NEW SOLUTIONS OF LORENTZ TRANSFORMATION II

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[1]Prerequisite for Special Theory of Relativity

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Abstract:-
A simple thought experiment with light clock, which is known in connection with the theory of relativity and its modifications, led me to create the last paper. Paper related to other mathematical solutions of Lorentz transformation. These solutions needed more elaboration and connections. Especially because the mathematical apparatus gains its meaning only when compared to the experiment. The following paper was conceived as a continuation of the previous one, so in this sense the numbering of chapters and, of course, the numbering of equations follows my previous work.

Keywords:-*Lorentz transformation, STR, Young´s experiment*

6. ACQUIRED SOLUTIONS, MUTUAL RELATIONS AND ELECTROMAGNETIC EXPRESSION

A prerequisite for further interpretation is the use of my previous paper New Solutions of the Lorentz Transformation [1]. I ended this paper with equations (4.27) and (4.28). If we rewrite these new solutions of the Lorenz transform, we see that the deformation of space and time shapes the same function and we mark it Θ with the indices + and – depending on the shape of the equation.

The solutions of the equations (4.27) and (4.28) can be rewritten as:

$$x' = \frac{1}{\Theta_{-\pm}} \cdot (x - v.t) \tag{6.1}$$

$$t' = \frac{1}{\Theta_{-\pm}} \cdot \left(t - \frac{vx}{c^2}\right) \tag{6.2}$$

Where we consider:

$$\Theta_{-\pm} = -\frac{v}{c}\cos\alpha \pm \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)} \tag{6.3}$$

Equation (6.1) corresponds to equation (1.1) into which we substitute equation (3.5) and equation (6.2) resulted from invariant. In the same way we can substitute equation (3.6) into equation (1.2) and by analogy deduce the remaining equations (ie for v = -v):

$$x = \frac{1}{\Theta_{+\pm}} \cdot (x' + v.t') \tag{6.4}$$

$$t = \frac{1}{\Theta_{+\pm}} \cdot \left(t' + \frac{vx'}{c^2}\right) \tag{6.5}$$

where

$$\Theta_{+\pm} = \frac{v}{c}\cos\alpha \pm \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)} \tag{6.6}$$

This essentially gives us four functions, between which there are certain transfer relationships. These relations are created by appropriate multiplication of the numerator and denominator:

$$\frac{1}{\Theta_{-+}} = \frac{1}{-\frac{v}{c}\cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}$$

$$\frac{1}{\Theta_{-+}} = \frac{\frac{v}{c}\cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}{\left[-\frac{v}{c}\cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}\right] * \left[\frac{v}{c}\cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}\right]}$$

$$\frac{1}{\Theta_{-+}} = \frac{\frac{v}{c}\cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha) - \left[\frac{v}{c}\cos\alpha\right]^2} = \frac{\frac{v}{c}\cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}}{1 - \frac{v^2}{c^2}}$$

therefore

$$\frac{1}{\Theta_{-+}} = \Theta_{++}\frac{c^2}{c^2 - v^2} \tag{6.7}$$

Then analogously

$$\frac{1}{\Theta_{--}} = \frac{1}{-\frac{v}{c}\cos\alpha - \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}} = \Theta_{+-}\frac{c^2}{c^2 - v^2} \tag{6.8}$$

$$\frac{1}{\Theta_{++}} = \frac{1}{+\frac{v}{c}\cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}} = \Theta_{-+}\frac{c^2}{c^2 - v^2} \tag{6.9}$$

$$\frac{1}{\theta_{+-}} = \frac{1}{+\frac{v}{c}\cos\alpha - \sqrt{1 - \frac{v^2}{c^2}(1 - \cos^2\alpha)}} = \theta_{--}\frac{c^2}{c^2 - v^2} \qquad (6.10)$$

