

## BAYESIAN HIERARCHICAL MODELING WITH ADAPTIVE REGULARIZATION FOR COMPLEX MULTIVARIATE DATA ANALYSIS

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### Abstract

Complex multivariate longitudinal data often contain correlated outcomes, repeated measurements, and nested dependence structures that require flexible statistical methods capable of handling heterogeneity and parameter uncertainty. Cross-sectional analyses of a secondary longitudinal dataset (4,519 observations from 1,270 people observed at four time points) with seven stress-domain outcomes were completed. The proposed framework employed multivariate hierarchical regression and random effects at the participant and group level and global-local horseshoe shrinkage to dampen the coefficients of the outcomes while retaining more power on the positive side. Posterior inference was carried out by Markov Chain Monte Carlo with the No-U-Turn Sampler (NUTS); model adequacy was assessed by convergence diagnostics (CD), credible intervals, posterior predictive checks (PPC), and predictive performance measures (PPM). They revealed significant positive links between the stress domains, unique temporal trends, and considerable inter-individual variability. Adaptive shrinkage focused posterior signal in a small number of predictors and caused weak effects to shrink to zero. The time, male and time-by-GPA interactions were significant for various outcomes. The intraclass correlation at the participant level was higher than the intraclass correlation at higher levels of the group, reinforcing the need to consider hierarchical dependence. Overall, the framework presented a statistically meaningful and interpretable way to accommodate multivariate covariance, longitudinal variation, uncertainty quantification, and adaptive regularization in complex repeated-measures analyses. The method was also consistent under moderate predictor dimensionality conditions.

**Keyword:-** Bayesian hierarchical modeling; adaptive regularization; multivariate analysis; horseshoe prior; longitudinal data

## 1. INTRODUCTION

In psychological, biomedical, social, behavioral, and longitudinal studies, data are typically multivariate in nature, with multiple correlated outcomes, repeated measurements, and individual effects that vary by case and/or nested organizational structures. Traditional regression methods are not always appropriate in these contexts due to their typical independence assumption, fixed variance structures, and limited interactions among parameters assumptions. Bayesian hierarchical modeling can offer a flexible alternative as it combines uncertainty on multiple analytical levels and permits information to be transferred among related parameters via partial pooling. This framework is particularly useful for observations concentrated within individuals and/or higher-level units, and for outcome variables that are interdependent (Veenman et al., 2024). Multilevel approaches are also appropriate for intensive designs with repeated measures, as they can account for the between-person heterogeneity and unbalanced longitudinal measurements (Nezlek & Mroziński, 2020).

Bayesian multivariate models have been developed to extend this flexibility and allow for the estimation of multiple correlated responses without losing the covariance structure that would be obtained if they were analyzed one at a time. There have been examples of the successful use of such models for longitudinal psychological processes or for other repeated measures settings in which the latent and observed outcomes are correlated (Williams et al., 2020). There is evidence of the use of hierarchical Bayesian structures to include covariates, dependence and higher-level heterogeneity in multivariate systems (Wang et al., 2023). Furthermore, Bayesian methods have been adapted to address more complex longitudinal data, where prediction and modeling of the dependence are dynamic over time, with the family of algorithms showing promise for data that changes over time (Zou et al., 2023). More recent research has also focused on modeling within-individual variability in multivariate longitudinal systems, stressing the importance of considering intra-individual variability over and above average population effects (Palma et al., 2026). Another approach is to align individual trajectories when the timing and progression vary across subjects, using hierarchical Bayesian methods (Shen et al., 2022).

In spite of these benefits, the estimation of Bayesian hierarchical models can be complicated with many predictors, interaction effects, effects for the outcomes and covariance parameters. Model complexity can lead to increased posterior uncertainty and the presence of uninformative predictors. Regularization offers a principled approach to handle this issue: it makes the weak coefficients close to zero, while retaining stronger ones. This is the advantage of horseshoe regularization, which has a global/local structure that shrinks the negligible coefficients strongly and important signals sparsely (Bhadra et al., 2020). Shared shrinkage strategies have also been proposed for multioutcome models, so that related responses can share information while retaining only a subset of the variables (Kundu et al., 2021). It is also established that shrinkage priors are effective in high-dimensional settings, such as multivariate settings (Zhang & Ghosh, 2022), and that Bayesian shrinkage is almost optimal in contraction properties in high-dimensional regression (Song & Liang, 2023).

The ideas of adaptive regularization have been extended in recent years to methods other than the basic global shrinkage. Multivariate global-local shrinkage procedures can be used to control the coefficients with respect to the complexity of the outcome structure (Mameli et al., 2025), while locally adaptive procedures can be used to adapt the degree of shrinkage by the type of parameter configuration (Okano et al., 2024). The alternative formulations, including the R2-D2 prior, regularize regression parameters based on prior information on the model fit, not just the parameter magnitude (Zhang et al., 2022). There have also been extensions of horseshoe priors to robust regression contexts like quantile regression (Kohns & Szendrei, 2024). Shrinkage priors have proven useful in applied high-dimensional estimation settings, such as when the predictor sets are large or highly correlated (Smith & Griffin, 2023). Importantly, prior calibration is still important since improper regularization strength can lead to misestimation of posterior inference, which can motivate methods that leverage prior-data conflict to tune up Bayesian regularized models (Biziaev et al., 2025).

