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*$(\hat{g}r)_h$ - SEPARATION AXIOMS

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Abstract

In this paper, we are trying to define some new types of separation axioms in hexa topological spaces using , *$(\hat{g}r)_h$ - continuous functions *$(\hat{g}r)_h$ - irresolute functions, *$(\hat{g}r)_h$ - open maps and *$(\hat{g}r)_h$ - closed maps. We also introduce the ideas of *$(\hat{g}r)_h$ -T_0 *$(\hat{g}r)_h$-T_1 and *$(\hat{g}r)_h$-T_2 spaces. Along with that, some basic properties of these spaces are also studied and discussed.

Keywords: *$(\hat{g}r)_h$ -T_0, *$(\hat{g}r)_h$ - T_1, *$(\hat{g}r)_h$ -T_2

1. Introduction

The concept of generalized topological space became more important, which was introduced by Császár. In this approach, instead of open sets, a family of subsets was considered to study basic separation axioms (Császár [3], 2004). Then, Roy [10] in 2010 introduced separation axioms R0 and R1 in generalized topological spaces and also explained some of their properties. Later, M.K. Min [5] in 2011 studied relative separation axioms and discussed their properties and relationships .In 2015, Rajendran V, Vijesh C and others [8, 9] introduced *(gr)-closed sets and *(gr)-continuous functions. Christal Bai. I and Shyla isac mary. T[2] introduced (gg)* Seperations axioms in 2020. R.V. Chandra and V. Pushpalatha [1] introduced hexa closed and hexa continuous. J.sathyamalar and C.Dhanapakyam[14,15] studied the notion of $*(\hat{g}r)_h$ - closed sets and $*(\hat{g}r)_h$ - continuous functions in hexa topological spaces. Separation axioms in hexa-topological spaces were studied by Nabeela Abu-Alkishik et al. [7] (2026).

2. Preliminaries

Definition 2.1: A subset A of a topological space (X,τ_h) is called :

1) r_h-closed [6] if $A=clh(int_h(A))$.
2) g_h- closed set [11] if $cl_h(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ_h).
3) hexa semi open [4] if $cl_h(int_h(A)) \subseteq A$.
4) gh-closed [11] set if $cl_h(A) \subseteq U$ whenever $A \subseteq U$ and U is hexa semi open in (X,τ_h).
5) $*(\hat{g}r)_h$ - closed [11] if $r_h\ cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}_h$-open in (X,τ_h) .

Definition 2.2: Let f: $(X,\tau_h)\to(Y,\sigma_h)$ be a mapping:

1) The function f is called hexa continuous [2] if the inverse image $f^{-1}(V)$ is a hexa closed set in (X,τ_h) whenever V is a hexa closed set in (Y,σ_h).
2) The function f is called g_h - continuous[12] if the inverse image $f^{-1}(V)$ is a g_h-closed set in (X,τ_h) whenever V is a hexa closed set in (Y,σ_h).
3) The function f is called $*(\hat{g}r)_h$ - continuous[12] if the inverse image $f^{-1}(V)$is $*(\hat{g}r)_h$- closed in (X,τ_h) whenever V is a hexa closed set in (Y,σ_h).

Definition 2.3:[11] Let (X, τ_h) be a hexa topological space and $A \subseteq X$. The $*(\hat{g}r)_h$ - cl (A) is defined by the intersection of all $*(\hat{g}r)_h$ - closed sets contained in A. That is $*(\hat{g}r)_h - cl(A) = \cap \{G : A \subseteq G$ and G is $*(\hat{g}r)_h$ - closed$\}$ then $*(g\ r)_h$ - closed of the subset A is denoted by $*(\hat{g}r)_h$-cl(A).

Definition 2.4: [12] A function f: $(X, \tau_h) \to (Y, \sigma_h)$ is said to be $*(\hat{g}r)h$- irresolute, if $f^{-1}(V)$ is $*(\hat{g}r)h$- closed set in (X, τ_h) for every $*(\hat{g}r)_h$ - closed set V in (Y, σ_h).

Definition 2.5: [13] A map f: $(X, \tau_h) \to (Y, \sigma_h)$ is said to be a $*(\hat{g}r)_h$ - closed map if the image of every hexa closed set in (X, τ_h) is $*(\hat{g}r)_h$ - closed in (Y, σ_h).

Definition 2.6:[13] A map f: $(X, \tau_h) \to (Y, \sigma_h)$ is said to be a $*(\hat{g}r)_h$ - open map if the image of every hexa open set in (X, τ_h) is $*(\hat{g}r)_h$ - open in (Y, σ_h).

