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STATISTICAL MODELING AND MULTI-OBJECTIVE OPTIMIZATION OF ANTI-LOCK BRAKING CONTROL PARAMETERS UNDER VARIABLE OPERATING CONDITIONS

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Abstract:
When a single control setting must be used for all operating conditions, trade-offs exist between anti-lock braking performance and characteristics of tires and the load carried by a vehicle. This research quantified the effects of anti-lock braking control parameters on multiple braking-distance responses and identified a balanced configuration across varying tire and load conditions. A quantitative computational framework was applied to two control parameters and five braking-distance responses using descriptive statistics, Pearson correlation, quadratic response-surface modeling, parameter sensitivity assessment, multi-response optimization, and Pareto analysis. Control-parameter influence varied across braking conditions. The quadratic models achieved the strongest fit for y4 ($R^2 = 0.9356$) and the weakest for y3 ($R^2 = 0.2666$). Equal-weight optimization identified $x1 = -1.2$ and $x2 = -2.8$ as the best overall compromise, producing a composite score of 0.0936. The selected configuration was Pareto-optimal, while 405 of the 10,101 evaluated configurations (4.0095%) were non-dominated. Results show that the braking responses vary between tires and loads and are used in the support condition-aware parameter calibration. The response-surface modeling together with a Pareto-based optimization approach is an interpretable tool to optimize conflicting anti-lock braking performance criteria.

Keywords: *anti-lock braking system, response-surface modeling, multi-objective optimization, Pareto analysis, braking performance*

1. Introduction:

ABS systems are key to modern vehicle safety as they control wheel behavior during heavy braking without compromising vehicle control. They rely on the interaction of controller settings, tire–road interaction, vehicle dynamics and operating state during a braking event. The response to the application of varying control can then be very different even for the same braking problem. Comparative studies on anti-lock braking systems have revealed that the braking response and system performance depend directly on the controller design, and hence the choice of the control parameters over the operating conditions is crucial (Moaaz et al., 2020). The study of adaptive and intelligent control methods is also explored to enhance performance when the system behaviour changes during operation, specifically in the context of anti-lock braking (Abougarair et al., 2022). The performance of the control system is closely related to the dynamic properties of the car and car–road interaction. Anti-lock braking control must be able to handle time delays and nonlinearity and still perform its braking function in a desirable manner. The time delay can affect the dynamics of the anti-lock braking system, as shown in the stability analysis, and its effect on the control behavior should be taken into account (Ye et al., 2022). Thus, evaluating the controller design and assessment should be done systematically with regard to the operating states of interest and not at one operating state. To highlight the necessity of evaluating the anti-lock braking performance at controller level, Unguritu and Nichitelea (2023) showed the importance of such an approach. Quantitative approaches, which correlate controller setting with resultant system responses, are thus important.

Vehicle-dynamics modeling offers a foundation for studying these relationships, without breaking the brake controller characteristic. Vehicle behavior can be modeled in one of two ways: either physically or data-driven, where the choice of model structure affects the way the real-world dynamic responses are expressed (James et al., 2020). Vehicle-dynamics models also give a structured representation of the longitudinal and lateral motions for use in modelling and evaluation of control systems (Das, 2021). The reliability of those assessments will depend upon the extent to which dynamic behavior variations are represented. Engineering decisions based on vehicle-dynamics predictions may be affected by uncertainty in the application, so it is crucial to treat varying conditions in a systematic manner in model-based assessment (Funfschilling & Perrin, 2019). Additionally, virtual validation has emerged as a relevant method to validate vehicle-dynamics models that are applied to implementing electronic stability control systems before putting them into practice (Torres et al., 2024).

The study of anti-lock braking control has been using increasingly sophisticated methods to deal with the nonlinear feedback behavior of vehicles. The anti-lock braking problem has been solved with nonlinear model predictive control (Pretagostini et al., 2019) based on wheel information to generate control decisions in a dynamic setting. Extensive study of anti-lock braking systems indicates that controller design includes the development of both traditional, intelligent, and advanced controllers for improved braking performance in varying conditions, pointing to ongoing research and development efforts (Zhu et al., 2023). These advances have highlighted the significance of control design, and at the same time introduced a need for understandable statistical analysis of the relationship between variations in individual control parameters and multiple braking responses.