And also:

$$\theta_{+-} = -\theta_{-+} \qquad (6.11)$$

$$\theta_{--} = -\theta_{++} \qquad (6.12)$$

The general assumption of this transformation is relations (1.4) and (1.5), which we renumber:

$$x' = ct' \qquad (6.13)$$
$$x = ct \qquad (6.14)$$

And here we remind the relationships for electrical intensity and magnetic induction in vacuum

$$E = cB \qquad (6.15)$$
$$E_0 = cB_0 \qquad (6.16)$$

Thus, if we compare equations (6.13) and (6.15), (6.14) and (6.16), then we can assume that it is at least a similar transformation. And if we substitute these new variables into (6.1), (6.2), (6.4) and (6.5), we can write:

$$E = \frac{1}{\theta_{-\pm}} \cdot (E_0 - v.B_0) \qquad (6.17)$$

$$B = \frac{1}{\theta_{-\pm}} \cdot \left(B_0 - \frac{vE_0}{c^2}\right) \qquad (6.18)$$

$$E_0 = \frac{1}{\theta_{+\pm}} \cdot (E + v.B) \qquad (6.19)$$

$$B_0 = \frac{1}{\theta_{+\pm}} \cdot \left(B + \frac{vE}{c^2}\right) \qquad (6.20)$$

If we generally consider that

$$E = vB \qquad (6.21)$$

and substitute into equation (6.19), then:

$$E_0 = \frac{1}{\theta_{+\pm}} \cdot (E + E)$$

$$E = E_0 \frac{\theta_{+\pm}}{2} \qquad (6.22)$$

If we substitute equations (6.17) and (6.18) to (6.21), then:

$$\frac{1}{\theta_{-\pm}} \cdot (E_0 - v.B_0) = v\frac{1}{\theta_{-\pm}} \cdot \left(B_0 - \frac{vE_0}{c^2}\right)$$

$$(E_0 - v.B_0) = v.\left(B_0 - \frac{vE_0}{c^2}\right)$$

$$E_0\left(1 + \frac{v^2}{c^2}\right) = 2v.B_0$$

$$E_0(c^2 + v^2) = 2vc^2.B_0$$

So we can express both E0 and B0 as follows:

$$E_0 = \frac{2vc^2}{(c^2 + v^2)}B_0 \qquad (6.23)$$

$$B_0 = \frac{(c^2 + v^2)}{2vc^2}E_0 \qquad (6.24)$$

And we can also rewrite the relation (6.18) by substituting it (6.23)

$$B = \frac{1}{\Theta_{-\pm}} \cdot \left(B_0 - \frac{v}{c^2}\left(\frac{2vc^2 . B_0}{c^2 + v^2} \right) \right)$$

$$B = \frac{1}{\Theta_{-\pm}} \cdot \left(B_0 - \left(\frac{2v^2 . B_0}{c^2 + v^2} \right) \right)$$

$$B = \frac{1}{\Theta_{-\pm}} \cdot \left(B_0 \left(\frac{c^2 - v^2}{c^2 + v^2} \right) \right)$$

Finally, we adjust, according to the transfer, relations (6.7) and (6.8)

$$B = \Theta_{+\pm} \frac{c^2}{c^2 - v^2}\left(B_0 \left(\frac{c^2 - v^2}{c^2 + v^2} \right) \right)$$

$$B = \Theta_{+\pm} B_0 \frac{c^2}{c^2 + v^2} \tag{6.25}$$

7. Volume energy density of the electric and magnetic fields

Now we express the volume energy density of the electric field [2]. Suppose that:

$$B = \frac{E}{v} \tag{7.1}$$

and also in general:

$$v = \frac{1}{\sqrt{\mu \varepsilon}} \tag{7.2}$$

$$w_E = \frac{1}{2}\vec{E}\vec{D} = \frac{1}{2}\varepsilon E^2 \tag{7.3}$$

And the the volume energy density of the magnetic field

$$w_M = \frac{1}{2}\vec{H}\vec{B} = \frac{1}{2}\frac{1}{\mu}B^2 \tag{7.4}$$

(7.1) substitute into (7.4)

$$w_M = \frac{1}{2}\frac{1}{\mu}\frac{E^2}{v^2}$$

And from (7.2) we calculate

$$\frac{1}{\mu} = \varepsilon v^2$$

And substitute.