Definition 2.7:[7] A hexa topological space (L,τ_H) is called

(1) Hexa -T_0 space if for any two distinct points h_1 ,$h_2 \in L$, there exists a H-open set contains only one point but not the other and it is denoted by H-T_0 space.
(2) A Hexa -T_1 space if for any two distinct points h_1 ,$h_2 \in L$, there exists two H-open sets U_{h1}, V_{h2} , such that $h_1 \in U_{h1}$, $h_2 \notin U_{h1}$ and $h_2 \in V_{h2}$, $h_1 \notin V_{h2}$ and it is denoted by H-T_1 space.
(3) A Hexa -T_2 space if for any two distinct points h_1 ,$h_2 \in L$, there exists two H-open sets U_{h1}, V_{h2} , such that $h_1 \in U_{h1}$, $h_2 \in V_{h2}$ and $U_{h1} \cap V^{h2}=\varphi$ and it is denoted by H-T_2 space.

3. $*(\hat{g}r)_h$ -T_k (k = 0 , 1, 2) SPACES

In this section, we are introducing some new type of separation axioms in hexa topological spaces, which we call $*(\hat{g}r)_h$ - T_k spaces for k = 0,1,2. Also, some basic properties of these spaces are discussed and studied.

Definition 3.1:A hexa topological space (X,τh) is said to be

1.$*(\hat{g}r)_h$ - T_0 if for each pair of distinct points x, y in X, there exists a $*(\hat{g}r)_h$ - open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.
2.$*(\hat{g}r)_h$ - T_1 if for each pair of distinct points x, y in X, there exist two $*(\hat{g}r)_h$ - open sets U and V such that x U but $y \notin U$ and $y \in V$ but $x \notin V$.
3.$*(\hat{g}r)_h$ - T_2 if for each pair of distinct points x, y in X, there exist two disjoint $*(\hat{g}r)_h$ - open sets U and V containing x and y respectively.
4.$*(\hat{g}r)_h$ - space if every $*(\hat{g}r)_h$ - open set of X is hexa open in X.

Theorem 3.2
(i) Every $*(\hat{g}r)_h$ - T_1 space is $*(\hat{g}r)_h$ - T_0.
(ii) Every $*(\hat{g}r)_h$ - T_2 space is $*(\hat{g}r)_h$ - T_1 space.

Proof :
(i) Let X be a $*(\hat{g}r)_h$ -T1 space. Let x, y ∈ X with x ≠ y. Since X is a $*(\hat{g}r)_h$ -T_1 space, there exists $*(\hat{g}r)_h$ - open sets U and V containing x and y such that x ∈ U and y ∉ U or x ∉ V and y ∈ V. Therefore X is a $*(\hat{g}r)_h$ -T_0 space.
(ii) Let X be a $*(\hat{g}r)_h$ - T_2 space. Let x, y ∈ X with x ≠ y. Since X is a $*(\hat{g}r)_h$ - T_2 space, there exists disjont $*(\hat{g}r)_h$ - open sets U and V containing x and y such that x ∈ U and y ∈ V. that is x ∉ V and y ∉ U. Therefore X is a $*(\hat{g}r)_h$ - T_1 space.

Theorem 3.3
A topological space X is $*(\hat{g}r)_h$ -T_0 iff $*(\hat{g}r)_h$ - closure of disjoint points are disjoint.
Proof:
Let X be a $*(\hat{g}r)_h$ -T_0 space. Let x, y ∈ X with x ≠ y. Since X is a $*(\hat{g}r)_h$ - T_0 space, there exists a $*(\hat{g}r)_h$ - open set U such that x ∈ U and y ∉ U or x ∉ U and y ∈ U.
Let us consider x ∈ U and y ∉ U. Then x ∉ X – U and y ∈ X – U. Since U is a $*(\hat{g}r)_h$ - open set, X – U is a $*(\hat{g}r)_h$ - closed set in X containing y but not x. But $*(\hat{g}r)_h$ - cl(y) ⊆ X – U. Since x ∉ X – U , x ∉ $*(\hat{g}r)_h$ - cl(y). But y ∈ $*(\hat{g}r)h$ - cl(y). Hence $*(\hat{g}r)_h$ - cl(x) ≠ $*(\hat{g}r)_h$ - cl(y). Similarly we can prove the other case. Hence $*(\hat{g}r)_h$ - closure of distinct points are distinct. Conversely suppose that $*(\hat{g}r)_h$ - cl(x) ≠ $*(\hat{g}r)_h$ - cl(y). If x ≠ y with x, y ∈ X. Then there exists atleat one point z ∈ X such that z ∈ $*(\hat{g}r)_h$ - cl(x) and z ∉ $*(\hat{g}r)_h$ - cl(y) or z ∉ $*(\hat{g}r)_h$ - cl(x) and z ∈ $*(\hat{g}r)_h$ - cl(y). Now let us consider z ∈ $*(\hat{g}r)_h$ - cl(x) and z ∉ $*(\hat{g}r)_h$ - cl(y). Suppose x ∈ $*(\hat{g}r)_h$ - cl(y). Then $*(\hat{g}r)_h$ - cl(x) ⊆ $*(\hat{g}r)_h$ - cl(y). Hence z ∈ $*(\hat{g}r)_h$ - cl(x) ⊆ $*(\hat{g}r)_h$ - cl(y) and so z ∈ $*(\hat{g}r)_h$ - cl(y). Which is a contradiction. Hence x ∉ $*(\hat{g}r)_h$ - cl(y). This implies x ∈ X – $*(\hat{g}r)_h$ - cl(y), which is a $*(\hat{g}r)_h$ - open set in X containing x but not y. Hence X is a $*(\hat{g}r)_h$ - T_0 space.