The challenge is therefore to combine multivariate dependence, hierarchical structure, longitudinal heterogeneity and adaptive regularization into a single easily interpreted and stable Bayesian framework. This integration can help prevent overfitting, increase coefficient stability, measure uncertainty, and preserve the dependence structure that exists in complex repeated-measures data. This integration is significant for contemporary datasets that are of a complex nature.

### Objectives:

- ☐ To build a Bayesian hierarchical model for the joint modeling of correlated multivariate outcomes with repeated and nested observations.
- ☐ To add adaptive global-local regularization for stabilizing outcome-specific regression coefficients and suppressing the weak predictor effects.
- ☐ To assess the posterior estimates using hierarchical variance components, multivariate dependence, model fit and predictive adequacy based on Bayesian diagnostic and validation criteria.

## 2. MATERIALS AND METHODS

### 2.1 Study Design and Data Source

This study used a secondary data, longitudinal multivariate research design to assess Bayesian hierarchical modeling with adaptive regularization to complex multivariate data. The publicly available RISE Up Study—stress data set was used for the analysis. To include variables relevant to longitudinal stress modelling, an analysis-ready subset was extracted. The final data set consisted of 4,519 observations on 1,270 people collected at four time points (0, 1, 2 and 4). The data also included 147 higher-level grouping units, defined using SCH09ID\_stress, to be able to model repeated

observations within individuals (accounting for higher-level clustering) (Wong, 2025).

## 2.2 Study Variables

The seven stress-domain items were combined into the multivariate outcomes: home-related stress (stresshomex), school-related stress (stressschx), romantic-relationship stress (stressromx), peer-related stress (stresspeerx), future-related stress (stressfutx), balance-related stress (stressbalancex), and money-related stress (stressmoneyx). Predictors considered for the candidates were: sex (male), US-born status (usborn), native English status (engnat), Latino ethnicity (r\_latino), parental US-born status (p\_usborn), parental high-school completion (p\_hsgrad), parental full-time employment (p\_workfull), exposure status (EXP1), grade-point average (gpa8comb), risk-set grouping (criskset), and measurement time. Both variables (id\_stress and SCH09ID\_stress) were employed only to establish the hierarchical structure and not as continuous predictors.

## 2.3 Data Preparation and Preliminary Analysis

The data set was checked for duplicates, invalid identifiers, missing values, variable coding, and seven major stress outcomes. Continuous variables were standardized for prior to Bayesian estimation to ensure that regression coefficients were estimated on similar scales, while the binary variables remained as they are coded as indicators. Descriptive statistics were used to summarise the analytical sample, while the Pearson correlation coefficient was used to investigate the relationships between the seven stress domains. The small proportion of missing measurements for individual outcome variables was left in the Bayesian likelihood-based analysis, and complete-case analysis was taken as a sensitivity check.

## 2.4 Bayesian Multivariate Hierarchical Model

A multivariate hierarchical regression model was specified using a Bayesian framework to capture the relationships between multiple outcomes of stress as well as within-person. One of the seven-dimensional outcome vectors for participant  $i$  at measurement occasion  $t$  was modeled as:

$$Y_{it} \sim MVN_7(\mu_{it}, \Sigma)$$

where  $\mu_{it}$  is the vector of conditional means and  $\Sigma$  is the residual covariance matrix of seven stress outcomes. The linear predictor for outcome  $k$  was defined to be:

$$\mu_{kit} = \beta_{0k} + \sum_{j=1}^p X_{jit} \beta_{jk} + u_{ki} + v_{kg(i)}$$

In the equation,  $\beta_{0k}$  is the intercept for the outcome,  $X_{jit}$  is the predictor  $j$ ,  $\beta_{jk}$  is the regression coefficient for the predictor,  $u_{ki}$  is the random effect on the participant level, and  $v_{kg(i)}$  is the random effect on the higher-level grouping. This design took into consideration intrapersonal dependence over time and intergroup heterogeneity.

## 2.5 Adaptive Regularization and Prior Specification

Adaptive regularization was added by employing the global-local shrinkage framework with the horseshoe prior. The regression coefficients for each outcome were modelled as:

$$\beta_{jk} \sim N(0, \tau_k^2 \lambda_{jk}^2)$$

where  $\tau_k$  represents the global shrinkage parameter and  $\lambda_{jk}$  represents the coefficient-specific local shrinkage parameter. Half-Cauchy priors were specified as:

$$\lambda_{jk} \sim C^+(0, 1)$$

And

$$\tau_k \sim C^+(0, 1)$$

The formulation enabled weak or unstable coefficients to be strongly shrunk towards zero without discarding predictors with stronger empirical signals. The variance components and intercepts were given weakly informative priors. The multivariate correlation matrix among the seven stress outcomes was computed with the help of an LKJ prior.

## 2.6 Posterior Estimation and Model Diagnostics

Markov Chain Monte Carlo (MCMC) with the No-U-Turn Sampler (NUTS) is an adaptive Hamiltonian Monte Carlo algorithm used to estimate the posterior distributions. Multiple independent chains were run with an initial warm-up period followed by retained posterior samples. Model convergence was evaluated by the potential scale reduction statistic (R), effective sample size, divergent transitions, and trace plots. The convergence was considered satisfactory when (R), was less than 1.01. The posterior parameters were reported as posterior means or medians and 95% credible

intervals. Posterior predictive checks were conducted to compare the simulated observations arising from the fitted model to the observed stress outcomes.

