EPH - International Journal of Mathematics and Statistics
ISSN (Online): 2208-2212
Volume 06 Issue 2 November2020

DOI:https://doi.org/10.53555/eijms.v6i1.50

INVERSE DOMINATION IN SOME OPRATIONS ON BIPOLAR FUZZY GRAPHS

Saqr H. AL-Emrany[1*] Mahiuob M. Q. Shubatah[2]
*[*1]Department of Mathematic, faculty of Art and Science University of Sheba Region, Mareb, Yemen*
[2]Department of mathematics, faculty of Science and Education, AL-Baydaa University, AL-Baydaa, Yemen

[*]Corresponding Author:-
E-mail: aborayan177299@gmail.com, E-mail: mahioub70@yahoo.com

Abstract:-
In this paper the concept of inverse domintion in some operations on bipolar fuzzy graphs was introduced and investigated the bound of γ^{8} of some operations on bipolar fuzzy graphs are obtained like union, join, Cartesian product, strong product and composition.

Keywords:-*Bipolar fuzzy graph, inverse domination number.*

AMS Mathematics Classification (2010), 03E72, 68R10, 68R05.

1 INTRODUCTION

Zhang[19] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. The fuzzy relations between fuzzy sets were also considered by Rosen-feld. He developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. The notion of bipolar fuzzy graphs was introduced by Akram[1]. The concept of domination in fuzzy graphs was investigated by A. Somasundaram and S. Somasundaram[15]. M. G. Karunambigai, M. Akram and K. Palanivel[11] investigated the concept of domination in bipolar fuzzy graphs. S. Ghobadi, N. D. Soner and Q. M. Mahioub introduced the concept of inverse domination number in fuzzy graphs[8]. Akram and Rabia Akmal introduced the concept of certain operations on bipolar fuzzy graph structures [5], also defined some operations on bipolar fuzzy graphs. The domination in operations on bipolar fuzzy graphs was investigated[18]. In this paper we introduce and invistegate the concept of inverse domination in some operations on bipolar fuzzy graphs. We obtain the bounds of the inverse domination number in some operations on bipolar fuzzy graphs like union, join, Cartesian product, strong product and composition.

2 Preliminaries

In this section we review some basic definitions related to bipolar fuzzy graphs and inverse domination in bipolar fuzzy graph.

A bipolar fuzzy graph *BFG*, G is of the form $G = (V,E)$, where

(1) $V = \{v_1,v_2,.....,v_n\}$ such that $\mu^+ : V \to [0,1]$ and $\mu^- : V \to [-1,0]$.

(2) $E \subset V \times V$, where $\rho^+ : V \times V \to [0,1]$ and $\rho^- : V \times V \to [-1,0]$ such that,

$$\rho^+ = \rho^+(v_i,v_j) \le \mu^+(v_i) \wedge \mu^+(v_j)$$

and

$$\rho^- = \rho^-(v_i,v_j) \ge \mu^-(v_i) \vee \mu^-(v_j)$$

for all $v_i,v_j \in V$. In a bipolar fuzzy graph G, when $\rho^+ = \rho^- = 0$ for some i and j, then there is no edge between v_i and v_j. Otherwise there exists an edge between v_i and v_j. A bipolar fuzzy graph, $G = (V,E)$ is said to be $semi-\rho^+$ 3
strong bipolar fuzzy graph if $\rho^+ = min(\mu^+{}_i,\mu^+{}_j)$ for every i and j. A bipolar fuzzy graph *BFG* $G = (V,E)$ is said to be $semi{-}\rho^-$ strong bipolar fuzzy graph if $\rho^- = max(\mu^-{}_i,\mu^-{}_j)$ for every i and j. Let $G = (V,E)$ be a bipolar fuzzy graph.
Then the cardinality of G is defined to be

$$|G| = \sum_{v_i \in V} \frac{1+\mu^+(v_i)+\mu^-(v_i)}{2} + \sum_{(v_i,v_j)\in E} \frac{1+\rho^+(v_i,v_j)+\rho^-(v_i,v_j)}{2}.$$

The vertex cardinality of G is defined by

$$|V| = \sum_{v_i \in V} \frac{1+\mu^+(v_i)+\mu^-(v_i)}{2}.$$

For all $v_i \in V$. is called the order of a bipolar fuzzy graph is denoted by $P(G)$. The edge cardinality of a bipolar fuzzy graph G is defined by

$$|E| = \sum_{(v_i,v_j)\in E} \frac{1+\rho^+(v_i,v_j)+\rho^-(v_i,v_j)}{2}.$$

For all $(v_i,v_j) \in E$ is called the size of a bipolar fuzzy graph is denoted by $q(G)$. An edge $e = (x,y)$ of a bipolar fuzzy graph is called strong edge if $\rho^+(x,y) = min\{\mu^+(x),\mu^+(y)\}$ and $\rho^-(x,y) = max\{\mu^-(x),\mu^-(y)\}$. The degree of a vertex can be generalized in different ways for a bipolar fuzzy graph $G = (V,E)$. The effective degree of a vertex v in a bipolar fuzzy graph $G = (V,E)$ is defined to be sum of the weights of the strong edges incident at v and it is denoted by $d_E(v)$. The minimum effective degree of G is $\delta_E(G) = min\{d_E(v)|v \in V\}$. The maximum effective degree of G is $\Delta_E(G) = max\{d_E(v)|v \in V\}$. The vertex v is adjacent to a vertex u if they reach between the strong edge. Two vertices v_i and v_j are said to be adjacent if neighbors in a bipolar fuzzy graph $G = (V,E)$ if either one of the following conditions holds,