Theorem 3.4
Let f: (X, τ_h) → (Y, σ_h) be an injective, $*(\hat{g}r)_h$ - irresolute map. If Y is a $*(\hat{g}r)_h$ -T_0 then X is a $*(\hat{g}r)_h$ - T_0 space.
Proof:
Let x, y ∈ X with x ≠ y. Since f is injective, f(x) ≠ f(y). As Y is a $*(\hat{g}r)_h$ - T_0 space, there exists a $*(\hat{g}r)_h$ - open set U of Y such that f(x) ∈ U and f(y) ∉ U or f(x) ∉ U and f(y) ∈ U. Since f is $*(\hat{g}r)_h$ - irresolute, $f^{-1}(U)$ is $*(\hat{g}r)h$ - open set in X such that x ∈ $f^{-1}(U)$ and y ∉ $f^{-1}(U)$ or x ∉ f^{-1} (U) and y ∈ $f^{-1}(U)$. Hence X is a $*(\hat{g}r)_h$ - T_0 space.

Theorem 3.5
Let f: (X, τ_h) → (Y, σ_h) be an injective, $*(\hat{g}r)_h$ - irresolute map. If Y is a $*(\hat{g}r)_h$ -T_2 then X is a $*(\hat{g}r)_h$ - T_2 space.
Proof:
Let x, y ∈ X with x ≠ y. Since f is injective, f(x) ≠ f(y). As Y is a $*(\hat{g}r)_h$ - T_0 space, there exists a $*(\hat{g}r)_h$ - open set U of Y such that f(x) ∈ U and f(y) ∈V and U∩ V =φ . Since f is $*(\hat{g}r)_h$ - irresolute, $f^{-1}(U)$, $f^{-1}(V)$ are $*(\hat{g}r)_h$ - open set in X such that x ∈ $f^{-1}(U)$ and y ∈ $f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V)$ =φ . Hence X is a $*(\hat{g}r)_h$ -T_2 space.

Theorem 3.6
Let f: (X, τ_h) → (Y, σ_h) be an bijective, $*(\hat{g}r)_h$ - continuous map. If Y is a hexa - T_1 space then X is a $*(\hat{g}r)_h$ - T_1 space.
Proof :
Let x_1, x_2 ∈ X be such that $x_1 \neq x_2$. Since f is bijective, there exists y_1, y_2 in Y with $y_1 \neq y_2$ such that y_1= f(x_1) and y_2= f(x_2). Also since Y is a hexa - T_1 space, there exists hexa open sets U, V of Y such that y_1∈ U and y_1 ∉ V or y_2 ∈ V and y_1 ∉ U. Since f is $*(\hat{g}r)_h$ - continuous, there exists $*(\hat{g}r)_h$ - open sets $f^{-1}(U)$, $f^{-1}(V)$ in X such that $x_1 \in f^{-1}(U)$ and x1 ∉ f^{-1}V) or x2 ∈ $f^{-1}(V)$ and x2 ∉ $f^{-1}(U)$. Hence X is a $*(\hat{g}r)_h$ - T_1 space.