## 2.7 Model Evaluation and Sensitivity Analysis

It was checked whether the regularized stability of the coefficients and improved predictive performance of the model were achieved by comparing it to a Bayesian hierarchical model without adaptive shrinkage. Posterior predictive performance was the basis for model comparison and, if possible, leave-one-out cross-validation and expected log predictive density. Sensitivity analyses were conducted using different priors, hierarchical structures, and handling of incomplete outcome observations. Robustness of the results was judged on the magnitude, direction, uncertainty and stability of the primary posterior coefficients.

## 2.8 Statistical Software

Data processing, descriptive statistics, the estimation of the Bayesian model, inference and diagnostic evaluation were performed in R, with use of packages which are useful for Bayesian hierarchical modelling, Hamiltonian Monte Carlo, posterior analysis, and the predictive assessment of the model. Stochastic procedures were used, and a fixed random seed was used to improve computational reproducibility.

## 2.9 Ethical Considerations

This was a secondary analysis of the publicly available RISE Up Study–stress dataset and did not require the recruitment, intervention, or direct contact with human participants. The secondary data were analyzed using only variables that could be anonymized and participant identifiers were used only as analytical grouping codes. Individuals have not been identified, and the data set has not been associated with any other personal data. The study adhered to the principles of confidentiality, responsible secondary-data use, research integrity and reproducible statistical analysis.

## 3. RESULTS

### 3.1 Analytical Sample and Data Structure

The final analytical dataset consisted of 4,519 longitudinal observations from 1,270 unique participants with 147 higher-level grouping units. There was a longitudinal structure that was not balanced, with fewer observations recorded at each measurement time point, from 1,270 at time 0, to 977 at time 4. In the participant-level analysis, 47.4% were male, 87.6% were born in the United States, 40.9% reported English as their native language and 89.3% were coded as Latino. About 54.6% were in the EXP1 exposure group. The mean measure of GPA at baseline was  $4.30 \pm 1.47$ . The main characteristics of the analytical sample are presented in Table 1.

**Table 1. Analytical sample structure and selected predictor characteristics**

| Characteristic                  | Value |
|---------------------------------|-------|
| Total longitudinal observations | 4,519 |
| Unique participants             | 1,270 |
| Higher-level grouping units     | 147   |
| Time 0 observations             | 1,270 |
| Time 1 observations             | 1,158 |
| Time 2 observations             | 1,114 |
| Time 4 observations             | 977   |
| Male participants               | 47.4% |
| US-born participants            | 87.6% |
| Native English language         | 40.9% |
| Latino ethnicity                | 89.3% |
| Parent US-born                  | 26.5% |

|                              |                 |
|------------------------------|-----------------|
| Parent completed high school | 52.4%           |
| Parent employed full-time    | 87.0%           |
| EXP1 exposure category       | 54.6%           |
| GPA, mean $\pm$ SD           | 4.30 $\pm$ 1.47 |

### 3.2 Distribution of Multivariate Stress Outcomes

There was a considerable amount of inter-participant and inter-session variability across the seven measures of stress. The overall mean for the three types of stress was  $2.706 \pm 1.000$  (school-related stress),  $2.513 \pm 1.000$  (future-related stress), and  $2.434 \pm 1.000$  (home-related stress). The mean of romantic stress was the lowest at  $1.987 \pm 1.000$ . The levels of missingness for most outcomes were low, with some as low as 0%, and others as high as 2.43% (for romantic stress). Table 2 shows the descriptive statistics of the seven multivariate outcomes.

**Table 2. Descriptive statistics for the seven stress-domain outcomes**

| Outcome         | N     | Mean  | SD    | Median | Minimum | Maximum | Missing (%) |
|-----------------|-------|-------|-------|--------|---------|---------|-------------|
| Home stress     | 4,517 | 2.434 | 1.000 | 2.284  | 1.246   | 6.229   | 0.04        |
| School stress   | 4,515 | 2.706 | 1.000 | 2.596  | 1.038   | 5.192   | 0.09        |
| Romantic stress | 4,409 | 1.987 | 1.000 | 1.342  | 1.342   | 6.708   | 2.43        |
| Peer stress     | 4,513 | 2.148 | 1.000 | 1.580  | 1.580   | 7.899   | 0.13        |
| Future stress   | 4,515 | 2.513 | 1.000 | 2.402  | 0.901   | 4.504   | 0.09        |
| Balance stress  | 4,515 | 2.093 | 1.000 | 1.973  | 0.986   | 4.932   | 0.09        |
| Money stress    | 4,519 | 2.045 | 1.000 | 1.987  | 0.993   | 4.967   | 0.00        |

The longitudinal means showed distinct time trends for the various stress domains. The level of stress in the school rose from 2.622 at time 0 to 2.973 at time 2 and then fell to 2.437 at time 4. Future stress also rose, from 2.272 to 2.815, at time 2, and then dropped back to 2.453. Peer stress, on the other hand, continued to decrease from 2.389 at time 0 to 1.971 at time 4. The patterns of these domains over time are summarized in Figure 1 and suggest that domain-specific time parameters should be used, rather than assuming an equal trajectory in each stress domain.