When more than one performance objective is to be considered, optimization is more complicated. Parameters that yield good performance under one set of operating conditions may not do so under a different set. Multi-objective optimization provides a way to determine parameter sets that optimize a set of responses instead of optimizing each response individually. The same principle has been increasingly applied in the field of automotive systems, particularly in optimizing multiple objectives across various levels of abstraction in electric-vehicle powertrain systems (Lin et al., 2025). With the electrification of vehicle architectures, braking has become more complex, and more and more performance requirements are involved in the development of braking systems, especially when regenerative braking is added to the system, which brings extra control and coordination considerations (Yang et al., 2024).

Although there have been many advances, there is still a gap in the interpretable statistical analysis of anti-lock braking control parameters for various braking objectives. Existing literature has focused on controller development, dynamic simulation, advanced control algorithms, or improvement to individual system performance. Few efforts have been made to unite three of the most important aspects of statistical association analysis, nonlinear response-surface characterization, parameter sensitivity and multi-response optimization within one framework. These types of integrations are helpful for determining how the control parameters affect multiple braking responses, whether those effects are the same under any given condition, and where acceptable compromises exist in the parameter space.

This gap is filled by the present study, which investigates the statistical modelling and multi-objective optimization of anti-lock braking control parameters under changing operating conditions. The goals of the study are to: Quantify relationships between anti-lock braking control parameters and several braking-performance objectives; Model response surfaces and determine the relative sensitivity of the different braking objectives to changes in the different anti-lock braking control parameters; Identify an equal-weight multi-response compromise solution and determine its Pareto optimality. These goals can be combined to give an interpretable description of parameter effects and to assess trade-offs between various requirements for braking performance.

2. Materials and Methods

2.1 Research Design

Anti-lock braking control parameters were used to evaluate the relationship between braking performance and various vehicle settings using a quantitative computational design. The data preparation, descriptive and correlation analysis, quadratic response surface modeling, parameter sensitivity analysis, multi-response optimization and Pareto analysis were included in the analytical framework. These same procedures were repeated for all the combinations of control parameters and braking responses available.

2.2 Data Source
Data came from the vehicle-dynamics resource developed by Thomaser et al. (2023). Two anti-lock braking control parameters (x1, x2) were included in the dataset, as were five braking-distance response variables (y1–y5). The response variables were different combinations of tire performance and vehicle load and were kept for the statistical modelling and optimisation analysis. The five conditions of the responses were as follows: y1 represented high-performance tires with a partially loaded vehicle, y2 represented medium-performance tires with a partially loaded vehicle, y3 represented under-performance tires with a partially loaded vehicle, y4 represented high-performance tires with a fully loaded vehicle, and y5 represented high-performance tires with a lightly loaded vehicle. The distances from the optimum of each of the responses were given in metres. .

2.3 Data Preprocessing
Other fields were excluded from the index due to the fact that they were not considered analytical variables. The rest of the data were checked for missing data, duplicate data and completeness of the control-parameter grid. The following descriptive statistics were computed for x1, x2, and y1, y2, y3, y4, and y5. Pearson correlation analysis was then used to quantify linear associations between the control parameters and the five braking-distance responses.

2.4 Quadratic Response-Surface Modeling
Two second-order response-surface models were fitted for each of the brake distance responses. The effects of x1 and x2, together with their interaction, and the quadratic terms of both were included in the models. Model adequacy was evaluated using R2, adjusted R2, root mean squared error, and mean absolute error. Model coefficients and the statistical significance of individual terms were also examined.