(1) $\rho^+(v_i,v_j) > 0$ and $\rho^-(v_i,v_j) > 0$,

(2) $\rho^+(v_i,v_j) = 0$ and $\rho^-(v_i,v_j) < 0$,

(3) $\rho^+(v_i,v_j) > 0$ and $\rho^-(v_i,v_j) = 0, v_i,v_j \in V$. And they are strong neighbor if $\rho^+(v,u) = min\{\mu^+(v),\mu^+(u)\}$ and $\rho^-(v,u) = max\{\mu^-(v),\mu^-(u)\}$. A vertex

subset $N(v) = \{u \in V : v \text{ adjacent to } u\}$ is called the open neighborhood set of a vertex v and $N[v] = N(v) \cup \{v\}$ is called the closed neighborhood set of v. The neighborhood degree of a vertex v in a bipolar fuzzy graph, $G = (V,E)$ is defined to be sum of the weights of the vertices adjacent to v, and it is denoted by $d_N(v)$, that is mean that $d_N(v) = |N(v)|$. The minimum neighborhood degree of G is $\delta_N(G) = min\{d_N(v)|v \in V\}$. The maximum neighborhood degree of G is $\Delta_N(G) = max\{d_N(v)|v \in V\}$. A bipolar fuzzy graph, $G = (V,E)$ is said to be complete bipolar fuzzy graph if $\rho^+(v_i,v_j) = min\{\mu^+(v_i),\mu^+(v_j)\}$, $\rho^-(v_i,v_j) = max\{\mu^-(v_i),\mu^-(v_j)\}$, for all $v_i,v_j \in V$ and is denoted by K_p. The complement of a bipolar fuzzy graph, $G = (V,E)$ is a bipolar fuzzy graph, $\overline{G} = (\overline{V},\overline{E})$ where

$(i)\ \overline{V} = V$;

$(ii)\ \overline{\mu^+} = \mu^+;\ \overline{\mu^-} = \mu^-$ for all vertices;

$(iii)\overline{\rho^+} = min\{\mu_i^+, \mu_j^+\} - \rho^+$ and $\overline{\rho^-} = max\{\mu_i^-, \mu_j^-\} - \rho^-$ for all $i,j = 1,2,3,...,n$. A bipolar fuzzy graph $G = (V,E)$ is said to bipartite if the vertex set V of G can be partitioned into two non empty subsets V_1 and V_2 such that

(i) $\rho^+(v_i,v_j) = 0$ and $\rho^-(v_i,v_j) = 0$ if $v_i,v_j \in V_1$ or $v_i,v_j \in V_2$;

(ii) $\rho^+(v_i,v_j) > 0$ and $\rho^-(v_i,v_j) < 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some i and j,(or); $\rho^+(v_i,v_j) = 0$ and $\rho^-(v_i,v_j) < 0$, if $v_i \in V_1$ and $v_j \in V_2$,(or); $\rho^+(v_i,v_j) > 0, \rho^-(v_i,v_j) = 0$, if $v_i \in V_1$ and $v_j \in V_2$ for some i and j. A bipartite bipolar fuzzy graph $G = (V,E)$ is said to be complete bipartite bipolar fuzzy graph if $\rho^+(v_i,v_j) = min\{\mu^+(v_i),\mu^+(v_j)\}$ and $\rho^-(v_i,v_j) = max\{\mu^-(v_i),\mu^-(v_j)\}$ for all $v_i \in V_1$ and $v_j \in V_2$. Its denoted by $K_{m,n}$, where $|V_1| = m, |V_2| = n$. A vertex $u \in V$ of a bipolar fuzzy graph, $G = (V,E)$ is said to be an isolated vertex if $\rho^+(v,u) = 0$ and $\rho^-(v,u) = 0$ for all $v \in V$. That is $N(u) = \varphi$. Thus an isolated vertex does not dominate any other vertex in G. A bipolar fuzzy graph, $G = (V,E)$ is said to be strong bipolar fuzzy graph, if $\rho^+ = min(\mu^+{}_i, \mu^+{}_j)$, for every i and j and $\rho^- = max(\mu^-{}_i, \mu^-{}_j)$ for all $(v_i,v_j) \in E$. A bipolar fuzzy graph,

$H = (X,Y)$ is said to be bipolar fuzzy subgraph of $G = (V,E)$ if $X \subseteq V$ and 5

$Y \subseteq E$. That is $\mu^{8+} \le \mu^+{}_i$, $\mu^- \ge \mu^-$ and $\rho^{8+} \le \rho^+$; $\rho^{8-} \ge \rho^-$. A vertxe subset D of V inBFG is called dominating set of bipolar fuzzy graph G, if for every $v \in V - D$ there exists $u \in D$, such that (uv) is strong edge. A subset D of V is called a dominating set in G if for every $v \in V\ D$, there exists $u \in D$ such that u dominates v. A dominating set D of BFG is called minimal dominating set if $D - \{u\}$ is not dominating set for every $u \in D$. A minimal dominating set D, with $|D| = \gamma(G)$ is denoted by $\gamma - set$. Let D be $\gamma - set$ of G, if $V - D$ contains $\gamma - set$, D^8 of (G), then D^8 is called an inverse dominating set with respect to D of G. An inverse dominating set D^8 of G is called minimal inverse dominating set if $D^8 - \{u\}$ is not inverse dominating setof G for every $u \in D^8$. The minimum cardinality among all minimal inverse dominating sets of G is called the inverse domination number of G and denoted by $\gamma^8(G)$.