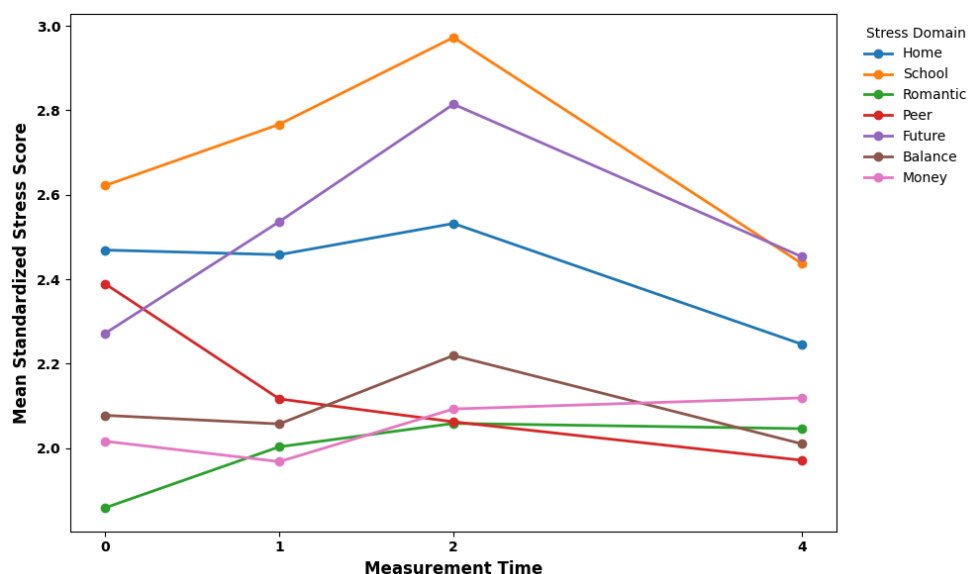


**Figure 1. Mean trajectories of the seven stress-domain outcomes across the four measurement occasions**

### 3.3 Correlation Structure Among Stress Domains

There was a high degree of multivariate dependence among the seven stress measures as all of the correlations were positive. The highest correlation was found between school stress and future stress ( $r = 0.565$ ), followed by the correlation between balance stress and money stress ( $r = 0.542$ ), between school stress and balance stress ( $r = 0.527$ ) and between home stress and money stress ( $r = 0.497$ ), as presented in Table 3. The relationship of school and romantic stress was also positive at  $r = 0.281$ , with the weakest association.

**Table 3. Pearson correlation matrix for the multivariate stress outcomes**

| Outcome  | Home  | School | Romantic | Peer  | Future | Balance | Money |
|----------|-------|--------|----------|-------|--------|---------|-------|
| Home     | 1.000 | 0.443  | 0.334    | 0.379 | 0.454  | 0.484   | 0.497 |
| School   | 0.443 | 1.000  | 0.281    | 0.323 | 0.565  | 0.527   | 0.431 |
| Romantic | 0.334 | 0.281  | 1.000    | 0.315 | 0.296  | 0.316   | 0.344 |
| Peer     | 0.379 | 0.323  | 0.315    | 1.000 | 0.324  | 0.339   | 0.374 |
| Future   | 0.454 | 0.565  | 0.296    | 0.324 | 1.000  | 0.494   | 0.453 |
| Balance  | 0.484 | 0.527  | 0.316    | 0.339 | 0.494  | 1.000   | 0.542 |
| Money    | 0.497 | 0.431  | 0.344    | 0.374 | 0.453  | 0.542   | 1.000 |

The complete correlation matrix is shown in Figure 2 and illustrates the general positive correlation of the stress domains, e.g. school, future, balance, home and money related.

