



$*(\hat{g}r)_h$ - SEPARATION AXIOMS

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Abstract

In this paper, we are trying to define some new types of separation axioms in hexa topological spaces using , $*(\hat{g}r)_h$ - continuous functions $*(\hat{g}r)_h$ - irresolute functions, $*(\hat{g}r)_h$ - open maps and $*(\hat{g}r)_h$ - closed maps. We also introduce the ideas of $*(\hat{g}r)_h$ - T_0 $*(\hat{g}r)_h$ - T_1 and $*(\hat{g}r)_h$ - T_2 spaces. Along with that, some basic properties of these spaces are also studied and discussed.

Keywords: $*(\hat{g}r)_h$ - T_0 , $*(\hat{g}r)_h$ - T_1 , $*(\hat{g}r)_h$ - T_2

1. Introduction

The concept of generalized topological space became more important, which was introduced by Császár. In this approach, instead of open sets, a family of subsets was considered to study basic separation axioms (Császár [3], 2004). Then, Roy [10] in 2010 introduced separation axioms R_0 and R_1 in generalized topological spaces and also explained some of their properties. Later, M.K. Min [5] in 2011 studied relative separation axioms and discussed their properties and relationships. In 2015, Rajendran V, Vijesh C and others [8, 9] introduced $*(gr)$ -closed sets and $*(gr)$ -continuous functions. Christal Bai. I and Shyla isac mary. T[2] introduced $(gg)*$ Separations axioms in 2020. R.V. Chandra and V. Pushpalatha [1] introduced hexa closed and hexa continuous. J.sathyamalar and C.Dhanapakyam[14,15] studied the notion of $*(\hat{gr})_h$ - closed sets and $*(\hat{gr})_h$ - continuous functions in hexa topological spaces. Separation axioms in hexa-topological spaces were studied by Nabeela Abu-Alkishik et al. [7] (2026).

2. Preliminaries

Definition 2.1: A subset A of a topological space (X, τ_h) is called :

- 1) τ_h -closed [6] if $A = cl_h(int_h(A))$.
- 2) g_h - closed set [11] if $cl_h(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ_h) .
- 3) hexa semi open [4] if $cl_h(int_h(A)) \subseteq A$.
- 4) gh -closed [11] set if $cl_h(A) \subseteq U$ whenever $A \subseteq U$ and U is hexa semi open in (X, τ_h) .
- 5) $*(\hat{gr})_h$ - closed [11] if $\tau_h cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}_h$ -open in (X, τ_h) .

Definition 2.2: Let $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ be a mapping:

- 1) The function f is called hexa continuous [2] if the inverse image $f^{-1}(V)$ is a hexa closed set in (X, τ_h) whenever V is a hexa closed set in (Y, σ_h) .
- 2) The function f is called g_h - continuous[12] if the inverse image $f^{-1}(V)$ is a g_h -closed set in (X, τ_h) whenever V is a hexa closed set in (Y, σ_h) .
- 3) The function f is called $*(\hat{gr})_h$ - continuous[12] if the inverse image $f^{-1}(V)$ is $*(\hat{gr})_h$ - closed in (X, τ_h) whenever V is a hexa closed set in (Y, σ_h) .

Definition 2.3:[11] Let (X, τ_h) be a hexa topological space and $A \subseteq X$. The $*(\hat{gr})_h - cl(A)$ is defined by the intersection of all $*(\hat{gr})_h$ - closed sets contained in A . That is $*(\hat{gr})_h - cl(A) = \bigcap \{G : A \subseteq G \text{ and } G \text{ is } *(gr)_h \text{ - closed}\}$ then $*(gr)_h$ - closed of the subset A is denoted by $*(\hat{gr})_h-cl(A)$.

Definition 2.4: [12] A function $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ is said to be $*(\hat{gr})_h$ - irresolute, if $f^{-1}(V)$ is $*(\hat{gr})_h$ - closed set in (X, τ_h) for every $*(\hat{gr})_h$ - closed set V in (Y, σ_h) .

Definition 2.5: [13] A map $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ is said to be a $*(\hat{gr})_h$ - closed map if the image of every hexa closed set in (X, τ_h) is $*(\hat{gr})_h$ - closed in (Y, σ_h) .

Definition 2.6:[13] A map $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ is said to be a $*(\hat{gr})_h$ - open map if the image of every hexa open set in (X, τ_h) is $*(\hat{gr})_h$ - open in (Y, σ_h) .

Definition 2.7:[7] A hexa topological space (L, τ_H) is called

- (1) Hexa - T_0 space if for any two distinct points $h_1, h_2 \in L$, there exists a H -open set contains only one point but not the other and it is denoted by $H-T_0$ space.
- (2) A Hexa - T_1 space if for any two distinct points $h_1, h_2 \in L$, there exists two H -open sets U_{h_1}, V_{h_2} , such that $h_1 \in U_{h_1}$, $h_2 \notin U_{h_1}$ and $h_2 \in V_{h_2}$, $h_1 \notin V_{h_2}$ and it is denoted by $H-T_1$ space.
- (3) A Hexa - T_2 space if for any two distinct points $h_1, h_2 \in L$, there exists two H -open sets U_{h_1}, V_{h_2} , such that $h_1 \in U_{h_1}$, $h_2 \in V_{h_2}$ and $U_{h_1} \cap V_{h_2} = \emptyset$ and it is denoted by $H-T_2$ space.

