surface water level,

$G_{sn}{}^i$ - conductivity of river-bed deposits, $H_{sn}{}^i$ and $G_{sn}{}^i$ may be time dependent,

- $Qci^n \square\square n^i \frac{\square H^n i}{\square t}$ - changes in the capacity for broken

filtration regime,
$\square_n{}^i$ - effective value of aquifer capacity (elastic or gravitational) has different values for confined and unconfined aquifers,

- $H_n{}^i$ - function of water pressure head in point *i* of aquifer *n.*

It's important to emphasize – the strict decision of this equation is possible for the approximation assuming that flows in water bearing aquifer have lateral character while in relatively water-resisting layer they are vertical [1]. The equation is solved by finite difference method, the author's algorithm of numerical solution allows to display on the model both stationary and nonstationary processes of geofiltration.
The result of numerical solution of equation system (1) in such settings is the field of groundwater head levels for each aquifer at assigned time moments according to accepted calculated hydrogeological schematization and model discreteness on the space and time.
Another result of numerical modeling is the possibility to calculate separate components of groundwater flow balance at every calculated time-step for any combination of model net points:

- the water exchange between the water-bearing aquifers;
- decreasing or increasing of aquifer capacitive reserves

(for the broken processes);

- groundwater/surface water interaction;

The described setting of groundwater flows math modelling is realized in the author's software "Aquasoft" [2].

III. THE PROBLEMS TO BE SOLVED FOR JOINT USE OF GROUND WATER

Now transboundary problems become especially acute for bordering territories of former soviet republics. But all studies have been focused mostly on hydrological objects. The first attempt to estimate the simultaneous exploitation of aquifers of near-border zones was made on the region Russian-Estonian boundary [3].

According described methodology the next problems should be solved for transboundary aquifers:

- assessment and forecasting of admissible depletion of groundwater,
- assessment and forecasting of admissible damage to underground component of river runoff as a result of longtime groundwater extraction [4],
- assessment and forecasting of contaminated areas extended in groundwater from possible pollution sources, and also tracking their relative dynamics.

To provide optimal schemes of shared groundwater use in accordance with suggested criteria the following data are output from the model database:

assessment of groundwater depletion:

- maps of aquifer water head levels for calculated time steps allowing to give quantitative assessment of direction, velocity and time of depression cones spreading towards administrative boundaries;
- graphs of level lowering in administrative boundaries net points of groundwater flow;
- graphs of the full amount of lateral groundwater flow if depression cone spreads on the territory of the neighboring country; *assessment of surface water depletion:*
- graph of value variations in water exchange in groundwater/surface water interaction in natural and broken conditions;
- values of damage to groundwater component of river runoff for all surface water sources; *assessment of groundwater contamination:*
- maps of time and areas of extending contaminated groundwater from the surface sources of pollution to estimate protection areas of exploited or planned water intake wells;
- identification of unknown location a possible source of contamination by groundwater flow velocity inversion if it's detected the contaminants in groundwater samples taken from aquifers.

In a number of cases, a major factor defining character and rate of migrant movement within groundwater flow is convectional mass transfer which develops according to geofiltration process. The most simple calculated scheme of the mass transfer process in homogeneous groundwater is so called "scheme of piston displacement", where it is considered that between "clean" and "contaminated" water there is the border section, continued throughout the reporting period. The pollution moves in space, each conditional water particle at the front of pollution is moving with a speed determined by the speed of geofiltration and volumetric porosity aquifers and layers active species –*S, S'*, (molecular diffusion, gidrodispersion and other mass transfer mechanisms do not count). Speeds of migrant V_X, V_Y, V_Z, based on directions *grad H* are calculated in each point of modelled area:

$$V_x = -\frac{k_x}{S}\frac{H(x+\Delta x, y) - H(x, y)}{\Delta x};$$

$$V_y = -\frac{k_y}{S}\frac{H(x, y+\Delta y) - H(x, y)}{\Delta y};$$

$$V_z = -\frac{k_z}{S'}\frac{H_{n+1}(x, y) - H_n(x, y)}{m'}$$

The time of displacement of front contamination in each nodal point is determined on the basis of the analysis of time values of displacement migrant to a given nodal point from the 10 neighboring in accordance with the direction of the velocity vector. Minimum of these values is calculated and this value is considered as the time of displacement front in a given nodal point from the nearest neighbor [2] along possible routes is shown in fig 1.