2.5 Parameter Sensitivity and Response-Landscape Analysis
Pearson correlation was used when the variables were continuous and numerical, and Spearman correlation when the variables were ordinal and suitable. The comparisons of the academic performance of binary and multi-category groups were conducted through independent-samples t-tests and one-way ANOVA. This was followed by multiple linear regression to establish the independent contribution of the predictors to the overall academic performance. The measure of multicollinearity was done with the help of variance inflation factors, and the level of statistical significance was set at $p < 0.05$. The special focus was on Last due to the close connection with the Overall.

2.6 Machine-Learning Model Development
All of the five responses of braking distance were rescaled to a common scale before optimization. The normalized responses were given equal weights, and the mean was computed to derive the composite criterion, with smaller values corresponding to better combined performance. To investigate the effect of individual responses, a weight sensitivity analysis was performed to analyse the preferred configuration of the parameter set. Finally, Pareto dominance analysis was used to find non-dominated configurations under the five different tire-load conditions.

3. Results
3.1 Descriptive Statistics of ABS Parameters and Braking Objectives
There were 10,101 combinations of the parameters used in the analysis, with x1 ranging from −5.0 to 6.0 and x2 ranging from −5.0 to 4.0. The mean values of the braking-objectives y1, y2, y3, y4 and y5 were 0.3119 m, 0.4461 m, 1.1031 m, 0.3632 m and 0.3249 m, respectively, and the corresponding standard deviations were 0.1559 m, 0.2450 m, 0.6470 m, 0.1684 m and 0.1823 m, respectively. The lowest score for each of the braking objectives was 0.0 m. Table 1 shows the descriptive statistics of the ABS control parameters and braking-distance objectives.

Table 1. Descriptive Statistics of ABS Control Parameters and Braking-Distance Objectives

Variable	N	Mean
x1	10,101	0.5000
x2	10,101	−0.5000
y1	10,101	0.3119
y2	10,101	0.4461
y3	10,101	1.1031
y4	10,101	0.3632
y5	10,101	0.3249

Note: y1–y5 are expressed as distance from the corresponding optimum in metres.

3.2 Correlation Analysis
The Pearson correlation coefficients indicated various relationships between the different ABS parameters and the braking goals, with the highest correlation being x1/y4 (0.8819). Its correlations with the remaining objectives ranged from −0.1039 to 0.1113. x2 was positively correlated with all five responses, with coefficients of 0.5952 for y1, 0.5985 for y2, 0.3960 for y3, 0.1690 for y4, and 0.7296 for y5. The Pearson correlations between the two ABS parameters and each of the braking objectives are presented in Table 2.

Table 2. Pearson Correlations Between ABS Control Parameters and Braking-Distance Objectives

Braking Objective	Correlation with x1	Correlation with x2

y1	0.0754	0.5952
y2	−0.1039	0.5985
y3	−0.0564	0.3960
y4	0.8819	0.1690
y5	0.1113	0.7296

The range of the response correlations was from −0.0079 to 0.8193. The highest coefficient occurred between y1 and y2 (r=0.8193), followed by y1 and y5 (r=0.7863) and y2 and y5 (r=0.7756). y3 and y4 showed the lowest correlation (r=-0.0079). The full correlation matrix for the ABS parameters and 5 braking objectives is presented in Figure 1.