3 Inverse Domination In Some Operation On Bipolar Fuzzy Graphs

Definition 3.1. *[18] Let $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ be two BFGs on V_1,V_2 respectively with $V_1 \cap V_2 = \varphi$. The union of G_1 and G_2 and defined by $G_1 \cup G_2$ is the bipolar fuzzy graph on $V_1 \cup V_2$ and defineds $G = G_1 \cup G_2 =$* $\{(\mu_1^+ \cup \mu_2^+, \mu_1^- \cup \mu_2^-), (\rho_1^+ \cup \rho_2^+, \rho_1^- \cup \rho_2^-)\}$. *Where*

$$(\mu_1^+ \cup \mu_2^+)(x) = \begin{cases} \mu_1^+(x) & if\ \ x \in V_1 \\ \mu_2^+(x) & if\ x \in V_2 \end{cases} \qquad (\mu_1^- \cup \mu_1^-)(x) = \begin{cases} \mu_1^-(x) & if\ x \in V_1 \\ \mu_2^-(x) & if\ x \in V_2 \end{cases}$$

$$(\rho_1^+ \cup \rho_2^+)(xy) = \begin{cases} \rho_1^+(xy) & if\ xy \in E_1 \\ \rho_2^+(xy) & if\ xy \in E_2 \\ 0\ otherwise \end{cases} \qquad (\rho_1^- \cup \rho_2^-)(xy) = \begin{cases} \rho_1^-(xy) & if\ xy \in E_1 \\ \rho_2^-(xy) & if\ xy \in E_2 \\ 0\ otherwise \end{cases}$$

Theorem 3.2. *Let G_1 and G_2 be two bipolar fuzzy graphs, then*

$$\gamma^8(G_1 \cup G_2) = \gamma^8(G_1) + \gamma^8(G_2).$$

6

Proof. Let $D_1{}^8$ and $D_2{}^8$ be an inverse dominating sets of G_1 and G_2 with respect to D_1 and D_2 respectivily, Then $D_1{}^8 \subseteq V_1 - D_1$ and $D_2{}^8 \subseteq V_2 - D_2$.

Therefore,$\gamma^8(G_1) \le |v_1 - D_1|$, $\gamma^8(G_2) \le |v_2 - D_2|$. Thus $D18 \cup D28 \subseteq (V1 - D1) \cup (V2 - D2)$. Thus

$$D_1^{`} \cup D_2^{`} \subseteq (V_1 - D_1) \cup (V_2 - D_2)$$

so

$$|{}^{D}{}_1{}^8 \cup {}^{D}{}_2{}^8| \le |(V_1 - D_1) \cup (V_2 - D_2)| = |V_1 - D_1| + |V_2 - D_2|.$$

Hance

$$\gamma^{`}(G_1 \cup G_2) = \gamma^{`}(G_1) + \gamma^{`}(G_2).$$

Examples 3.1: consider the bipolar fuzzy graphs G_2 and G_1 gevin in the figures 3.1a and 3.1b.

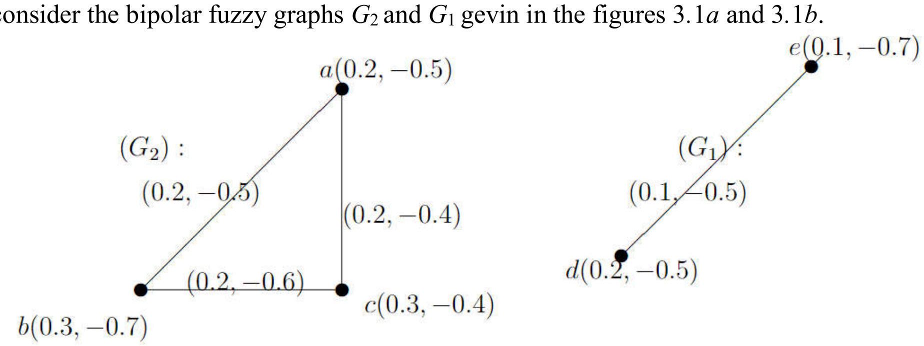

Fig.3.1a Fig.3.1b

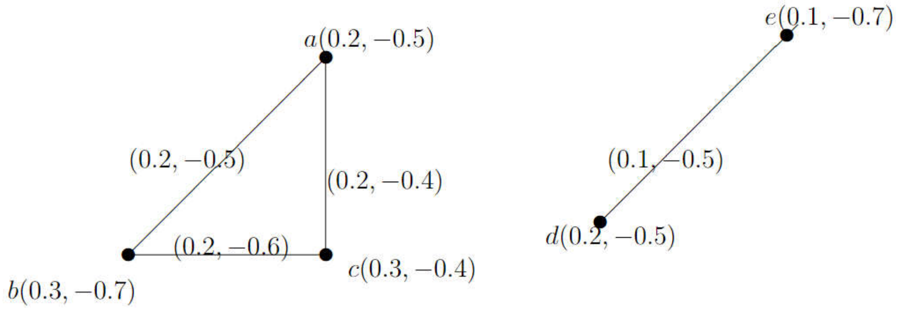

***Fig* 3.1*c* : ($G_1 \cup G_2$)**

The union of G_1 and G_2 is showen In figure (3.1*c*)we see that D_1 is a $\gamma - set$ of G_1, D_2 is a $\gamma - set$ of G_2 and $D = D_1 \cup D_2$ is a $\gamma - set$ of $G_1 \cup G_2$ with $|D| = |D_1 \cup D_2| = |D_1|+|D_2| = \{|a|\}+\{|e|\}$ Therefore, the inverse domination of G_1 and G_2 are $|D_1^8| = \{|d|\}$ and $|D_2^8| = \{|b,c|\}$ respectively. The inverse dominating set of $(G_1 \cup G_2)$ is $|D_1^8| + |D_2^8| = (\{|b,c|\} + \{|d|\})$.