$$w_M = \frac{1}{2}\varepsilon v^2 \frac{E^2}{v^2} = \frac{1}{2}\varepsilon E^2 \tag{7.5}$$

For the total volume energy density we can write that

$$w_E + w_M = \varepsilon E^2 \tag{7.6}$$
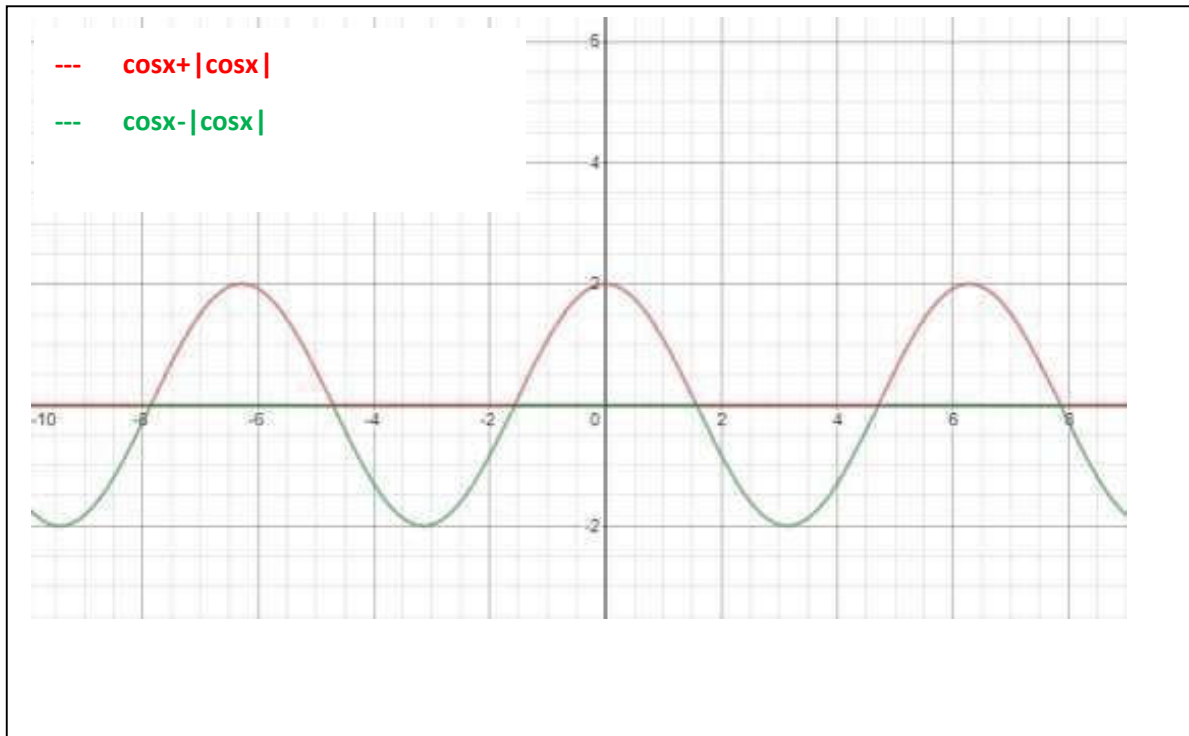
8. Average value of function Θ

Consider equation (6.22) where we are interested in the function (6.6)

