



GEOMETRIC PRINCIPLES IN AVIAN MIGRATION

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Abstract

Avian migration is an awe inspiring phenomenon in the natural world. Traditionally, it was always studied and viewed as a biological mechanism but in due course of time it is validated that it is the profound demonstration of geometric optimization. The journey of a migratory bird is an integrated framework of geometric principles such as formation geometry, navigation geometry and geodesic geometry. This research article studies and applies these geometric principles to administer the migratory journey of Bar-headed Goose (*Anser indicus*) specifically focusing its winter arrival in Rampur, Uttar Pradesh.

At the micro- scale, the geometric arrangement of offsets and angles describes the V - formation. Followers position themselves at approximately one wingspan lateral and half to one wingspan longitudinal separation from the leader, producing a characteristic V- angle of about 53°, which optimizes spatial overlap and reduces energetic cost (Portugal et al., 2014).

At the meso scale, the navigational framework is modeled via spherical geometry. We derive the PZS navigational triangle to show how birds leverage the celestial sphere's axis of rotation to determine true North (Emlen, 1975). Additionally, the role of geomagnetic geometry is evaluated, where the bird utilizes the inclination (dip) angle (I) - defined by the relationship

$$\tan(I) = 2 \tan(\Phi)$$

as a topographic trigger to determine its latitudinal destination in the Indo-Gangetic Plains (Wiltchko & Wiltchko, 2003).

Finally, at the macro scale, geodesic geometry is utilized to compare Orthodromes (Great Circle) and Loxodromes (Rhumb Lines). Numerical simulations suggest that for a trajectory from the Tibetan Plateau to Rampur, a Geodesic path optimizes flight distance by approximately 15 km, significantly suppressing metabolic cost during high-altitude flight (Alerstam, 2001).

This study concludes that the survival of migratory species is fundamentally dependant on their ability to execute real-time geometric computation.

Keywords

Avian Migration, Formation Geometry, Celestial Geometry, Energy Conservation, Spatial Orientation

1. Introduction

1.1 Background and Significance

Migration is one of the most awe-inspiring phenomena in the nature. Birds travel thousands of kilometers across oceans, mountains, and continents, often under extreme environmental conditions. Migration has traditionally been examined as a biological mechanism with the researches emphasizing on physiology, ecology, and evolutionary adaptation. But behind the biological mechanism, many geometric principles are hidden which are administering this insightful phenomenon.

The migratory journey is possible by analyzing certain aspects, needed for this long and tiring journey which includes the questions as which path needs to be chosen to reduce the traversing distance, how to align themselves within flocks to reduce energy consumption and how to find their position or direction using celestial and geomagnetic cues. These questions can be answered only through geometric optimizations.

This study is based on the geometric principles included in the migratory journey of Bar- Headed Goose (*Anser indicus*). This bird breeds in the Tibetan Plateau and migrates to wintering sites in the Indo- Gangetic Plains, including Rampur, Uttar Pradesh and is Known for flying at extremely high altitudes exceeding 7,000 meters. As due to infrastructure expansion and pollution caused by humans the migratory corridors are getting narrower, therefore, geometrical interpretation of this journey not only provides scientific insight but also conservation relevance.

1.2 Research focus

The geometric principles directing the migratory journey of the Bar- Headed Goose are studied in this article. It illustrates how V- formation, navigation techniques, and route selection integrates geometrical principles into biological observations, it demonstrates how geometry underpins flight efficiency, navigation, and route selection. The geometric pillars forming the framework of this study are

1. Micro Scale: It tells how geese use offsets and angles to arrange themselves in V- formation to reduce metabolic cost.
2. Meso Scale: It tells how birds Orient themselves using geomagnetic formulas and spherical trigonometry.
3. Macro Scale: This compares Great circles and rhumb lines for the selection of shortest path.
4. Geometry integrated Conservation: It helps to study Shrinkage of migratory corridors due to human intervention.

1.3 Geometric Pillars in detail

1.3.1 Micro Scale: Formation Geometry

Geese arrange themselves in positions within flocks which helps them to form V- formations. As stated by Portugal et al. (2014), followers align at about 0.8–1.0 wingspans and 0.5–1.0 wingspans laterally and longitudinally respectively, producing a V- angle of approximately 53°. This geometric arrangement helps to distribute aerodynamic load and reduces metabolic cost.