Definition 3.3. *[18] Let* $A_1 = (\mu_1^+, \mu_1^-)$ *and* $A_2 = (\mu_2^+, \mu_2^-)$ *be bipolar fuzzy subset of* V_1 *and* V_2 *in which* $V_1 \cap V_2 = \varphi$ *and let* $B_1 = (\rho_1^+, \rho_1^-)$ *and* $B_2 = (\rho_2^+, \rho_2^-)$ *be bipolar fuzze subset of* $V_1 \times V_2$ *and* $V_2 \times V_1$ *respectivily then we denoted the join of two bipolar fuzzy graphs* G_1 *and* G_2 *defined by* $G_1 + G_2 = (A_1 + A_2, B_1 + B_2)$
and defined as follows

$$\begin{cases} (\mu_1^+ + \mu_2^+)(x) = min\{\mu_1^+(x), \mu_2^+(x)\} \\ (\mu_1^- + \mu_2^-)(x) = max\{\mu_1^-(x), \mu_2^-(x)\} \end{cases} \quad if x \in V_1 \cup V_2$$

$$\begin{cases} (\rho_1^+ + \rho_2^+)(xy) = min\{\rho_1^+(x), \rho_2^+(y)\} \\ (\rho_1^- + \rho_2^-)(xy) = max\{\rho_1^-(x), \rho_2^-(y)\} \end{cases} \quad if xy \in E_1 \cup E_2$$

$$\begin{cases} (\rho_1^+ + \rho_2^+)(xy) = min\{\rho_1^+(x), \rho_2^+(y)\} \\ (\rho_1^- + \rho_2^-)(xy) = max\{\rho_1^-(x), \rho_2^-(y)\} \end{cases} \quad if xy \in E'$$

Where E' *is the set of all edge joining the vertices of* V_1 *and* V_2.

8

Theorem 3.4. *Let* G_1 *and* G_2 *be two bipolar fuzzy graphs, then*

$$\gamma'(G_1 + G_2) = max\{\gamma'(G_1), \gamma'(G_2)\}.$$

Proof. Let D'_1 and D'_2 be an inverse dominating sets of G_1 and G_2 with respect to D_1 and D_2 respectivily, Then $D'_1 \subseteq V_1 - D_1$ and $D'_2 \subseteq V_2 - D_2$. Therefore, $\gamma'(G_1) \leq P_1 - \gamma(G_1)$ and $\gamma'(G_2) \leq P_2 - \gamma(G_2)$. Then

$$D18 + D28 \subseteq (V1 - D1) + (V2 - D2)$$

so

$$|D'_1 + D'_2| \leq |(V_1 - D_1) + (V_2 - D_2)|.$$

Where $P = P_1 + P_2$ Hance

$$\gamma'(G) = \gamma'(G_1 + G_2) \leq |(V_1 - D_1) + (V_2 - D_2)|$$

By theorem (3.1) in [18] $\gamma(G_1 + G_2) = min\{\gamma(G_1), \gamma(G_2)\}$.
Then

$$\gamma'(G_1 + G_2) \leq |V_1 + V_2| - (|D_1| \wedge |D_2|)$$

$$\gamma'(G_1 + G_2) \leq P - (D_1) \wedge D_2)) = |D'_1| \vee |D'_2.$$

Hance

$$\gamma'(G_1 + G_2) = max\{\gamma'(G_1), \gamma'(G_2)\}.$$

□

Examples 3.2: consider the bipolar fuzzy graphs G_2 and G_1 gevin in the figures 3.2*a* and 3.2*b*.

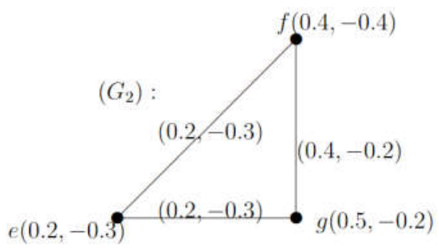

Fig 3.2*a*

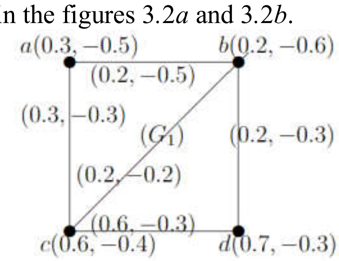

Fig 3.2*b*.

$D_2 = \{f\}$ is minmum cardinalty of G_2 $\qquad$ $D_1 = \{a,c\}$ is minmum cardinalty of G_1

$\gamma(G_2) = |D_2| = \{|f|\}$ $\qquad$ $\gamma(G_1) = |D_1| = \{|a,c|\}$

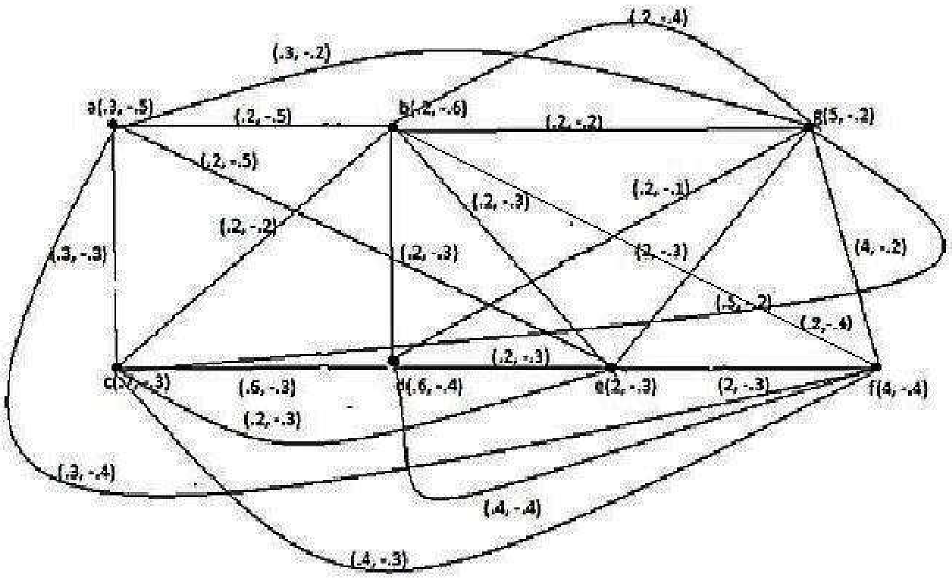

Fig 3.2*c* : ($G_1 + G_2$)