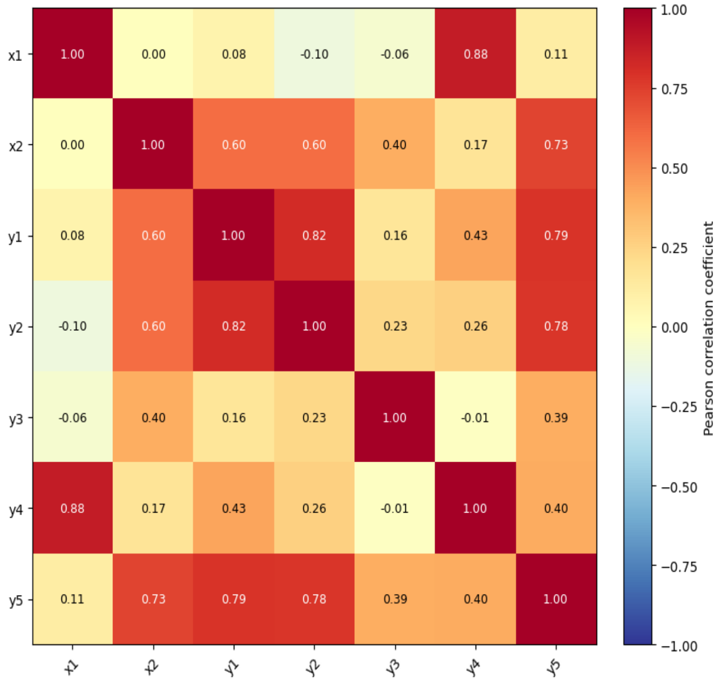

Figure 1. Correlation Structure of ABS Control Parameters and Braking-Distance Objectives.

3.3 Quadratic Model Performance and Parameter Sensitivity

There were different goodness-of-fit values for the quadratic response-surface models of the five responses. The highest R2was obtained for y4 (0.9356), followed by y1 (0.8177), y2 (0.8091), y5 (0.7443), and y3 (0.2666). RMSE values were 0.0639, 0.1289, 0.3161, 0.0597, and 0.0896 m for y1 through y5, respectively. The MAE values were between 0.0442 and 0.2501 m. The sensitivity estimates for y1 and y2 were almost equal (x1 = 82.19% of the relative sensitivity of y1, and x2 = 63.94% of the sensitivity of y3, and 66.55% of the sensitivity of y5). The model-performance statistics and relative parameter sensitivities are listed for all five responses in Table 3.

Table 3. Quadratic Response-Surface Model Performance and Relative ABS Parameter Sensitivity

Response	R^2	Adjusted R^2	RMSE
y1	0.8177	0.8176	0.0639
y2	0.8091	0.8090	0.1289
y3	0.2666	0.2663	0.3161
y4	0.9356	0.9355	0.0597
y5	0.7443	0.7442	0.0896
Response	R^2	Adjusted R^2	RMSE
y1	0.8177	0.8176	0.0639
y2	0.8091	0.8090	0.1289

3.4 Observed Braking-Performance Landscapes

The minima observed were at different points in the ABS parameter space. The observed landscapes of braking performance and the minimum point of each response are illustrated in Figure 2. Figure 2A shows that the minimum y1 value of 0.0 m occurred at x1 = −0.3 and x2 = −3.0. Figure 2B shows that the minimum y2 value of 0.0 m occurred at x1 = 0.3 and x2 = −2.4. Figure 2C shows that the minimum y3 value of 0.0 m occurred at x1 = 4.2 and x2 = −2.8. Figure 2D shows that the minimum y4 value of 0.0 m occurred at x1 = −3.5 and x2 = −3.9. Figure 2E shows that the minimum y5 value of 0.0 m occurred at x1 = −0.6 and x2 = −2.7.