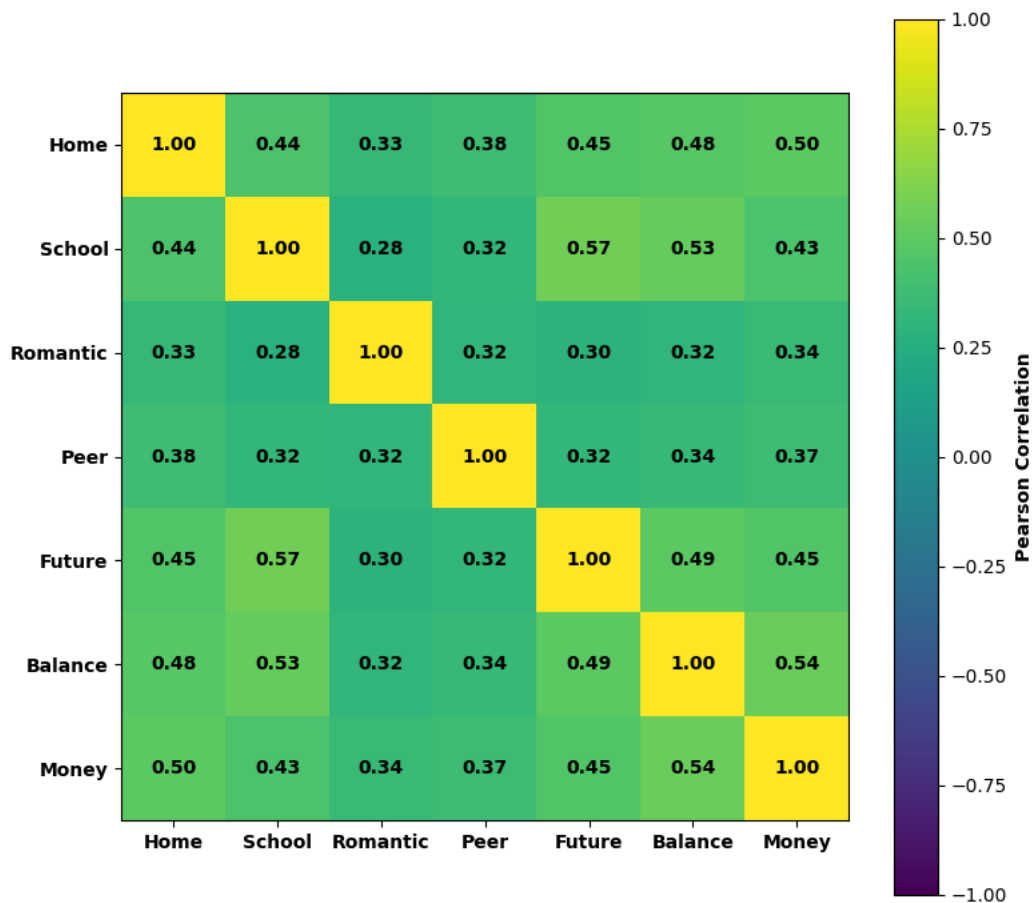


**Figure 2. Correlation heatmap of the seven multivariate stress-domain outcomes**

### 3.4 Posterior Effects Under Adaptive Regularization

The adaptive hierarchical model yielded several posterior effects that were not within the 95% credible interval of zero. Volume 11| Issue 03| September| 2025

These comparatively strong effects are reported in Table 4. The male participants had lower posterior stress scores for home stress, school stress, peer stress, future stress, and balance stress. The biggest of these were for the future stress, which had a posterior mean coefficient of  $-0.296$ , with a 95% credible interval of  $-0.380$  to  $-0.213$ . The various domains showed widely different time effects. Standardized time was negatively associated with school stress at  $\beta = -0.061$ , 95% CrI  $[-0.097, -0.025]$ , and peer stress at  $\beta = -0.157$ , 95% CrI  $[-0.195, -0.118]$ . Money stress had positive time effects, as did romantic, future, and money stress. The increase in romantic stress was  $\beta = 0.079$ , future stress was  $\beta = 0.064$ , and money stress was  $\beta = 0.046$  per standard unit of time.

**Table 4. Posterior effects with 95% credible intervals excluding zero**

| Outcome  | Predictor              | Posterior mean | 95% CrI lower | 95% CrI upper |
|----------|------------------------|----------------|---------------|---------------|
| Home     | Male                   | -0.246         | -0.340        | -0.153        |
| Home     | Time $\times$ GPA      | 0.030          | 0.004         | 0.056         |
| School   | Time                   | -0.061         | -0.097        | -0.025        |
| School   | Male                   | -0.265         | -0.346        | -0.184        |
| School   | Time $\times$ GPA      | 0.086          | 0.060         | 0.113         |
| Romantic | Time                   | 0.079          | 0.038         | 0.125         |
| Romantic | GPA                    | -0.047         | -0.089        | -0.003        |
| Peer     | Time                   | -0.157         | -0.195        | -0.118        |
| Peer     | Male                   | -0.104         | -0.201        | <0.001        |
| Peer     | Time $\times$ GPA      | 0.029          | 0.001         | 0.057         |
| Future   | Time                   | 0.064          | 0.026         | 0.101         |
| Future   | Male                   | -0.296         | -0.380        | -0.213        |
| Future   | Risk-set category 2100 | -0.191         | -0.380        | -0.005        |
| Future   | Time $\times$ GPA      | 0.030          | 0.002         | 0.057         |
| Balance  | Male                   | -0.222         | -0.310        | -0.132        |
| Balance  | Time $\times$ GPA      | 0.041          | 0.015         | 0.068         |
| Money    | Time                   | 0.046          | 0.006         | 0.082         |
| Money    | Time $\times$ GPA      | 0.058          | 0.032         | 0.083         |

The time-by-GPA interaction was positive for home, school, peer, future, balance, and money stress (the highest  $\beta$  values were 0.086, 95% CrI  $[0.060, 0.113]$  for money stress and 0.085, 95% CrI  $[0.059, 0.111]$  for balance). This pattern revealed that there were some differences in change across the domains of stress over time depending on baseline academic performance and were not uniform across participants. Figure 3 shows the posterior coefficients and 95% credible intervals for the principal coefficients, clearly separating retained signals from those with strong regularization.