The join of G_1 and G_2 given in figure (3.2*c*)we see that D_1 is a $\gamma - set$ of G_1, D_2 is a $\gamma - set$ of G_2 and $D = D_1 + D_2$ is a $\gamma - set$ of $G_1 + G_2$ with $|D| = min|D_1|,|D_2| = \{|a,c|\},\{|f|\}$ Therefore, the inverse domination of G_1 and G_2 are $D_1^8 = \{b,d\}$ and $D_2^8 = \{e,g\}$ respectively are minimal dominating sets of $G = G_1{+}G_2$ and $\gamma^8 = max\{|D_1^8|,|D_2^8| = max\{\{|b,d|\},\{|e,g|\}\} = \{|e,g|\}$. 10

Definition 3.5. *Let $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ be two BFG on V_1, V_2 respectively with $V_1 \cap V_2 = \varphi$. The Cartesian product of G_1 and G_2 and by*
$G_1 \times G_2$ is the bipolar fuzzy graph on $V_1 \times V_2$ and defined by $G = G_1 \times G_2 =$

$$\{(\mu_1^+ \times \mu_2^+, \mu_1^- \times \mu_2^-), (\rho_1^+ \times \rho_2^+, \rho_1^- \times \rho_2^-)\},\ such\ that$$

$$\begin{cases} (\mu_1^+ \times \mu_2^+)(xy) = min\{\mu_1^+(x), \mu_2^+(y)\} \\ (\mu_1^- \times \mu_2^-)(xy) = max\{\mu_1^-(x), \mu_2^-(y)\} \end{cases} \quad \forall x, y \in V_1 \times V_2$$

$$\begin{cases} (\rho_1^+ \times \rho_2^+)((xy_1)(xy_2)) = min\{\mu_1^+(x), \rho_2^+(y_1y_2)\} \\ (\rho_1^- \times \rho_2^-)((xy_1)(xy_2)) = max\{\mu_1^-(x), \rho_2^-(y_1y_2)\} \end{cases} \quad \forall x \in V_1, y_1y_2 \in E_2$$

$$\begin{cases} (\rho_1^+ \times \rho_2^+)((x_1y)(x_2y)) = min\{\rho_1^+(x_1x_2), \mu_2^+(y)\} \\ (\rho_1^- \times \rho_2^-)((x_1y)(x_2y)) = max\{\rho_1^-(x_1x_2), \mu_2^-(y)\} \end{cases} \quad \forall y \in V_2, x_1x_2 \in E_1.$$

Theorem 3.6. *Let G_1 and G_2 be two bipolar fuzzy graphs on V_1 and V_2 respectivily, with $V_1 \cap V_2 = \varphi$, then*

$$\gamma'(G_1 \times G_2) = max\{|D'_1 \times V_2|, |V_1 \times D'_2|\}.$$

Proof. Let D'_1 and D'_2 be an inverse dominating sets of G_1 and G_2 with respect to D_1 and D_2 respectivily, Then $D'_1 \subseteq V_1 - D_1$ and $D'_2 \subseteq V_2 - D_2$. Therefore $\gamma'(G_1) \le P_1 - \gamma(G_1)$ and $\gamma'(G_2) \le P_2 - \gamma(G_2)$. Then

$$D'_1 \times D'_2 \subseteq (V_1 - D_1) \times (V_2 - D_2).$$

So

$$|D'_1 \times D'_2| \le |(V_1 - D_1) \times (V_2 - D_2)|.$$

Thus

$$\gamma'(G) = \gamma'(G_1 \times G_2) \le |(V_1 - D_1) \times (V_2 - D_2)|.$$

By theorem (3.2) in[18] $\gamma(G_1 \times G_2) = min\{|D_1 \times V_2|,|V_1 \times D_2|\}$. Then

$$\gamma'(G_1{\times}G_2) \le |V_1{\times}V_2| {-} \gamma(G_1{\times}G_2) = |V_1{\times}V_2| {-} (|D_1{\times}V_2| \wedge |V_1{\times}D_2|) = |D'_1 \times V_2| \vee |V_1 \times D'_2|.$$

Hance

$$\gamma'(G_1 \times G_2) = max\{|D'_1 \times V_2|,|V_1 \times D'_2|\}. \qquad \square$$

Examples 3.3: consider the bipolar fuzzy graphs G_2 and G_1 gevin in the figures 3.3*a* and 3.3*b*.

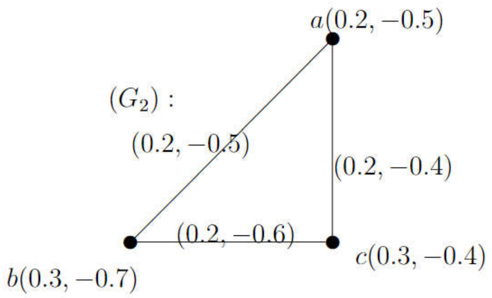

Fig 3.3a

$D_2 = \{a\} \Rightarrow D^{\backprime}_2(G_2) = \{b, c\}$

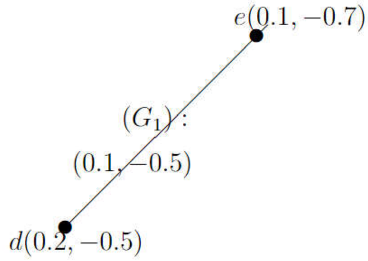

Fig 3.3b.