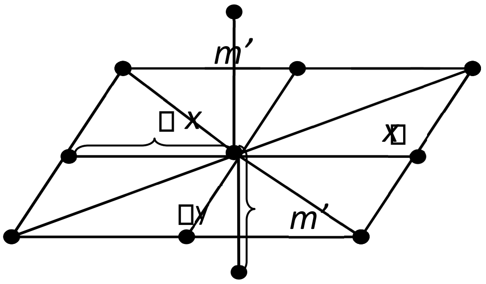

Figure 1: Design scheme of the migrant displacement routes

Isochrons of travel time values give the configuration of pollution front and the areas of extend of contamination areas, as shown in fig 2.

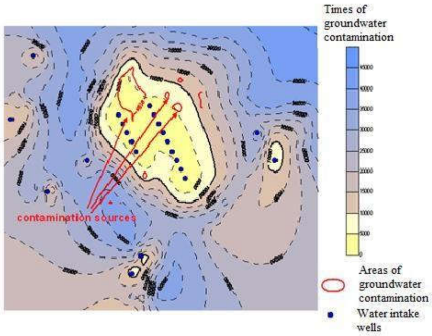

Figure 2: Map of time and areas extending from initial sources of contamination.

IV. NUMERICAL MODELS

A. Russian-Kazakhstan model

The described metodology was used to assess the joint exploitation of aquifers of the Belgorod region of Russia and Kharkov region of Ukraine as well as Russia and Kazakhstan.

The model area of Russia-Kazakhstan bordering zone is a part of West-Siberian artesian basin of 800×550 km in plane [5]. Three-layer hydrogeological schematization including first from surface Atlim aquifer and two aquifers of cretaceous deposits has been accepted for mathematical model with dimensions 170 × 140 node points in aquifer. The aquifer's borders are natural borders of aquifer outcrop on earth surface. All node points have exact geographical coordinates and modelling was carried out for not rectangular (quadrangular) blocks. Natural conditions are assumed to be on 1950 year. Broken conditions reflect the exploitation of cretaceous aquifers until 2000 year The water head map of upper cretaceous horizon is shown in fig 3. The agreement of model results and observed data is satisfactory.

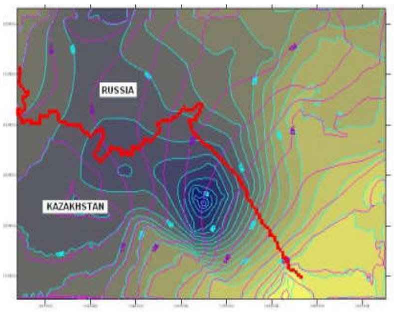

Figure 3: Map of the model water heads for broken groundwater flow conditions.

The total groundwater extraction in city of Kazakhstan from two aquifers with value 530 000m^3/day creates depression cone which extends abroad on Russian territory. The groundwater level along the administrative border is gradually reduced to 30-40 meters as shown in fig 4. The debit of lateral groundwater flow, directed from Russia to Kazakhstan with capture along border, increases from 60 up 135 thousands m^3/day.

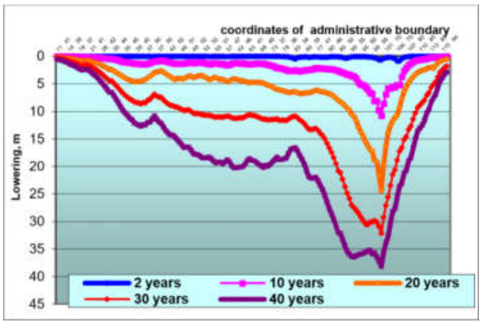

Figure 4: Lowering of groundwater level along administrative boundary for some time steps.