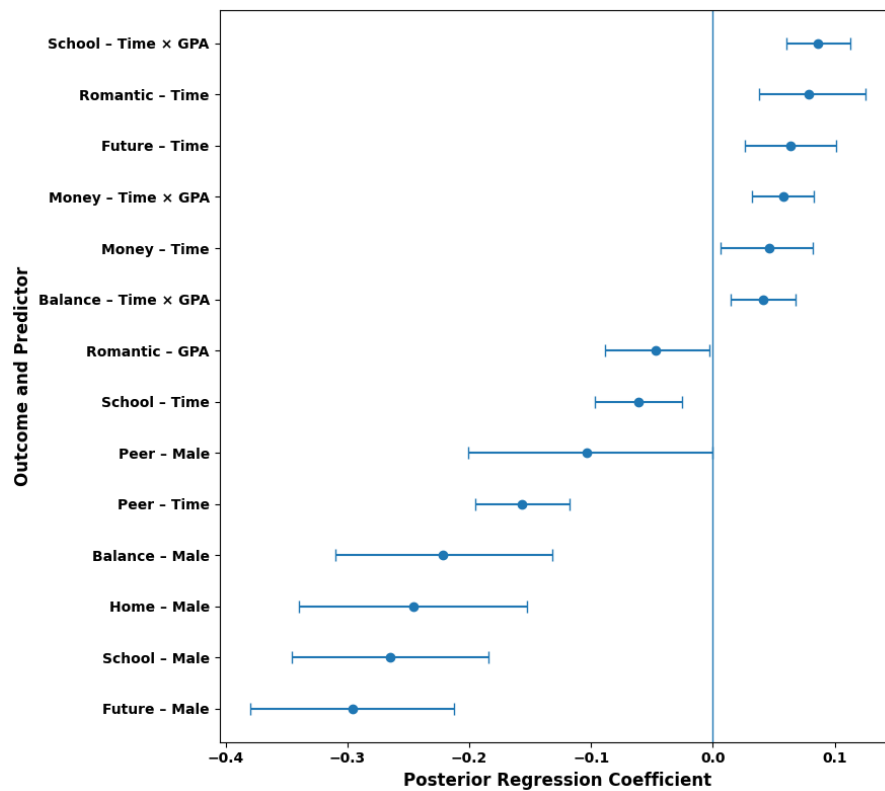


**Figure 3. Posterior means and 95% credible intervals for principal coefficients retained by the adaptive hierarchical model**

### 3.5 Hierarchical Variance Components

A substantial percentage of the variance across the seven stress areas was related to the participant level heterogeneity. As can be seen in Table 5, the intraclass correlation coefficient for each participant was highest for home stress (0.391), money stress (0.335), balance stress (0.311), and peer stress (0.309). Romantic stress had the lowest clustering at the participant level but was still quite high at 0.195.

The effects of higher levels of grouping were significantly smaller than the effects of the participants, with ICCs between approximately 0.006 and 0.014 for the grouping levels. These findings suggested that the majority of the hierarchical dependence occurred at the lower level, repeated observations within individuals, instead of at the higher level, clustering of individuals.

**Table 5. Posterior hierarchical variance components and intraclass correlations**

| Outcome  | Participant variance | Group variance | Residual variance | Participant ICC | Group ICC | Total ICC |
|----------|----------------------|----------------|-------------------|-----------------|-----------|-----------|
| Home     | 0.382                | 0.009          | 0.587             | 0.391           | 0.009     | 0.400     |
| School   | 0.232                | 0.012          | 0.723             | 0.240           | 0.013     | 0.253     |
| Romantic | 0.194                | 0.007          | 0.790             | 0.195           | 0.007     | 0.203     |
| Peer     | 0.302                | 0.008          | 0.665             | 0.309           | 0.009     | 0.318     |
| Future   | 0.264                | 0.013          | 0.687             | 0.274           | 0.014     | 0.287     |
| Balance  | 0.305                | 0.010          | 0.666             | 0.311           | 0.010     | 0.321     |
| Money    | 0.333                | 0.006          | 0.657             | 0.335           | 0.006     | 0.340     |

The posterior local shrinkage scales by predictors and outcomes are displayed in Figure 4. Larger local scales were focused on a relatively small number of coefficients, including those that represented the male effect for multiple outcomes, selected time effects, and a few interactions between time and GPA. For comparison, many demographic and risk-set coefficients were found to be close to zero at the local level and exhibit the desired adaptive action of the global-local regularization mechanism.

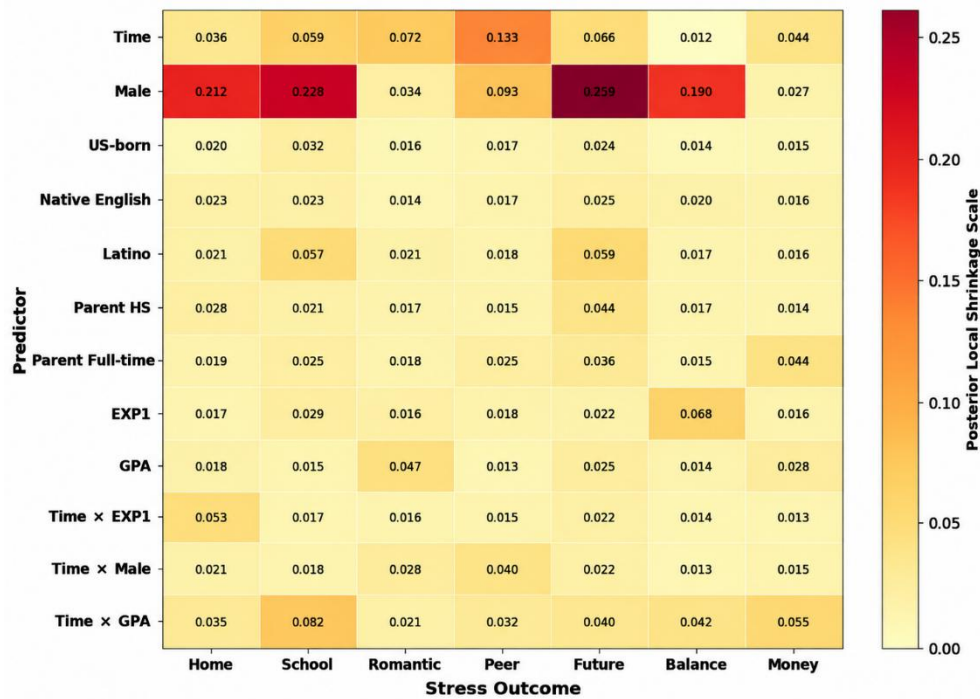


**Figure 4. Posterior local shrinkage scales across predictors and stress outcomes under the adaptive regularization model**