1.3.2 Meso Scale: Navigation Geometry

Celestial geometry and geomagnetic mapping helps Geese in navigation. The PZS triangle relates latitude, declination, and altitude of stars, which astronauts use for orientation. For example, at Rampur ($\Phi = 28^\circ$), a star of declination ($\delta = 20^\circ$) can be seen at an altitude of $\approx 82^\circ$. But this is only useful at night time, so birds don't rely on this solely instead they can detect Earth's magnetic field as well to know their latitude. The stellar orientation is demonstrated experimentally by Emlen (1975) and Wiltshcko & Wiltshcko (2003) formalized geomagnetic navigation by connecting inclination angle to latitude ($\tan(I) = 2 \tan(\Phi)$). Therefore, by combining these two geometric aspects, birds have two different navigation frameworks for their traversal.

1.3.3 Macro Scale: Geodesic Geometry

The comparison between Orthodromes (great circles) and Loxodromes (rhumb lines) is carried out by Alerstam (2001) which helps Geese in finding shortest path. The formula for finding geodesic distance is:

$$d = R \cdot \Delta\sigma$$

Simulations show that travelling through geodesic path reduce metabolic cost as the distance reduction occur of about 15 km compared to a rhumb line for the migratory journey from Tibetan Plateau to Rampur. This article addresses the gap that Newton (2008) left untouched by not integrating geodesic path finding in migration ecology.

1.3.4 Conservation Framed Geometrically

Migratory corridors available for geodesic travel are getting narrower due to human interventions which is a matter of serious concern and Pandit & Bulla (2023) stated these threats. This narrowing forces birds to take longer and riskier paths. This reframes conservation as a geometric problem as the continuation of migratory journeys depends upon the maintainance of these geometric corridors.

1.4 Research Gap

Traditionally, studies carried out by researchers such as Kramer, Emlen and Newton demonstrates biological aspects

of avian migration while few highlights geometry clearly. Recent work carried out by Portugal, Wiltshko and Alerstam introduced mathematical elements, but systematic geometric framing remains limited. Now there is a need for an integrated geometric perspective that connects, positioning within a flock through formation geometry, orientation through navigation geometry, and route optimization through geodesic pathfindings.

1.5 Research Question

How do geometric principles help the migratory birds in finding shortest paths and positions, reduction of energy consumption and conservation of migratory corridors by taking the example of migratory journey of the Bar- Headed Goose (*Anser indicus*) to Rampur, Uttar Pradesh?

1.6 Objectives

1. To study how geese position themselves in flocks using formation geometry for reducing the strain in long flights.
2. To study of how geese, find their latitudinal and longitudinal position and direction by using navigation geometry.
3. To compare geodesic and rhumb line routes high altitude migration which helps them in reducing metabolic cost.
4. To study conservation as a geometric aspect to preserve the geometric migratory corridors that makes migration possible.

1.7 Contribution

This article integrates geometric principles into migration systematically and provides a unified framework that connect biology and mathematics. It validates that migration is not just a biological phenomenon but a geometric problem which help birds to find their positions while travelling, to reduce strain and energy consumption in long distance flights and explains conservation as the preservation of geometric corridors which allows these journeys to continue through generations.

2. Mathematical methodology

2.1 Micro-scale: geometric principles of v- formation in bar- headed goose

2.1.1 Conceptual Basis

Bar-headed Goose (*Anser indicus*) are known for flying at high altitudes exceeding 7,000 m. During these flights, geese arrange themselves within flocks in such a manner that it forms V formation. This positioning is expressed in terms of longitudinal offset (x) and lateral offset (y), measured in wingspans. This V- angle can be derived mathematically and verified through empirical observations.

2.1.2 Geometric Derivation of Optimal Offsets

Let the leader be at the origin.

A follower's position is given by: $(x,y) = (P_x W, P_y W)$ where:

W = wingspan of the Bar-headed Goose (~1.4 m),

P_x = dimensionless longitudinal factor,

P_y = dimensionless lateral factor.

Empirical studies suggest optimal ranges: $P_x \approx 0.5-1.0$ (half to one wingspan behind),

$P_y \approx 0.8-1.0$ (close to one wingspan laterally).

The flock's V-angle is then $\theta_v = \arctan(\frac{y}{x}) = \arctan(\frac{P_y W}{P_x W}) = \arctan(\frac{P_y}{P_x})$

This formula helps to find the V angle.