Let's take a closer look at the speed of light in vacuum. We substitute $v = c$ into equation (6.6), ie:

$$\Theta_{c+\pm} = \frac{c}{c}\cos\alpha \pm \sqrt{1 - \frac{c^2}{c^2}(1 - \cos^2\alpha)}$$

$$\Theta_{c+\pm} = \cos\alpha \pm \sqrt{\cos^2\alpha}$$

$$\Theta_{c+\pm} = \cos\alpha \pm |\cos\alpha| \tag{8.1}$$

From this it can be seen that the solution is one or the other halfwave of the $2cos\alpha$ function, which would be generated by adding these halfwaves. Then if we start from the relation (6.22), we can write:

$$E(c) = E_0 \frac{2cos\alpha}{2} = E_0 cos\alpha \qquad\qquad \text{tj. for v=c}$$

After generalization

$$E(v) = E_0 \frac{\left[\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - cos^2\alpha)}\right] + \left[\frac{v}{c}cos\alpha - \sqrt{1 - \frac{v^2}{c^2}(1 - cos^2\alpha)}\right]}{2}$$

$$E(v) = E_0 \frac{v}{c} cos\alpha \qquad\qquad\qquad (8.2)$$

The same result gives calculation of the average value of the probability function. The function $\Theta\pm$ is actually an expression of two functions, each of which is equally probable, with a probability of 50% and 50%, as there is no indication that it is different. Thus, the average value can be calculated from these functions, which according to [3] is equal to:

$$\Theta_{EX} = \sum_{i=1,2} \frac{1}{2}\Theta_i$$

$$\Theta_{EX} = \frac{1}{2}\left[\frac{v}{c}cos\alpha + \sqrt{1 - \frac{v^2}{c^2}(1 - cos^2\alpha)}\right] + \frac{1}{2}\left[\frac{v}{c}cos\alpha - \sqrt{1 - \frac{v^2}{c^2}(1 - cos^2\alpha)}\right]$$

$$\Theta_{EX} = \frac{v}{c} cos\alpha \qquad\qquad\qquad (8.3)$$

Thus, the expression (8.2) corresponds to (8.3)

We actually use the average value Θ_{EX} together for two different functions that can refer to two different spacetime points simultaneously. This could therefore be a possible explanation for Young's experiment, where we consider that the photon occurs in both slots simultaneously. However, if we observe it, it will cease to be probable at that moment and continue its further probable existence away from the point of observation (ie the analogy of the collapse of the wave function)

9. Application to Young's experiment
We will remind [4] the analysis of this experiment in the picture.
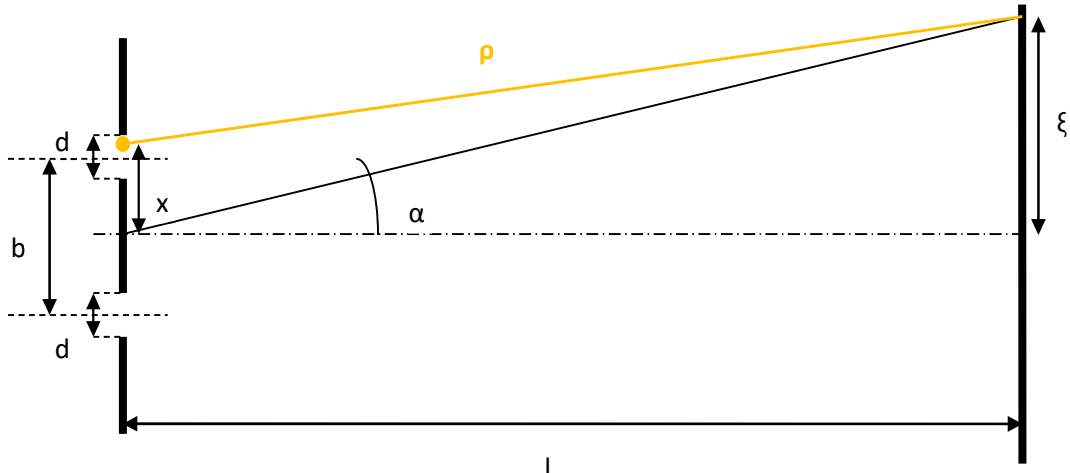
The electric intensity of a cylindrical wave can be described by the function:

$$E(\rho) = \frac{E_0}{\sqrt{\rho}}\left[e^{i\,(\omega t - k\rho)}\right] \tag{9.1}$$