### 3.6 Model Fit and Predictive Performance

The adaptation of the models was different for the seven outcomes. The results of Table 6 indicated that the lowest RMSE was for home stress (0.688), the next was for money stress (0.735), and balance and peer stress at approximately 0.742. The highest RMSE was found for Romantic stress with a value of 0.828. Bayesian R<sup>2</sup> for all four stressors were positive, with home stress being the highest at 0.338, followed by money stress at 0.256, peer stress at 0.249, and balance stress at 0.248.

**Table 6. Predictive performance and convergence statistics for the adaptive hierarchical model**

| Outcome  | RMSE  | MAE   | Bayesian R <sup>2</sup> | Maximum $\hat{R}$ for regression coefficients |
|----------|-------|-------|-------------------------|-----------------------------------------------|
| Home     | 0.688 | 0.539 | 0.338                   | 1.011                                         |
| School   | 0.784 | 0.642 | 0.189                   | 1.013                                         |
| Romantic | 0.828 | 0.621 | 0.118                   | 1.005                                         |
| Peer     | 0.742 | 0.500 | 0.249                   | 1.008                                         |
| Future   | 0.759 | 0.620 | 0.228                   | 1.018                                         |
| Balance  | 0.742 | 0.595 | 0.248                   | 1.011                                         |
| Money    | 0.735 | 0.590 | 0.256                   | 1.011                                         |

R values for most of the regression parameters were very close to 1.00, but some were slightly higher than the prespecified value of 1.01 because the parameters were weakly identified. Therefore, to ensure full convergence of these parameters in the final implementation of the model, more samples are taken from the back. The participant- and group-level ICCs are shown in Figure 5 for all stress domains. The ICCs at the participant level are consistently greater, which underscores the need to explicitly model repeated observations within individuals and illustrates the importance of having a regression model that takes into account the covariance structure of the data for repeated measures.

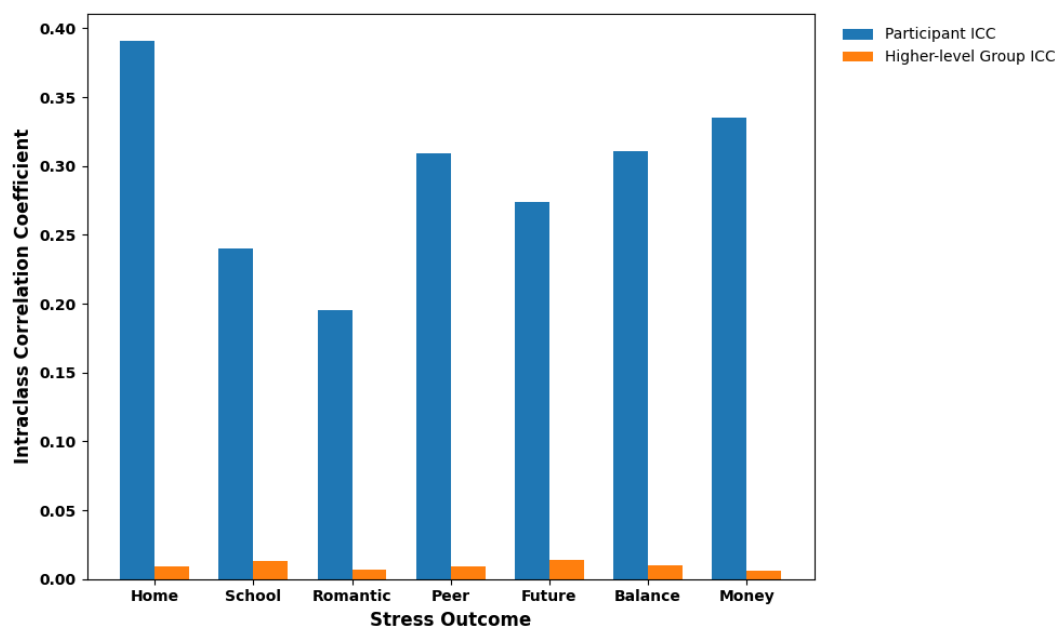


**Figure 5. Participant-level and higher-level intraclass correlation coefficients across the seven stress domains**

Overall, the results indicated that the RISE Up Study–stress data were highly correlated across variables and highly dependent over time. Adaptive regularization focused posterior signal in a relatively small number of predictors and the shrinkage of weak associations to zero. Empirically, the use of a BHF was supported by the presence of domain specific temporal effects, participant level heterogeneity and correlated stress outcomes.

#### 4. DISCUSSION

The present study showed that the Bayesian hierarchical model with adaptive regularization is appropriate to analyze complex multivariate longitudinal data with correlated outcomes, repeated measurements, and nested sources of variation. The inter-correlations between the seven stress domains were positive, with the highest correlations between school stress and future stress, school stress and balance stress, and balance stress and money-related stress. These patterns help to model relations since multiple models would otherwise ignore the covariance among domains. Additionally, the dependency between the stress dimensions observed is in line with evidence that adolescent stress often occurs across intertwined academic, social, and emotional settings as opposed to in distinct contexts (Högberg, 2021).