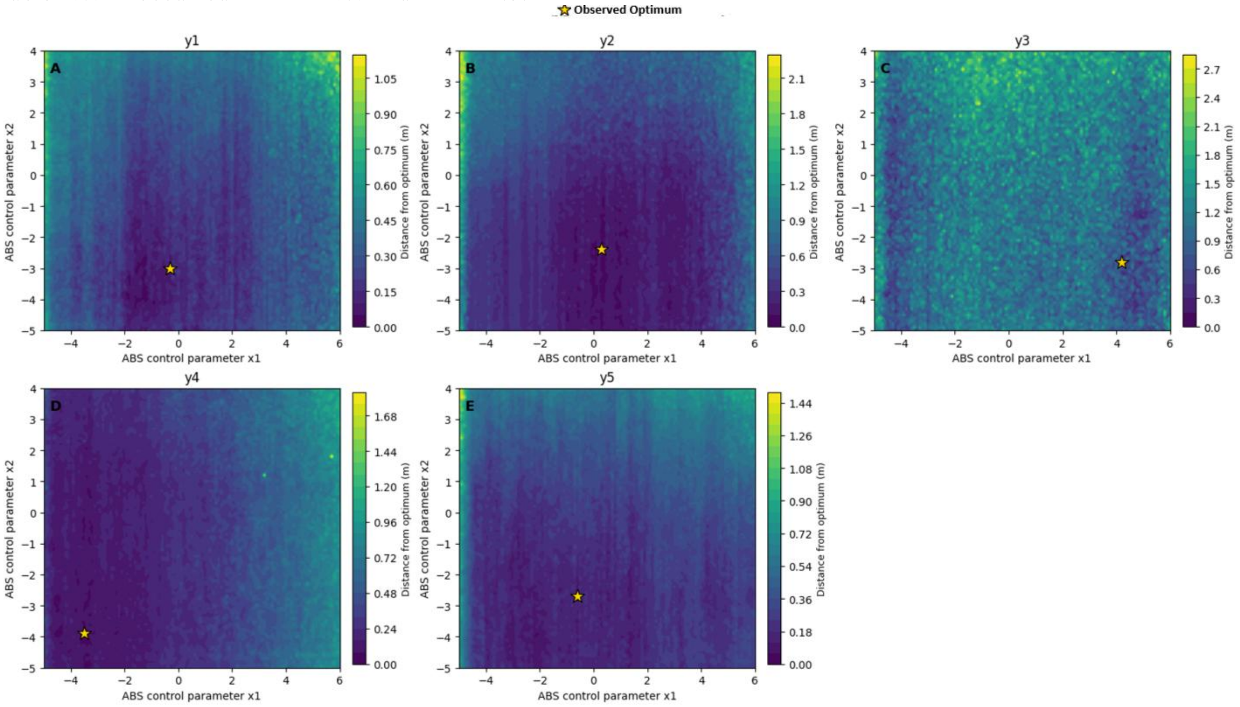

Figure 2. Observed Braking-Performance Landscapes Across Vehicle Configurations: (A) y1 Braking Objective; (B) y2 Braking Objective; (C) y3 Braking Objective; (D) y4 Braking Objective; (E) y5 Braking Objective

3.5 Multi-Response Optimization and Pareto Analysis

The equal-weight multi-response analysis showed that the combination x1 = −1.2 and x2 = −2.8 had the lowest composite value. At this point, y1 was 0.0091 m, y2 was 0.1046 m, y3 was 0.7469 m, y4 was 0.1519 m, and y5 was 0.0832 m. The normalized composite score was 0.0936. The best equal-weight configuration and the Pareto-analysis summary are presented in Table 4.

Table 4. Multi-Response Optimal ABS Configuration and Pareto Summary

Parameter/Metric	Equal-Weight Compromise
x1	−1.2
x2	−2.8
y1	0.0091
y2	0.1046
y3	0.7469
y4	0.1519
y5	0.0832
Composite Score	0.0936
Pareto Optimal	Yes
Pareto-Optimal Configurations	405

Pareto-Optimal Proportion (%)	4.0095

Note: y1–y5 represent distance from the corresponding optimum in metres.

After applying Pareto analysis, 405 configurations were identified as non-dominated, which is 4.0095% of the configurations that were evaluated. This Pareto set was the equal-weight configuration with x1 = −1.2 and x2 = −2.8. The Pareto-optimal configurations are plotted against the investigated space of ABS parameters and the position of the equal-weight compromise is plotted in Figure