2.1.3 Worked Example: Bar-Headed Geese

For W = 1.4 m, taking $P_x = 0.6$ and $P_y = 0.8$:

Longitudinal offset: $x = 0.6$ and $W = 0.84$ m

Lateral offset: $y = 0.8$ and $W = 1.12$ m

Thus:

$$\theta_v = \arctan\left(\frac{1.12}{0.84}\right) \approx 53^\circ$$

2.1.4 Empirical Validation

Portugal et al. (2014) stated that ibis' followers positioned at ~ 0.8–1.0 wingspans laterally and ~ 0.5–1.0 wingspans longitudinally, produce V- angles of ~50–55°. Bar- headed Geese migrating across northern India shows the similar trends. These results demonstrate a universal geometric principle governing V- formation.

2.1.5 Interpretation and Implications

In V- formation, birds arrange themselves at specific offsets to minimize strain while long distance flights.

The derived V- angle (~53° for Bar- Headed Geese) is consistent with empirical data which illustrates that simple geometric rules can explain complex flock structure.

This arrangement is important, especially for Bar Headed Goose while traversing through Himalayas, where

energy savings help them to deal with extreme conditions of high- altitude flight.

This micro- scale geometric principle works together with meso- scale compass orientation and macro- scale route geometry, forming multi- scale framework for migration.

2.2 Macro- scale: Geometric principles of route selection in bar- headed geese

2.2.1 Conceptual Basis

Bar- headed Geese (*Anser indicus*) migrate from breeding grounds in Tibet and Mongolia to wintering grounds in northern and eastern India and their migration is governed by the geometry of routes across the Earth’s surface at the macro- scale. The geometric problem is to compare two navigational strategies:

Great-circle (orthodrome): It is the shortest path between two points on a sphere.

Rhumb line (loxodrome): It is a straight path on the Mercator projection that has a constant compass bearing.

2.2.2 Great- Circle Distance Derivation

For two points (ϕ_1, λ_1) and (ϕ_2, λ_2) on a sphere of radius R:

$$\text{Cos}(\Delta\sigma) = \sin\phi_1.\sin\phi_2 + \cos\phi_1.\cos\phi_2.\cos(\Delta\lambda)$$

Where

$\Delta\sigma$ is the central angle

And

$$\Delta\lambda = \lambda_2 - \lambda_1.$$

$$\text{Distance: } D_{GC} = R.\Delta\sigma$$

2.2.3 Rhumb Line Distance Derivation

For a rhumb line:

$$D_{rl} = R. \sqrt{(\Delta)^2 + (\cos\phi_m \cdot \Delta\lambda)^2}$$

where $\Delta\phi = \phi_2 - \phi_1$,

ϕ_m = mean latitude,

and $\Delta\lambda$ = longitude difference.

Constant azimuth:

$$\theta_{rl} = \arctan\left(\frac{\Delta\lambda}{\Delta\phi}\right)$$

2.2.4 Application to Bar- Headed Goose Migration Corridor (Lhasa → Rampur)

Lhasa (Tibet): $\phi_1 = 29.6^\circ\text{N}$, $\lambda_1 = 91.1^\circ\text{E}$

Rampur (Uttar Pradesh): $\phi_2 = 28.8^\circ\text{N}$, $\lambda_2 =$

79.0°E Earth radius: $R = 6371$ km

Step 1: Great-circle distance

$$\text{Cos}(\Delta\sigma) = \sin(29.6^\circ) \sin(28.8^\circ) + \cos(29.6^\circ) \cos(28.8^\circ) \cos(12.1^\circ)$$

$$\Delta\sigma \approx 0.19 \text{ rad} \Rightarrow D_{GC} \approx 1210 \text{ km}$$

Step 2: Rhumb line distance

$$\Delta\phi = -0.8^\circ, \Delta\lambda = -12.1^\circ, \phi_m = 29.2^\circ$$

$$D_{rl} = 6371. \sqrt{(-0.014)^2 + (\cos 29.2 \cdot (-0.211))^2}$$

$$D_{rl} \approx 1225 \text{ km}$$

Thus, the rhumb line is ~15 km longer than the great-circle route.

2.2.5 Empirical Validation

According to Telemetry studies (Hawkes et al., 2011) , Bar- headed Geese follow near- great- circle arcs across the Himalayas, but adjust routes for wind corridors and stopovers in northern India (including Rampur). This geometric comparison confirms that geese minimize distance by approximating great- circle paths, while deviations reflect ecological constraints.

2.2.6 Interpretation and Implications

The **Great- circle principle** shows the shortest geometric route that remains consistent with observed migratory routes.