$D_1 = \{e\} \Rightarrow D^{\backprime}_1(G_1) = \{d\}$

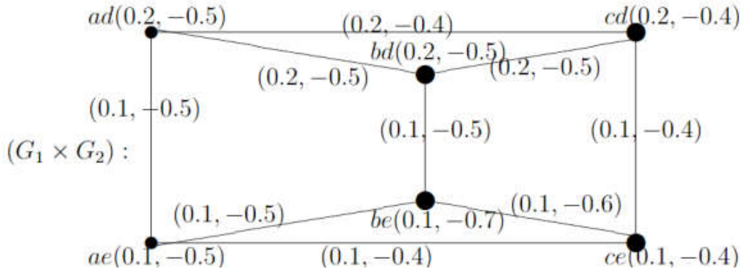

Fig 3.3c.

The Cartesian product of G_1 and G_2 given in figure ($3.3c$) we see that D_1 is a $\gamma-set$ of G_1, D_2 is a $\gamma-set$ of G_2 and D is a $\gamma-set$ of $G_1 \times G_2$ with
$|D| = min|V_1 \times D_2|,|D_1 \times V_2|$ Therefore, the inverse domination of G_1 and G_2 are $D^{\backprime}_1 = \{d\}$ and $D^{\backprime}_2 = \{b,c\}$ respectively. The inverse domination number of $(G_1 \times G_2)$ is $max\{\{|da,db,dc|\},\{|ce,cd,be,bd|\}\} = \{|ce,cd,be,bd|\}$.

Definition 3.7. *[18] Let $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ be two BFG on V_1,V_2 respectively with $V_1 \cap V_2 = \varphi$. The strong product of G_1 and G_2 and defined by $G_1 \otimes G_2$ is the bipolar fuzzy graph on $V_1 \otimes V_2$ and defined as*
$G = G_1 \otimes G_2 = (V_1 \otimes V_2, E_1 \otimes E_2)$, sach that

$$\begin{cases} (\mu_1^+ \otimes \mu_2^+)(xy) = min\{\mu_1^+(x), \mu_2^+(y)\} \\ (\mu_1^- \otimes \mu_2^-)(xy) = max\{\mu_1^-(x), \mu_2^-(y)\} \end{cases} \forall (x,y) \in V_1 \times V_2$$

$$\begin{cases} (\rho_1^+ \otimes \rho_2^+)((xy_1)(xy_2)) = min\{\mu_1^+(x), \rho_2^+(y_1y_2)\} \\ (\rho_1^- \otimes \rho_2^-)((xy_1)(xy_2)) = max\{\mu_1^-(x), \rho_2^-(y_1y_2)\} \end{cases} \forall x \in V_1, (y_1y_2) \in E_2$$

$$\begin{cases} (\rho_1^+ \otimes \rho_2^+)((x_1y)(x_2y)) = min\{\rho_1^+(x_1x_2), \mu_2^+(y)\} \\ (\rho_1^- \otimes \rho_2^-)((x_1y)(x_2y)) = max\{\rho_1^-(x_1x_2), \mu_2^-(y)\} \end{cases} \forall y \in V_2, (x_1x_2) \in E_1$$

$$\begin{cases} (\rho_1^+ \otimes \rho_2^+)((x_1y_1)(x_2y_2)) = min\{\rho_1^+(x_1x_2), \rho_2^+(y_1y_2)\} \\ (\rho_1^- \otimes \rho_2^-)((x_1y_1)(x_2y_2)) = max\{\rho_1^-(x_1x_2), \rho_2^-(y_1y_2)\} \end{cases} \forall (y_1y_2) \in E_2, (x_1x_2 \in E_1)$$

Theorem 3.8. *Let G_1 and G_2 be two bipolar fuzzy graphs on V_1 and V_2 respectivily, with $V_1 \cap V_2 = \varphi$ then*

$$\gamma^{\backprime}(G_1 \otimes G_2) = |D^{\backprime}_1 \times D^{\backprime}_2|.$$

Proof. Let $D^{\backprime}_1$ and $D^{\backprime}_2$ be an inverse dominating sets of G_1 and G_2 with respect to D_1 and D_2 respectivily, Then $D^{\backprime}_1 \subseteq V_1 - D_1$ and $D^{\backprime}_2 \subseteq V_2 - D_2$. Therefore, $\gamma^{\backprime}(G_1) \leq P_1 - \gamma(G_1)$ and $\gamma^{\backprime}(G_2) \leq P_2 - \gamma(G_2)$. Then

$$D^{\backprime}_1 \otimes D^{\backprime}_2 \subseteq (V_1 - D_1) \otimes (V_2 - D_2).$$

So

$$|D^{\backprime}_1 \otimes D^{\backprime}_2| \leq |(V_1 - D_1) \otimes (V_2 - D_2)|.$$

Thus

$\gamma^{\backprime}(G) = \gamma^{\backprime}(G_1 \otimes G_2) \leq P - \gamma(G_1 \otimes G_2) = |(V_1 - D_1) \otimes (V_2 - D_2)|$. By theorem (3.5) in [18], $\gamma(G_1 \otimes G_2) = |D_1 \times D_2|$, then

$\gamma^{\backprime}(G_1 \otimes G_2) \leq P - \gamma(G_1 \otimes G_2) = |V_1 \times V_2| - (|D_1 \times D_2|) \leq |V_1 \times V_2 - D_1 \times D_2| = |D^{\backprime}_1 \times D^{\backprime}_2|$. 13

Hance

$$\gamma^{\backprime}(G1 \otimes G2) = |D18 \times D28|. \qquad \square$$

Examples 3.4: consider the bipolar fuzzy graphs G_2 and G_1 gevin in the figures 3.4*a* and 3.1*b*.