Thus there is a gradual depletion of the aquifer.

This model displays the process of geofiltration in deep aquifers, so their effect on water exchange with surface water can be neglected.

B. Russian-Ukraine model

Russian-Ukraine model was created for the territory 248×276 km of Belgorod and Kharkov regions and takes into consideration four aquifers and three semipermeable layers on depth with 1 km step in plan.

The surface water network shows the major rivers in the modeled area in 2000 nodal points. Levels in the rivers set of topographic maps of scale 1: 100 000.

Natural conditions were accepted for 1970 and broken conditions were reproduced from 1971 up to 2009 with one year time step.

Maps modelling groundwater levels for one of aquifer in natural and disturbed conditions are shown in fig 5a,5b.

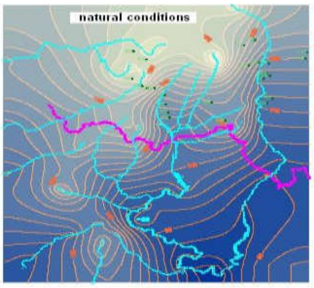

Figure 5a: Map of groundwater levels of Quaternary aquifer for natural conditions in the Russian-Ukrainian model.

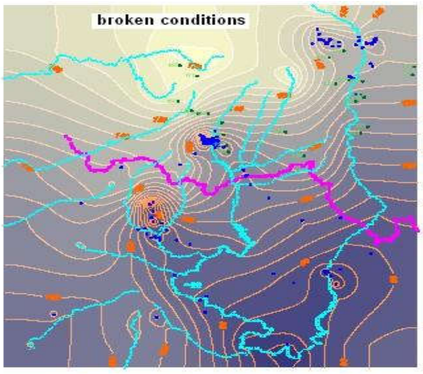

Figure 5b: Map of groundwater levels of Quaternary aquifer disturbed conditions in the Russian-Ukrainian model. *yellow lines – isolines of groundwater levels,* ***light blue lines - surface water network, pink line is part of the administrative border between Russia and Ukraine, blue point are water wells in this aquifer***

Comparing maps of groundwater levels can be concluded that the groundwater flow character has not changed. However, with increasing of water discharge begin to form cones of depression

The maximum decrease of the groundwater level within the depression cone for the most loaded horizon is 55 meters for Belgorod city and 80 meters for Kharkov city in 1980-1982.

However, decreasing exploitation of groundwater the depth of depression cone decreases and ground water level gradually increases. Fig 6 shows the change of groundwater level in the nodal points of the model within the depression during broken conditions in aquifer.

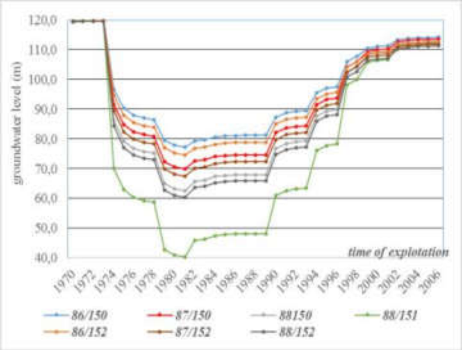

Figure 6. Groundwater level change in the nodal points within the cone of depression over time

As can be seen from the graphs the depth of depression cone increases with rate of pump over time of exploitation of water intakes, after 1990 year pump of rate decreases, ground water level gradually recovers.

Reduction of groundwater discharge almost restores the natural conditions of groundwater flow, but excess of water extraction can cause the increase in size of depression cones and considerable depletion of aquifer. Fig 7 shows the lowering of the groundwater level in 1992 compared to the natural regime of geofiltration in the exploited aquifer. The contours of the depression cones is limited by lowering the level by 1 meter. The red dots on the figure are marked water intake wells.

Figure. 8 shows the lowering of the water table in 1992 compared to natural regime of geofiltration in the exploited aquifer. Thus during the greatest load on the aquifer joint depression cone is formed that has spread into the territory of both States. Even more load can cause the increase in size of depression cones and significant depletion of aquifer.