Which we can rewrite according to our equations (8.2) or (8.3) to:

$$E(\rho) = \frac{E_0}{\sqrt{\rho}}\left[\frac{v}{c}\cos{(\omega t - k\rho)}\right] \tag{9.2}$$

Then the contribution to the electric intensity on the screen at the location with the co-ordinate ξ belonging to the elementary source of width dx at the co-ordinate x is

$$dE_x(\xi) = \frac{E_0}{\sqrt{\rho}}\left[\frac{v}{c}\cos{(\omega t - k\rho)}\right]dx \tag{9.3}$$

Where E0 is a constant, ρ is the distance of the line on the screen from the line source and can be expressed by l, ξ, x as:

$$\rho = \sqrt{l^2 + (\xi - x)^2} \tag{9.4}$$

The following adjustments are made in the calculations:

$$\rho = \sqrt{l^2 + (\xi - x)^2} = \sqrt{l^2 + \xi^2 - 2\xi x - x^2} = \sqrt{l^2 + \xi^2}\sqrt{1 - \frac{2\xi x}{l^2 + \xi^2} + \frac{x^2}{l^2 + \xi^2}}$$

$$\rho = \sqrt{l^2 + \xi^2}\sqrt{1 - \frac{2\xi x}{l^2 + \xi^2} + \frac{\xi^2 x^2}{(l^2 + \xi^2)^2}}$$

$$\rho = \sqrt{l^2 + \xi^2}\left(1 - \frac{\xi x}{l^2 + \xi^2}\right)$$

$$\rho = \sqrt{l^2 + \xi^2} - x\frac{\xi}{\sqrt{l^2 + \xi^2}}$$

Applies

$$sin\alpha = \frac{\xi}{\sqrt{l^2 + \xi^2}}$$

Then

$$\rho = \sqrt{l^2 + \xi^2} - x sin\alpha$$

The total electrical intensity at the screen coordinate ξ is given by the sum of the intensity contributions from all linear slot sources. Thus:

$$E(\xi) = \int_{1\,slit} dE_x(\xi) + \int_{2\,slit} dE_x(\xi) \tag{9.5}$$

$$E(\xi) = \int_{\frac{b}{2}-\frac{d}{2}}^{\frac{b}{2}+\frac{d}{2}} \frac{E_0}{\sqrt{l}}\left[\frac{v}{c}\cos\left[\omega t - k\left(\sqrt{l^2+\xi^2} - x\sin\alpha\right)\right]\right]dx$$

$$+ \int_{-\frac{b}{2}-\frac{d}{2}}^{-\frac{b}{2}+\frac{d}{2}} \frac{E_0}{\sqrt{l}}\left[\frac{v}{c}\cos\left[\omega t - k\left(\sqrt{l^2+\xi^2} - x\sin\alpha\right)\right]\right]dx$$

$$\int_{\frac{b}{2}-\frac{d}{2}}^{\frac{b}{2}+\frac{d}{2}} \frac{E_0}{\sqrt{l}}\left[\frac{v}{c}\cos\left[\omega t - k\left(\sqrt{l^2+\xi^2} - x\sin\alpha\right)\right]\right]dx$$

$$= \int_{\frac{b}{2}-\frac{d}{2}}^{\frac{b}{2}+\frac{d}{2}} \frac{E_0}{\sqrt{l}}\left[\frac{v}{c}\cos\left[\omega t - k\sqrt{l^2+\xi^2} + kx\sin\alpha\right]\right]dx$$

$$= -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}\left[\sin\left[\omega t - k\sqrt{l^2+\xi^2} + k(\frac{b}{2}+\frac{d}{2})\sin\alpha\right] - \sin\left[\omega t - k\sqrt{l^2+\xi^2} + k(\frac{b}{2}-\frac{d}{2})\sin\alpha\right]\right] \tag{9.6}$$