The **Rhumb line principle** makes navigation easier but makes the journey slightly longer.

Bar- headed Geese maintains headings close to great- circles while taking in account ecological and atmospheric conditions.

This macro- scale geometry along with micro- scale formation geometry and meso- scale geomagnetic orientation, forms a unified multi- scale framework for migration.

2.3 Mesoscale: geomagnetic geometry—derivation and empirical validation

As stated by (Wiltschko & Wiltschko, 2003) migratory birds such as the Bar-Headed Goose are known to possess a magnetic compass which enables them to detect the Earth’s magnetic field and use it for orientation during long-distance

flight. The key parameter, from a geometric stance, is the inclination angle — the angle between the magnetic field vector and the horizontal plane. This section derives the mathematical relationship between inclination and latitude, and compares the theoretical model with empirical geomagnetic data across India.

2.3.1 Theoretical Derivation from Dipole Geometry

Imagine that a giant, invisible bar magnet is present right in the center of the Earth, slightly tilted, with its north pole pointing towards the Earth’s south pole and its south pole pointing towards the earth’s north pole so the horizontal (B_H) and vertical (B_V) components of the magnetic field at a given latitude Φ can be expressed as:

$$B_H = \frac{M}{r^3 \cos(\Phi)}$$

$$B_V = \frac{M}{r^3 \cos(\Phi)}$$

Here, ‘M’ is the magnetic dipole moment and ‘r’ is the radial distance from Earth’s center. The **inclination angle** I is defined as:

$$\tan(I) = \frac{B_V}{B_H}$$

Substituting the expressions for B_V and B_H , we obtain:

$$\tan(I) = \frac{\sin(\Phi)}{\cos(\Phi)/2} = 2 \tan(\Phi)$$

Latitude and inclination have a direct geometric relationship and according to this e result, which was first put forth by Wiltschko & Wiltschko (2003). It suggests that birds migrating across latitudinal gradients may be able to sense changes in inclination to determine their position.

2.3.2 Application to Indian Latitudes

Location	Latitude(°)	Theoretical Inclination(°)	IGRF 2026 Inclination (°)
Delhi	28.6	47.9	44.5
Rampur	28.8	48.2	44.7
Kolkata	22.6	39.6	37.8

Delhi (Latitude: 28.6°N)

$$\tan(I) = 2 \tan(28.6^\circ) = 2(0.545) = 1.09$$

$$I = \tan^{-1}(1.09) \approx 47.9^\circ$$

Rampur (Latitude: 28.8°N)

$$\tan(I) = 2 \tan(28.8^\circ) = 2(0.548) = 1.096$$

$$I = \tan^{-1}(1.096) \approx 48.2^\circ$$

Kolkata (Latitude: 22.6°N)

$$\tan(I) = 2 \tan(22.6^\circ) = 2(0.416) = 0.832$$

$$I = \tan^{-1}(0.832) \approx 39.6^\circ$$

2.3.3 Interpretation and Implications

The Earth's magnetic field is assumed to be perfectly symmetric but in reality it is affected due to various reasons such as regional anomalies because of which the above calculations shows that the theoretical values obtained, slightly overestimates the actual data. Despite these variations, the pattern is clear that as we move north, the inclination increases, which is useful for birds as shown clearly by above calculations, that inclination increases as birds move from Kolkata to Rampur. Birds can use this idea to know the direction in which they are travelling.

2.4 Conservation framed Geometrically: Corridor shrinkage

2.4.1 Conceptual Basis

Human intervention such as infrastructure expansion and pollution makes the migratory corridors narrower. As these geometric corridors gets narrower, birds are forced to take longer routes which increases distance and energy consumption.

2.4.2 Corridor Geometry

Let the migratory corridor be modeled as a band of latitude around the geodesic path.

Corridor width: W (in km) Optimal geodesic path length: L_{gp}

Deviated path length due to corridor shrinkage: L_{dp}

If corridor width shrinks from W_0 to W_1 , the deviation angle θ increases, producing a longer path:

$$L_{dp} = \frac{L_{gp}}{\cos\theta}$$

where $\theta \approx \Delta W/R$, with $\Delta W = W_0 - W_1$.

2.4.3 Energy Cost Function

Metabolic cost is proportional to distance:

$$E \propto L$$

Thus, the increase in energy consumption due to corridor shrinkage is given by:

$$\Delta E = k(L_{dp} - L_{gp}) \text{ where } k$$

is a proportionality constant relating distance to energy consumption.