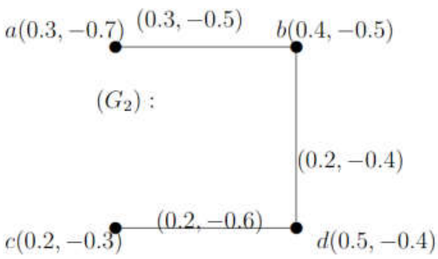

Fig 3.4a

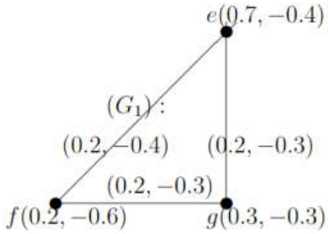

Fig 3.4b.

$D_2 = \{a\} \Rightarrow D_2^{\backprime}(G_2) = \{b, c\}$ $\qquad$ $D_1 = \{e\} \Rightarrow D_1^{\backprime}(G_1) = \{d\}$

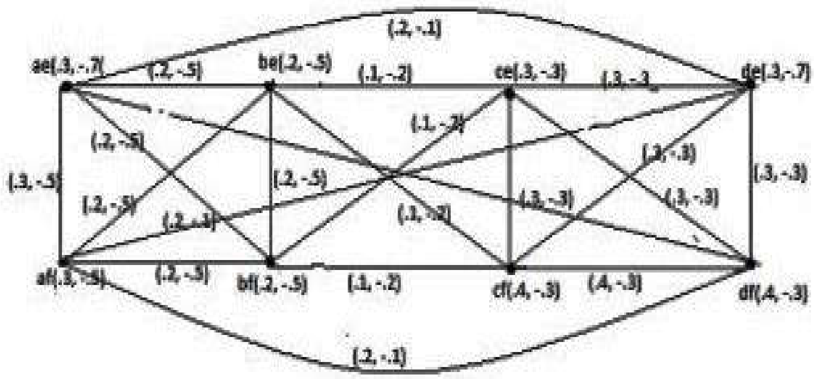

Fig 3.4*c* : *G*₁ ⊗ *G*₂

The strong product of G_1 and G_2 given in figure (3.4*c*) we see that D_1 is a $\gamma - set$ of G_1, D_2 is a $\gamma - set$ of G_2 and D is a $\gamma - set$ of $G_1 \otimes G_2$ with $|D| = |D_1 \times D_2|$ Therefore, the inverse domination of G_1 and G_2 are $D_1^{\backprime} = \{e,g\}$ and $D_2^{\backprime} = \{b,d\}$ respectively. The inverse domination number of $(G_1 \otimes G_2)$ is $|D_1^{\backprime} \times D_2^{\backprime}| = |de,dg,eb,bg|$

Definition 3.9. *[18] Let $G_1 = (V_1,E_1)$ and $G_2 = (V_2,E_2)$ be two BFG on V_1,V_2 respectively with $V_1 \cap V_2 = \varphi$.The composition of G_1 and G_2 and defined by $G_1 \circ G_2$ is the bipolar fuzzy graph on $V_1 \circ V_2$ and defined as $G = G_1 \circ G_2 = (G_1[G_2]) = (V_1 \circ V_2,E_1 \circ E_2)$ sach that*

$$\begin{cases} (\mu_1^+ \circ \mu_2^+)(xy) = min\{\mu_1^+(x), \mu_2^+(y)\} \\ (\mu_1^- \circ \mu_2^-)(xy) = max\{\mu_1^-(x), \mu_2^-(y)\} \end{cases} \quad \forall (x,y) \in V_1 \times V_2$$

$$\begin{cases} (\rho_1^+ \circ \rho_2^+)((xy_1)(xy_2)) = min\{\mu_1^+(x), \rho_2^+(y_1y_2)\} \\ (\rho_1^- \circ \rho_2^-)((xy_1)(xy_2)) = max\{\mu_1^-(x), \rho_2^-(y_1y_2)\} \end{cases} \quad \forall x \in V_1, (y_1y_2) \in E_2$$

$$\begin{cases} (\rho_1^+ \circ \rho_2^+)((x_1y)(x_2y)) = min\{\rho_1^+(x_1x_2), \mu_2^+(y)\} \\ (\rho_1^- \circ \rho_2^-)((x_1y)(x_2y)) = max\{\rho_1^-(x_1x_2), \mu_2^-(y)\} \end{cases} \quad \forall y \in V_2, (x_1x_2) \in E_1$$

$$\begin{cases} (\rho_1^+ \circ \rho_2^+)((x_1y_1)(x_2y_2)) = min\{\rho_1^+(x_1x_2), \mu_2^+(y_1 \wedge \mu_2^+(y_2))\} \\ (\rho_1^- \circ \rho_2^-)((x_1y_1)(x_2y_2)) = max\{\rho_1^-(x_1x_2), \mu_2^-(y_1 \vee \mu_2^-(y_2))\} \end{cases} \quad \forall (y_1y_2) \in V_2, (x_1x_2) \in E_1,$$

Such that $(y_1 \neq y_2)$.

Theorem 3.10. *Let G_1 and G_2 be two bipolar fuzzy graphs, then*

$$\gamma^{\backprime}(G_1 \circ G_2) = |D_1^{\backprime} \times D_2^{\backprime}|.$$

Proof. Let $D_1^{\backprime}$ and $D_2^{\backprime}$ be an inverse dominating sets of G_1 and G_2 with respect to D_1 and D_2 respectivily. Then $D_1^{\backprime} \subseteq V_1 - D_1$ and $D_2^{\backprime} \subseteq V_2 - D_2$. Therefore, $\gamma^{\backprime}(G_1) \leq P_1 - \gamma(G_1)$ and $\gamma^8(G_2) \leq P_2 - \gamma(G_2)$ and by theorem (3.3) in [18]. $\gamma^{\backprime}(G_1 \circ G_2) \leq P - \gamma(G_1 \circ G_2) = P - |D_1 \times D_2| = |V_1 \times V_2| - |D_1 \times D_2|$,
Such that $P = |V_1 \circ V_2| = |V_1 \times V_2|$.
Hance

$$\gamma^{\backprime}(G_1 \circ G_2) \leq |(V_1 - D_1) \times (V_2 - D_2)| = |D_1^{\backprime} \times D_2^{\backprime}|. \qquad \square$$

Examples 3.5: consider the bipolar fuzzy graphs G_2 and G_1 gevin in the figures 3.5*a* and 3.5*b*.