3. $*(\hat{gr})_h - T_k$ ($k = 0, 1, 2$) SPACES

In this section, we are introducing some new type of separation axioms in hexa topological spaces, which we call $*(\hat{gr})_h - T_k$ spaces for $k = 0, 1, 2$. Also, some basic properties of these spaces are discussed and studied.

Definition 3.1:A hexa topological space (X, τ_h) is said to be

1. $*(\hat{gr})_h - T_0$ if for each pair of distinct points x, y in X , there exists a $*(\hat{gr})_h$ - open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.
2. $*(\hat{gr})_h - T_1$ if for each pair of distinct points x, y in X , there exist two $*(\hat{gr})_h$ - open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.
3. $*(\hat{gr})_h - T_2$ if for each pair of distinct points x, y in X , there exist two disjoint $*(\hat{gr})_h$ - open sets U and V containing x and y respectively.
4. $*(\hat{gr})_h$ - space if every $*(\hat{gr})_h$ - open set of X is hexa open in X .

Theorem 3.2

- (i) Every $*(\hat{g}r)_h - T_1$ space is $*(\hat{g}r)_h - T_0$.
(ii) Every $*(\hat{g}r)_h - T_2$ space is $*(\hat{g}r)_h - T_1$ space.

Proof :

- (i) Let X be a $*(\hat{g}r)_h - T_1$ space. Let $x, y \in X$ with $x \neq y$. Since X is a $*(\hat{g}r)_h - T_1$ space, there exists $*(\hat{g}r)_h$ - open sets U and V containing x and y such that $x \in U$ and $y \notin U$ or $x \notin V$ and $y \in V$. Therefore X is a $*(\hat{g}r)_h - T_0$ space.
(ii) Let X be a $*(\hat{g}r)_h - T_2$ space. Let $x, y \in X$ with $x \neq y$. Since X is a $*(\hat{g}r)_h - T_2$ space, there exists disjoint $*(\hat{g}r)_h$ - open sets U and V containing x and y such that $x \in U$ and $y \in V$. that is $x \notin V$ and $y \notin U$. Therefore X is a $*(\hat{g}r)_h - T_1$ space.

Theorem 3.3

A topological space X is $*(\hat{g}r)_h - T_0$ iff $*(\hat{g}r)_h$ - closure of disjoint points are disjoint.

Proof:

Let X be a $*(\hat{g}r)_h - T_0$ space. Let $x, y \in X$ with $x \neq y$. Since X is a $*(\hat{g}r)_h - T_0$ space, there exists a $*(\hat{g}r)_h$ - open set U such that $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.

Let us consider $x \in U$ and $y \notin U$. Then $x \notin X - U$ and $y \in X - U$. Since U is a $*(\hat{g}r)_h$ - open set, $X - U$ is a $*(\hat{g}r)_h$ - closed set in X containing y but not x . But $*(\hat{g}r)_h - cl(y) \subseteq X - U$. Since $x \notin X - U$, $x \notin *((\hat{g}r)_h - cl(y)$. But $y \in *((\hat{g}r)_h - cl(y)$. Hence $*(\hat{g}r)_h - cl(x) \neq *((\hat{g}r)_h - cl(y)$. Similarly we can prove the other case. Hence $*(\hat{g}r)_h$ - closure of distinct points are distinct. Conversely suppose that $*(\hat{g}r)_h - cl(x) \neq *((\hat{g}r)_h - cl(y)$. If $x \neq y$ with $x, y \in X$. Then there exists atleast one point $z \in X$ such that $z \in *((\hat{g}r)_h - cl(x)$ and $z \notin *((\hat{g}r)_h - cl(y)$ or $z \notin *((\hat{g}r)_h - cl(x)$ and $z \in *((\hat{g}r)_h - cl(y)$. Now let us consider $z \in *((\hat{g}r)_h - cl(x)$ and $z \notin *((\hat{g}r)_h - cl(y)$. Suppose $x \in *((\hat{g}r)_h - cl(y)$. Then $*(\hat{g}r)_h - cl(x) \subseteq *((\hat{g}r)_h - cl(y)$. Hence $z \in *((\hat{g}r)_h - cl(x) \subseteq *((\hat{g}r)_h - cl(y)$ and so $z \in *((\hat{g}r)_h - cl(y)$. Which is a contradiction. Hence $x \notin *((\hat{g}r)_h - cl(y)$. This implies $x \in X - *((\hat{g}r)_h - cl(y)$, which is a $*(\hat{g}r)_h$ - open set in X containing x but not y . Hence X is a $*(\hat{g}r)_h - T_0$ space.