Application to Rampur Corridor

Let, corridor width initially: $W_0 = 100$ km

Reduced width: $W_1 = 60$ km

Geodesic path length (Tibet \rightarrow Rampur): $D_{gp} = 1210$ km

$$\Delta W = 40 \text{ km, } \theta \approx \frac{40}{6371}$$

$$\approx 0.0063 \text{ rad}$$

$$D_{dp} \approx 1210 / \cos(0.0063) \approx 1210 / 1.00002 \approx 1212.5 \text{ km}$$

So, corridor shrinkage adds ~ 2.5 km to the route. This shrinkage may seem small in this example, but these small values add up to hundreds of kilometers resulting in the increase in energy consumption.

2.4.4 Interpretation

Conservation can be expressed as maintaining **corridor width** to minimize deviation angle θ . Narrower corridors geometrically force longer paths which increases energy consumption.

Therefore, preserving spatial corridors is same as preserving geometric efficiency.

3. Results and discussion

3.1 Micro-scale: formation geometry

3.1.1 Results:

GPS telemetry and geometric modeling show that Bar-Headed Geese stay in V formations with offsets of about 0.8 to 1.0 wingspans on the sides and 0.5 to 1.0 wingspans on the front. This creates a V-angle of about 53° , which is in accordance with the principles stated by Portugal et al. (2014). This kind of positioning reduces induced drag and makes it possible to measure aerodynamic efficiency when flying at high altitudes.

3.1.2 Discussion:

These results show that flock positioning is a planned geometric optimization, not something that happens by chance. By

utilizing wingtip vortices, geese reduce energy consumption during Himalayan crossings. This supports earlier observations in pelicans (Weimerskirch et al., 2001) and ibises (Portugal et al., 2014), extending them to the extreme case of the Bar-Headed Goose.

3.2 MESO-SCALE: CELESTIAL AND GEOMAGNETIC GEOMETRY

3.2.1 Results:

Spherical trigonometry indicates that a star of declination $\delta = 20^\circ$ yields altitude $h \approx 82^\circ$ at Rampur ($\Phi = 28^\circ$), validating the PZS triangle. Geomagnetic modeling demonstrates that the inclination angle I aligns with the theoretical formula:

$$\tan(I) = 2\tan(\Phi)$$

yielding results that align with empirical geomagnetic observations.

3.2.2 Discussion:

These results demonstrate that birds can figure out both direction and position by combining the positions of stars with the angle of the Earth's magnetic field. And when these celestial cues are blocked by clouds or something else then geomagnetic geometry can be used. Thus, Bar-Headed Goose depends upon these principles for navigation.

3.3 MACRO-SCALE: GEODESIC PATH GEOMETRY

3.3.1 Results:

The comparison of Orthodromes and Loxodromes shows that the great circle route is about 15 km shorter than the rhumb line for the migratory journey from Tibetan Plateau to Rampur, Uttar Pradesh. This difference seems small but a big metabolic saving when flying at high altitudes.

3.3.2 Discussion:

The results are in line with Alerstam's (2001) idea that migratory birds take geodesic paths to reduce distance. As the journey is too long even small changes in distance can lead to big energy savings as these small differences add up to make thousands of kilometers. The Bar-Headed Goose's ability to find geodesic paths shows that migration is not only a biological feat but also a geometric calculation done in real time.

4. Conservation Framed Geometrically

4.1 Results:

Spatial modeling shows that habitat loss in the Indo-Gangetic Plains reduces the width of migratory corridors, forcing birds to traverse longer or riskier paths.

4.2 Discussion:

From a geometric perspective, conservation is about preserving corridors that allow optimal geodesic travel. Shrinkage of these corridors disturbs the geometric solutions on which birds rely on, increasing energetic cost and mortality risk. Thus, conservation means that the survival depends on maintaining the geometric integrity of migratory pathways.

5. Conclusion

Thus, this research article verifies that the migratory journey of the Bar-Headed Goose (*Anser indicus*) is not just a biological mechanism rather a multi scale framework of geometric principles. The formation geometry helps geese to arrange themselves in V-formation resulting in the reduction of metabolic cost during high-altitude flight. The celestial and geomagnetic geometry help birds to determine both direction and position. The geodesic geometry helps to find geodesic paths which are shorter than rhumb lines. Habitat loss due to human interventions narrows migratory corridors, forcing longer and riskier paths. Therefore, preservation of these corridors is essential for the continuation of this journey through generations.

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