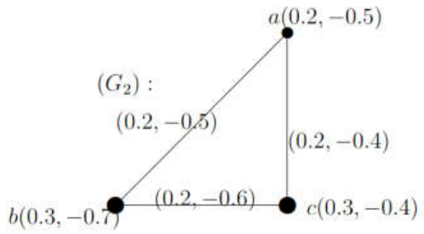

Fig 3.5*a*

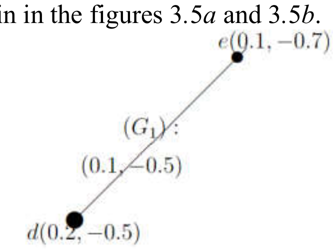

Fig 3.5*b*.

$D_2 = \{a\} \Rightarrow D_2^{\backslash}(G_2) = \{b, c\}$

$D_1 = \{e\} \Rightarrow D_1^{\backslash}(G_1) = \{d\}$

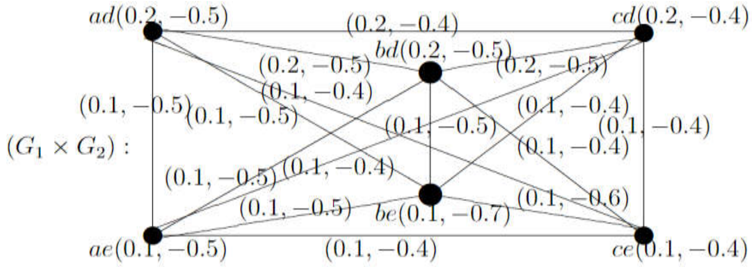

***Fig* 3.5*c* : $G_1 \circ G_2$**

The composition of G_1 and G_2 given in figure (3.5*c*) we see that D_1 is a γ−*set* of G_1, D_2 is a $\gamma - set$ of G_2 and D is a $\gamma - set$ of $G_1 \circ G_2$ with $|D| = |D_1 \times D_2|$ Therefore, the inverse domination of G_1 and G_2 are $D_1^{\backslash} = \{d\}$ and $D_2^{\backslash} = \{b,c\}$ respectively. The inverse domination number of $(G_1 \circ G_2)$ is $|D_1^{\backslash} \times D_2^{\backslash}| = |db,cb|$ 16

References

[1] M. Akram,"Bipolar fuzzy graphs", Information sciences, 181, (2011), 55485564.
[2] M. Akram and W. A. Dudek, "Regular bipolar fuzzy graphs", Neural Computing and Applications, 1, (2012), 197-205.
[3] M. Akram, "Bipolar fuzzy graphs with applications", Knowledge Based Systems, 39, (2013), 1-8.
[4] M. Akram, W. A. Dudek and S. Sarwar, "Properties of bipolar fuzzy hypergraphs", Italian J. Pure Appl. Math. 31, (2013), 426–458.
[5] M. Akram and Rabia Akmal, "Certain operations on bipolar fuzzy graph",Structures applications and applied Mathematics, An International Journal, 11(1), (2016), 443-468.
[6] J. John Stephan, A. Muthaiyan and N. Vinoth Kumar," domination in operations on intuitionistic fuzzy graphs", International Journal of Computer Applications, 96(20), (2014), 1-5.
[7] G. S. Domke, J. E. Donbar and L. R. Markus, "The inverse domination number of a graph", ARS combinatoria, 72, (2004), 149-160 .
[8] S. Ghobadi, N. D. Soner and Q. M. Mahyoub, "Inverse dominating Set in fuzzy graphs", Journal of Proceedings of the Jangjeon Mathematical Soci ety, 11(1), (2008), 75-81.
[9] F. Haray, "Graph theory", Addison Wesley, Third printing, October,(1972).
[10] V. R. Kulli and S. C. Sigarkanti, "Inverse domination in graphs", reprint extract from: Nat. acad. Sci. Letters, 14, (1991), 473-475.
[11] M. G. Karunambigai, M. Akram, and K. Palanivel, "domination in bipolar fuzzy graphs", International conference on fuzzy systems, (2013), 7-10.
[12] A. Kaufman, Introduction a La Theorie des Sous- emsembles Flous, Paris, Masson etcie Editeurs, (1973).
[13] R. Parvathi and G. Thamizhendhi, "domination in intuitionistic fuzzy graphs", Fourteenth Int. Conf. on IFSs, Sofia, NIFS,16 (2), (2010), 39-49.
[14] A. Rosenfeld, Fuzzy graphs, In. L. A. Zadeh, K. S. Fu and M. Shimura, (Eds). "Fuzzy Sets and their Applications", Academic Press, (1975), 77-95.
[15] A. Somasundaram and S. Somasundaram, "Domination in fuzzy graphs-I",Pattern Recognition Letters, 19, (1998), 787-791.
[16] A. Somasundaram, "Domination in fuzzy graphs-II", Journal of FuzzyMathematics, 20, (2000), 281-289.
[17] A. Somasundaram, "Domination in products of fuzzy graphs", Inter, Jornal of Uncertainty fuzziness and Knowledge-Based systems, 13(2), (2005), 195204.
[18] D. Umamageswari and P. Thangaraj, "Domination in operation on bipolar fuzzy graphs", International Journal of Computational Research and Development (IJCRD), 2(2), (2017), 56-65.
[19] W. R. Zhang,"Bipolar fuzzy sets and relations", A computational framework for cognitive modeling and multi agent decision analysis, Proceedings of IEEE. Conf. (1994), 305-30