Figure 7: Contours of depression cones in transboundary groundwater flow in the Russian-Ukraine region in 1992 year.

The process of water exchange of groundwater and surface water in the disturbed conditions depends on the value of extracted water, and the depletion of aquifers has large impact on the degree of depletion of surface water. This task is described in detail in [4] when modeling the Western part of Moscow artesian basin. It is known, along almost the entire length of the river occurs discharge of groundwater to surface water and rivers are the natural drainage network. In some areas due to existing geological conditions the rivers are supply sources for groundwater. The whole process can be evaluated quantitatively in the model.

Table 1 shows the summary data for the major urban water withdrawals in comparison with changes in total water exchange of the river network and groundwater throughout the model. The surface water network is confined to the Quaternary aquifer, water wells are placed in the Albian-Cenomanian aquifer

TABLE 1

MODIFICATION OF GROUNDWATER/SURFACE WATER INTERACTION IN COMPARISON WITH WATER DISCHARGE

(cub.m/day)

Some time step (year)	**Recharge from rivers**	**Drainage into rivers**	**General water discharge**
1970	301315	-3203539	0
1980	380198	-3092887	-339487
1991	416498	-2821991	-406736
1996	399894	-2756362	-377620
2009	383275	-2634687	-411719

It is shown with increasing amount of exploitation of groundwater discharge of surface water to groundwater increases and the amount of drain decreases. However, the rivers retain their draining properties.

A more specific analysis of changes in the interaction of ground and surface water is shown for Quaternary aquifer node points on the river fragment for the natural and broken geofiltration conditions in Fig 8.

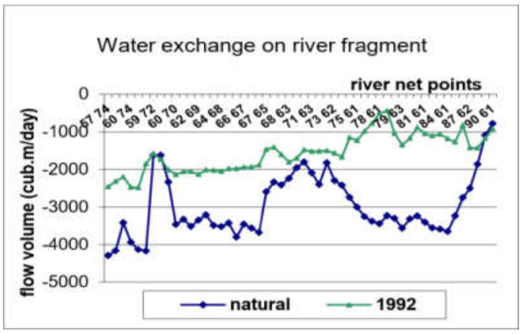

Figure 8. Ground/surface water interaction for time of maximum water discharge compared to natural conditions.

These lines represent the flow rates in the water exchange of each unit grid area of the model. The blue line is the magnitude of the discharge of groundwater to surface water under natural conditions of geofiltration. Green line – the same water exchange in 1992. Negative values reflect the fact of discharge. The magnitude of discharge become smaller, what confirms the character of changes in the overall balance of the model

V. CONCLUSIONS

Mathematical models as a tool of hydrogeological and ecological forecasting must be general monitoring subsystem. In this case, as the hydrogeological services and Government agencies get the real tool to assess the current and projected state of the underground hydrosphere, as well as for the effective management of anthropogenic impact.

Acknowledgement

This study has been performed under the framework of projects carried out using a grant from the Russian Fund of Fundamental Investigations. The author is grateful to the leader of the Russian project Prof. I. Zektser and the leader of the Ukrainian project Prof. V. Shestopalov and also to Dr. U. Rudenko for the actual material they have provided.

References

[1]. Gavich I. K. 1997) Hydrogeodynamics, A. A. Balkema Publishers.
[2]. Polshkova I. N. 1994. Specifics of implementing of special math software system for automated grid models of basins and deposits of underground waters. Dr. Sci. Eng. Diss. Moscow; (in Russian).
[3]. Mironova, A. 2006. Transboundary problems associated with groundwater exploitation near the Russian-Estonian state boundary. Water resources and the regime of water bodies. 33 (4): 423-432 (in Russian).
[4]. Polshkova I. N. April 2015. Estimation of damage to underground component of rivers due to regional water withdrawals at mathematic models. Modern Environmental Science and Engineering. 1(1): 27-34
[5]. Krashin, I.I. Numerical modeling for regional estimation of the ground water conditions and resources in south-east part of
[6]. West Siberian Artesian Basin. Exploration and Protection of Underground Resources, 2006; Nov.2: 68-75 (in Russian).