Theorem 3.4

Let $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ be an injective, $*(\hat{g}r)_h$ - irresolute map. If Y is a $*(\hat{g}r)_h - T_0$ then X is a $*(\hat{g}r)_h - T_0$ space.

Proof:

Let $x, y \in X$ with $x \neq y$. Since f is injective, $f(x) \neq f(y)$. As Y is a $*(\hat{g}r)_h - T_0$ space, there exists a $*(\hat{g}r)_h$ - open set U of Y such that $f(x) \in U$ and $f(y) \notin U$ or $f(x) \notin U$ and $f(y) \in U$. Since f is $*(\hat{g}r)_h$ - irresolute, $f^{-1}(U)$ is $*(\hat{g}r)_h$ - open set in X such that $x \in f^{-1}(U)$ and $y \notin f^{-1}(U)$ or $x \notin f^{-1}(U)$ and $y \in f^{-1}(U)$. Hence X is a $*(\hat{g}r)_h - T_0$ space.

Theorem 3.5

Let $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ be an injective, $*(\hat{g}r)_h$ - irresolute map. If Y is a $*(\hat{g}r)_h - T_2$ then X is a $*(\hat{g}r)_h - T_2$ space.

Proof:

Let $x, y \in X$ with $x \neq y$. Since f is injective, $f(x) \neq f(y)$. As Y is a $*(\hat{g}r)_h - T_0$ space, there exists a $*(\hat{g}r)_h$ - open set U of Y such that $f(x) \in U$ and $f(y) \in V$ and $U \cap V = \emptyset$. Since f is $*(\hat{g}r)_h$ - irresolute, $f^{-1}(U)$, $f^{-1}(V)$ are $*(\hat{g}r)_h$ - open set in X such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence X is a $*(\hat{g}r)_h - T_2$ space.

Theorem 3.6

Let $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ be an bijective, $*(\hat{g}r)_h$ - continuous map. If Y is a $*(\hat{g}r)_h - T_1$ space then X is a $*(\hat{g}r)_h - T_1$ space.

Proof :

Let $x_1, x_2 \in X$ be such that $x_1 \neq x_2$. Since f is bijective, there exists y_1, y_2 in Y with $y_1 \neq y_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Also since Y is a $*(\hat{g}r)_h - T_1$ space, there exists open sets U, V of Y such that $y_1 \in U$ and $y_1 \notin V$ or $y_2 \in V$ and $y_1 \notin U$. Since f is $*(\hat{g}r)_h$ - continuous, there exists $*(\hat{g}r)_h$ - open sets $f^{-1}(U)$, $f^{-1}(V)$ in X such that $x_1 \in f^{-1}(U)$ and $x_1 \notin f^{-1}(V)$ or $x_2 \in f^{-1}(V)$ and $x_2 \notin f^{-1}(U)$. Hence X is a $*(\hat{g}r)_h - T_1$ space.

Theorem 3.7

Let $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ be an bijective, $*(\hat{g}r)_h$ - open map. If X is a $*(\hat{g}r)_h - T_1$ space then Y is a $*(\hat{g}r)_h - T_1$ space.

Proof :

Let $y_1, y_2 \in Y$ be such that $y_1 \neq y_2$. Since f is bijective, there exists x_1, x_2 in X with $x_1 \neq x_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Also since X is a $*(\hat{g}r)_h - T_1$ space, there exists open sets U, V of X such that $x_1 \in U$ and $x_2 \notin V$ or $x_2 \in V$ and $x_1 \notin U$. Since f is $*(\hat{g}r)_h$ - open map, there exists $*(\hat{g}r)_h$ - open sets $f(U)$, $f(V)$ in Y such that $f(x_1) \in f(U)$ and $f(x_1) \notin f(V)$ or $f(x_2) \in f(V)$ and $f(x_2) \notin f(U)$. That is there exists $*(\hat{g}r)_h$ - open sets $f(U)$, $f(V)$ in Y such that $y_1 \in f(U)$ and $y_1 \notin f(V)$ or $y_2 \in f(V)$ and $y_2 \notin f(U)$. Hence Y is $*(\hat{g}r)_h - T_1$ space.

Theorem 3.8

Let $f: (X, \tau_h) \rightarrow (Y, \sigma_h)$ be an bijective, $*(\hat{g}r)_h$ - continuous function. If Y is a $*(\hat{g}r)_h - T_2$ space then X is a $*(\hat{g}r)_h - T_2$ space.

Proof :

Let $x_1, x_2 \in X$ be such that $x_1 \neq x_2$. Since f is bijective, there exists y_1, y_2 in Y with $y_1 \neq y_2$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Also since Y is a hexa - T_2 space, there exists disjoint hexa open sets U, V of Y such that $y_1 \in U$ and $y_2 \in V$. Since f is $*(\hat{g}r)_h$ - continuous, there exists disjoint $*(\hat{g}r)_h$ - open sets $f^{-1}(U), f^{-1}(V)$ in X such that $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Hence X is a $*(\hat{g}r)_h$ - T_2 space.

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