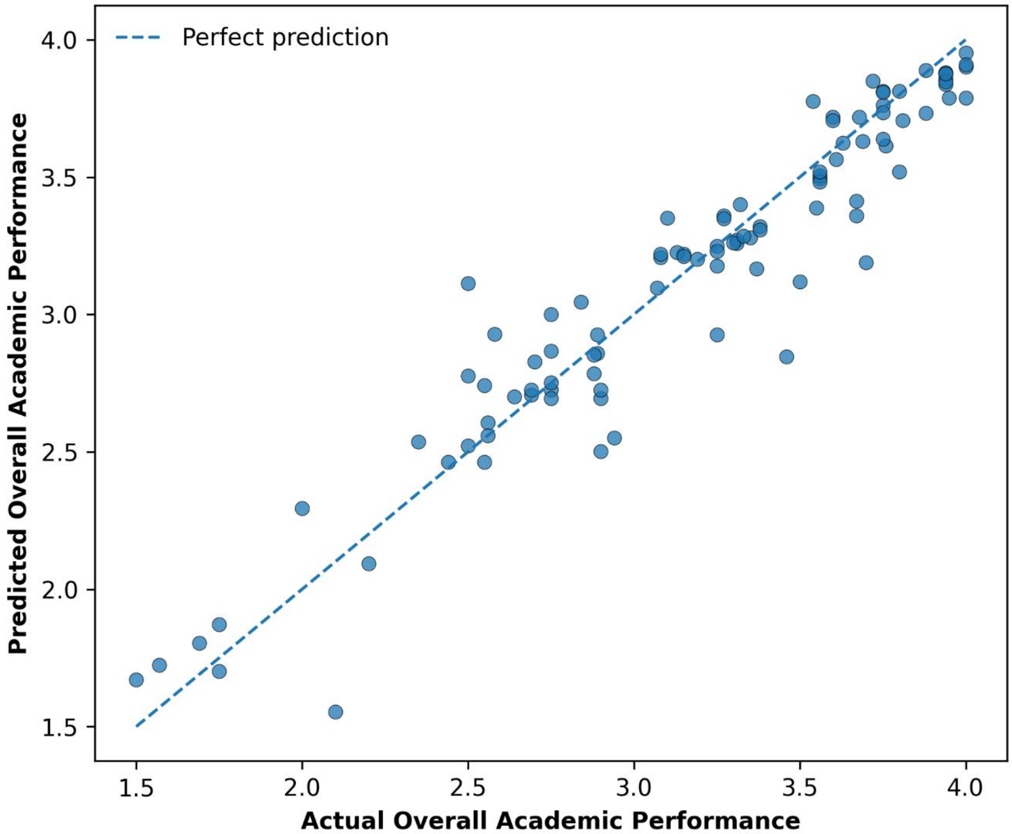

Figure 3. Pareto-Optimal ABS Configurations and Equal-Weight Multi-Response Compromise

4. Discussion

The results indicate that the impact of both anti-lock braking system control parameters differs significantly with tire and vehicle load conditions. Variable x1 had the strongest linear association in the case of y4 (fully loaded, high performance tires), whereas x2 was more influential in the case of the lightly loaded, high and moderately loaded tires (represented by y5 and y3, respectively) and contributed strongly under the partially loaded, high performance (y5) and under the moderately loaded, high performance tire conditions (y3). The results for the response surface were similar: the quadratic model fit the data for y4 very well, while the other data for under-performance tires with partial vehicle load (y3) fit poorly. This suggests that the braking-response field is less uniform in poor tire performance and that there are different operating conditions that are not equally well modelled with a simple model.

The differences in relation to the tires are particularly interesting. If the loading is partially loaded, y1, y2 and y3 refer to high, medium and under performance tires, respectively. The behaviour of y3 is quite different, not only from the other models, but also from the optimum found for the other models, x1 = 4.2 and x2 = −2.8, which suggests that the behaviour of the tire changes the relationship between the control parameters and the braking response. Other research has been driven by the desire to adapt braking behaviour to varying conditions; hence, an adaptive anti-lock (AAL) braking strategy that adjusts the control action based on the system state, instead of a fixed parameterization, has been used (Kang et al., 2024). The research on variable-friction braking has also demonstrated the significant impact that tire–road interaction changes can have on anti-lock braking performance, and therefore the need for condition-sensitive control responses (Qasem et al., 2025).