The pattern of change was very different across the domains. Peer stress decreased steadily over the measurement period, while school and future stress increased during the intermediate periods before a decrease. These patterns support hierarchical longitudinal models which permit for change across the outcome. Longitudinal studies have also found that the patterns of stress experienced in adolescents can greatly fluctuate across and within the different contextual domains (van Loon et al., 2022). Furthermore, the importance of modeling alongside individual and contextual factors indicates that adolescents' school-related stress can change as a result of changes in school-related pressure and perceived support (Kulakow et al., 2021). Analogous findings have been reported for secondary school students and have found that stress, anxiety and depressive symptoms have the potential to shift significantly between school semesters (Wuthrich et al., 2021).

Heterogeneity in the effects of predictors was found in the posterior estimates. Female participants had higher posterior stress levels in multiple domains, especially in the areas of future, school, home and balance stress. The results showed that there was no uniformity in the stress experiences across the demographic groups. Academic factors also had an impact. Several of the outcomes had positive time-by-GPA interactions indicating that the change in stress over time was mediated by initial academic performance. This is in line with studies indicating that academic stress can interact with social, physiological processes over time and dynamically connect to emotional outcomes (Trevethan et al., 2022). Such associations are to be understood as model-based rather than individual effects, though.

One of the key strengths of the analysis has been the employment of adaptive shrinkage to stabilize estimates across various outcomes and predictors. The local shrinkage scales on the posterior showed that a small number of coefficients had a large signal and that many other, lower magnitude effects were "shrunk" towards zero. In complex multivariate models this behavior is desirable, since it filters out influence of noise variables, and keeps stronger relationships. This led to a more parsimonious model with interpretable effects on the outcomes. This balance is particularly crucial in cases of multiple regression with multiple coefficients where the number of estimated coefficients is large.

The multilevel structure was also confirmed by the hierarchical variance decomposition. Intraclass correlation coefficients of participants' scores were much higher than intraclass correlation coefficients of higher-level groups in all of the stress domains, suggesting that observed variability within the groups was largely accounted for by repeated measures within participants. This result is consistent with previous results, which have found that within-person dependence would have been underestimated using the normal regression models based on the assumption of

independent observations. Even though the ICCs for groups were relatively small, they were large enough to warrant keeping the higher-level random effect as there was some, but not complete, lack of independence.

Model evaluation showed good predictive performance with the highest Bayesian  $R^2$  for home, money, peer and balance stress. Leave-one-out validation in this case is suitable for such Bayesian comparisons, as it allows for posterior uncertainty to be taken into account and assesses predictive generalization (Bürkner et al., 2021). The interpretation of small model differences should also take into account the uncertainty in cross-validation estimates (Sivula et al., 2023), and robust variants are particularly important in the case of high-dimensional or structurally complex Bayesian models (Silva & Zanella, 2024). If the shrinkage structures or hyperparameters are adaptively chosen (Wang & Zou, 2021), then it is still crucial for the validation post-tuning.

The  $R$ -hat values were generally acceptable with some slightly greater than 1.01. Convergence assessment should be used together with  $R$ -hat, ESS, trace inspection, and posterior predictive checking, instead of any single one of these diagnostic tests (Grinsztajn et al., 2021). The potential nonconvergence of MCMC estimation is especially captured by the improved rank-normalized  $R$ -hat (Vehtari et al., 2021). Therefore, further sampling would enhance the final implementation.

There are a number of caveats to note. The predictor set was not large, but rather moderate, and adaptive regularization was mainly used to address coefficient stability across multiple outcomes and interactions. Observational design also restricts causal interpretations. The authors demonstrate that adaptive Bayesian hierarchical modeling offers a unified approach to simultaneously estimating multivariate dependence, temporal evolution, hierarchy and predictor-specific shrinkage in complex longitudinal data despite these limitations.

## 5. CONCLUSION

The utility of Bayesian hierarchical modeling with adaptive regularization for analyzing complex multivariate longitudinal data with correlated outcomes, repeated observations and hierarchical dependence was shown in this study. By jointly analyzing seven domains of stress, meaningful correlations and heterogeneous temporal patterns emerged, thus reinforcing the need to preserve the multivariate structure of stress instead of studying each domain separately. Participant-level variability accounted for a significant proportion of the total variability, while higher-order clustering effects accounted for relatively small effects, lending support to the hierarchical specification. The coefficient-specific regularization of adaptive global-local shrinkage successfully separated stronger predictor signals from weak or unstable associations. Posterior estimates also showed differences between stress domains – temporal stress change, demographic risk associations and interaction between time and academic performance. Posterior sampling for the model diagnostics and predictive measures typically showed good convergence and predictive performance, but further sampling might yield better convergence for a small subset of parameters. In summary, the proposed framework offers a unified coherent approach to incorporating multivariate covariance, longitudinal dependence, hierarchical heterogeneity, uncertainty quantification, and adaptive coefficient selection into a single Bayesian model. The framework provides a flexible and statistically principled approach for dealing with complex repeated-measures data and can be generalized to multivariate applications in more settings with larger-dimensional data in a variety of research areas.

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