And the second integral:

$$\int_{-\frac{b}{2}-\frac{d}{2}}^{-\frac{b}{2}+\frac{d}{2}} \frac{E_0}{\sqrt{l}}\left[\frac{v}{c}\cos\left[\omega t - k\left(\sqrt{l^2+\xi^2} - x\sin\alpha\right)\right]\right]dx$$

$$= \int_{-\frac{b}{2}-\frac{d}{2}}^{-\frac{b}{2}+\frac{d}{2}} \frac{E_0}{\sqrt{l}}\left[\frac{v}{c}\cos\left[\omega t - k\sqrt{l^2+\xi^2} + kx\sin\alpha\right]\right]dx$$

$$= -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}\left[\sin\left[\omega t - k\sqrt{l^2+\xi^2} + k(-\frac{b}{2}+\frac{d}{2})\sin\alpha\right] - \sin\left[\omega t - k\sqrt{l^2+\xi^2} + k(-\frac{b}{2}-\frac{d}{2})\sin\alpha\right]\right] \tag{9.7}$$

Let's mark a constant

$$k_0 = \omega t - k\sqrt{l^2+\xi^2} \tag{9.8}$$

We add equations (9.6) and (9.7), substitute (9.8) and then we can write:

$$E(\xi) = -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}\Big[\sin\Big[k_0 + k(\frac{b}{2}+\frac{d}{2})\sin\alpha\Big]$$
$$- \sin\Big[k_0 + k(\frac{b}{2}-\frac{d}{2})\sin\alpha\Big] + \sin\Big[k_0 + k(-\frac{b}{2}+\frac{d}{2})\sin\alpha\Big]$$
$$- \sin\Big[k_0 + k(-\frac{b}{2}-\frac{d}{2})\sin\alpha\Big]\Big]$$

$$E(\xi) = -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}\Big[\sin\Big[k_0 + k(\frac{b}{2}+\frac{d}{2})\sin\alpha\Big]$$
$$- \sin\Big[k_0 + k(\frac{b}{2}-\frac{d}{2})\sin\alpha\Big] + \sin\Big[k_0 - k(\frac{b}{2}-\frac{d}{2})\sin\alpha\Big]$$
$$- \sin\Big[k_0 - k(\frac{b}{2}+\frac{d}{2})\sin\alpha\Big]\Big]$$

The rule applies

$$sinx - siny = 2 \cos \left(\frac{x+y}{2}\right) \sin \left(\frac{x-y}{2}\right)$$

Then:

$$E(\xi) = -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}\left[2\cos(k_0)\sin\left[k\left(\frac{d}{2}+\frac{b}{2}\right)sin\alpha\right] + 2\cos(k_0)\sin\left[k\left(\frac{d}{2}-\frac{b}{2}\right)sin\alpha\right]\right]$$

Or:

$$E(\xi) = -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}2\cos(k_0)\left[\sin\left[k\left(\frac{d}{2}+\frac{b}{2}\right)sin\alpha\right] + \sin\left[k\left(\frac{d}{2}-\frac{b}{2}\right)sin\alpha\right]\right] \qquad (9.9)$$

We centered from parameter cos (k_0), which represents the phase shift, so we assign this member an average value, which is:

$$\langle\cos(k_0)\rangle = \frac{1}{\sqrt{2}} \qquad (9.10)$$

We can also write:

$$\langle E(\xi)\rangle = -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}\sqrt{2}\left[\sin\left[k\left(\frac{d}{2}+\frac{b}{2}\right)sin\alpha\right] + \sin\left[k\left(\frac{d}{2}-\frac{b}{2}\right)sin\alpha\right]\right] \qquad (9.11)$$