This type of difference was also seen when comparing y1, y4 and y5, which all have high-performance tires, but are partially loaded, fully loaded and lightly loaded. The best parameter conditions were found to be different for all three conditions, and y4 was most sensitive to x1. This substantiates that load is not just a secondary issue, but it also affects the braking-response optimum shape and position. Anti-lock braking (AAB) experimental research with intelligent active force control has also shown that the performance of the anti-lock braking (AAB) controller is dependent on system conditions and the interaction between the input signals to the controller and the vehicle response (Al-Mola et al., 2023).

The adaptive estimation of the vehicle velocity has also been used to enhance the anti-lock braking system by continuously updating the dynamic information fed into the anti-lock braking system controller when braking (Rafatnia & Mirzaei, 2021).
Another key finding of the multi-response analysis is the extent of break-in that occurred. A second important finding from the multi-response analysis is the degree of break-in. None of the 5 individual optima was the optimum combination of all the vehicle settings. The equal-weight compromise at $x1 = -1.2$ and $x2 = -2.8$ was the result of the lowest normalized composite score, and it was inside the Pareto-optimal set. Accordingly, this enhances the applicability of multi-objective treatment in cases where multiple braking conditions need to be taken into account. Moreover, multi-objective design of braking system parameters is demonstrated by Wu et al. (2021) to be advantageous since there may be trade-offs between the various performance criteria. The multi-objective tuning of anti-lock braking controllers, which optimizes multiple control objectives, instead of tuning a response alone, has also been employed (Domingues et al., 2019).
The Pareto analysis reduced the number of potential designs from 10,101 to 405 with no design showing any dominance. The number of Pareto-optimal solutions is small compared to the size of the parameter space, suggesting that much of the parameter space can be discarded when the five braking responses are considered together. This is useful due to the fact that calibration can be done on a limited area of candidate settings, instead of the entire area of control space. Previous studies on road surface recognition and sliding-mode braking control have demonstrated that optimizing the parameters under changing operating conditions is beneficial for braking optimization (Zhou et al., 2025). In the case of electro-hydraulic anti-lock braking systems, redundancy control also makes it important to ensure good braking performance for different states of the system, excluding just one particular state (Liu et al., 2022).
So, the implications are both statistical and engineering in nature. The RSMs made for the more regular braking landscapes were very clear, and the sensitivity analysis determined which of the control parameters under each condition was most influential. The variations between the different tire and load conditions demonstrated that the calibration of the parameters should be load- and condition-dependent. When all five conditions are equally important, the equal-weight solution can be used, and if the engineering priorities are different, then one can choose among the alternative non-dominated solutions of the Pareto set.
There are a couple of caveats. Only two control parameters were tested; the quadratic model was not as good for the irregular y3 landscape. The equal-weight formulation is also one particular set of preferences and might not match all practical calibration objectives. Further research is needed to add more control parameters, use more flexible models of the nonlinear response surface for the irregular surface, and validate some of the Pareto-optimal configurations by a hardware-in-the-loop validation or experimental braking tests. The resulting conclusions would also be valuable if the study were extended to a wider range of conditions of road friction and other load states.

5. Conclusion

The performance of the anti-lock brakes (ABS) was clearly dependent on both tire and vehicle-load conditions, indicating that control-parameter effects were not constant but depended on the conditions. The two control parameters had varying degrees of influence in the five braking responses and the quadratic response-surface models performed well in most cases, but were not as well suited for the under-performance tire condition. The resulting response landscapes also demonstrated that individual optima for the braking response were found at different combinations of the parameters, further emphasizing the need for the multi-response formulation. The outcome of the equal-weight optimization was $x1 = -1.2$ and $x2 = -2.8$, which gave a composite score of 0.0936. This setup was Pareto-optimal as well, and only 405 of the 10,101 possible setups were not dominated. The results presented here illustrate the usefulness of using response-surface modelling along with sensitivity analysis and Pareto-based optimization in an interpretable anti-lock braking system calibration process. The outcome of this work gives a practical application for choosing the control setting under conflicting braking situations and can be extended to more scenarios and more control variables.

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