Theorem 3.7
Let f: (X, τ_h) → (Y, σ_h) be an bijective, $*(\hat{g}r)_h$ - open map. If X is a hexa - T_1 space then Y is a $*(\hat{g}r)_h$ - T_1 space.
Proof :
Let y_1, y_2 ∈ Y be such that $y_1 \neq y_2$. Since f is bijective, there exists x_1, x_2 in X with $x_1 \neq x_2$ such that y_1= f(x_1) and y_2= f(x_2). Also since X is a hexa - T_1 space, there exists hexa open sets U, V of X such that x_1 ∈ U and x_2 ∉ V or x_2 ∈ V and x1 ∉ U. Since f is $*(\hat{g}r)_h$ - open map, there exists $*(\hat{g}r)_h$ - open sets f(U), f(V) in Y such that f(x_1) ∈ f(U) and f(x_1) ∉ f(V) or f(x_2) ∈ f(V) and f(x_2) ∉ f(U). That is there exists $*(\hat{g}r)_h$ - open sets f(U), f(V) in Y such that y_1∈ f(U) and y_1 ∉ f(V) or y_2 ∈ f(V) and y_2 ∉ f(U). Hence Y is $*(\hat{g}r)_h$ - T_1 space.

Theorem 3.8
Let f: (X, τ_h) → (Y, σ_h) be an bijective, $*(\hat{g}r)_h$ - continuous function. If Y is a hexa - T_2 space then X is a $*(\hat{g}r)_h$ - T_2 space.

Proof :

Let x_1, x_2 ∈ X be such that x1 ≠ x2. Since f is bijective, there exists y_1, y_2 in Y with $y_1 \neq y_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Also since Y is a hexa - T_2 space, there exists disjoint hexa open sets U, V of Y such that $y_1 \in U$ and $y_2 \in V$. Since f is $*(\hat{g}r)_h$ - continuous, there exists disjoint $*(\hat{g}r)_h$ - open sets $f^{-1}(U)$, $f^{-1}(V)$ in X such that $x_1 \in f^{-1}(y_1) \in f^{-1}(U)$ and $x_2 \in f^{-1}(y_2) \in f^{-1}(V)$. Hence X is a $*(\hat{g}r)_h$ - T_2 space.

References:

1. R.V. Chandra and V. Pushpalatha , "Introduction to Hexa Topological Space (6-Tuple Topology) ", International Journal of Management, Technology And Engineering , 10(I) , 16-18, 2020.
2. Christal Bai. I and Shyla isac mary. T, On (gg)* Seperations axioms, Journal of Mathematical computional science, 10(3), 2020,474-478
3. A. Cs´asz´ar, Separation axioms for generalized topologies, Acta Math. Hungar., 104(1-2) (2004), 63-69.
4. T. Kalaiselvi and G. Sindhu, Introduction To Wh Closed Sets In Hexa Topological Spaces,International Journal of Creative Research Thoughts, 11(4), 2023 220-227 .
5. W. K. Min, Mixed weak continuity on generalized topological spaces, Acta Math. Hungar., 132(4) (2011), 339-347.
6. R. Mohanapriya and S.Santhiya, A New Notion of Hexa Regular Closed Sets via Hexa Topological Space, Spaces, 2023.
7. Nabeela Abu-Alkishik1,∗ , Diana Mahmoud2 , Maryam Alholi3 , Hamza Qoqazeh4 , Sanaa Khataybeh "Some Topological Properties in View of Hexa Topological Spaces"International Journal of Analysis and Applications ISSN: 2291-8639 Nov. 8, 2025.
8. Rajendran.V, Indirani.K, Vijesh.C and Suresh.N "On *(gr)-Closed Sets in Topological Spaces" nternational Journal of Mathematics Trends and Technology, 24 (2) – August 2015.
9. Rajendran V, Indirani K, Sathishmohan P, Vijesh C, Suresh N"On *(gr)-Continuous Functions and Contra*(gr)-Continuous Functions in Topological Spaces International Journal of Applied Research , 1 (10), 2015 ,940-944
10. B. Roy, On generalised R0 and R1 spaces, Acta Math. Hungar., 127(3) (2010), 291-300.
11. Sathya Malar. J and Dhanapakyam .C *(ĝr)h - Closed Sets In Hexa Topological Spaces ICMSAI-2025,Gobi Arts and Science college.
12. Sathya Malar. J and Dhanapakyam .C A New class of continuous functions in hexa topological spaces JTSEIT ,13(2), 2025,125-132.
13. Sathya Malar. J and Dhanapakyam .C ,*(ĝr)h - Homeomorphisms In Hexa Topological Spaces, International Journal of creative research thoughts (IJCRT), 14(7), 2026, 264-271.

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