The following applies

$$sinx + siny = 2 \sin \left(\frac{x+y}{2}\right) \cos \left(\frac{x-y}{2}\right)$$

Then:

$$\langle E(\xi)\rangle = -\frac{v}{c}\frac{E_0}{ksin\alpha\sqrt{l}}\sqrt{2}\left[2\sin\left[k\left(\frac{d}{2}\right)sin\alpha\right]\cos\left[k\left(\frac{b}{2}\right)sin\alpha\right]\right] \qquad (9.12)$$

If we assign

$$\Psi = k\left(\frac{d}{2}\right)sin\alpha \qquad (9.13)$$

$$\Phi = k\left(\frac{b}{2}\right)sin\alpha \qquad (9.14)$$

Substituting (9.13), (9.14) into (9.12) we get

$$\langle E(\xi)\rangle = -\frac{v}{c}\frac{E_0 d}{\sqrt{l}}\sqrt{2}\frac{\sin\Psi\cos\Phi}{\Psi} \qquad (9.15)$$

According to (7.6), the total average volume energy density is then equal to:

$$\langle w_E\rangle + \langle w_M\rangle = \varepsilon\langle E(\xi)\rangle^2 \qquad (9.16)$$

$$\langle w_E\rangle + \langle w_M\rangle = \varepsilon\frac{v^2}{c^2}\frac{E_0{}^2 d^2}{l}2\frac{\sin^2\Psi\cos^2\Phi}{\Psi^2} \qquad (9.17)$$
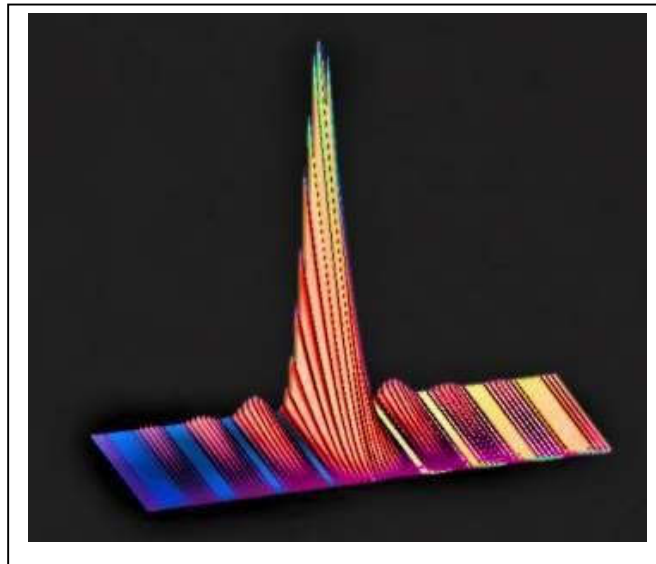
We can mark

$$\langle w_0\rangle = 2\varepsilon\frac{E_0{}^2 d^2}{l} \qquad (9.18)$$

Then

$$\langle w_E\rangle + \langle w_M\rangle = \frac{v^2}{c^2}w_0\frac{\sin^2\Psi\cos^2\Phi}{\Psi^2} \qquad (9.19)$$

The shape of this graph may look like this for certain Ψ and Φ:

It can be seen that the outline of the back of this 3d model is made up of the light speed function. The forward value decreases with the square of speed. In the front, it is zero, for zero speed. Thus, we calculated the possible solution of Young's experiment from the Lorentz transformation.

Odkazy:

[1]. https://www.sjpub.org/sjp/abstract/sjp-220.html
[2]. http://reseneulohy.cz/1404/objemova-hustota-energie-el-a-mag-pole
[3]. https://en.wikipedia.org/wiki/Expected_value?oldid=940213941
[4]. http://physicstasks.eu/1698/youngs-experiment-including-impact-of-diffraction