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INVENTORY MODEL WITH ADVERTISEMENT-DEPENDENT DEMAND

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Abstract
In the present study we develop an inventory model in which demand is influenced by advertisement activities. The advertisement effect is represented by a power function of advertisement cost and its frequency. Demand is assumed to be both inventory-dependent and advertisement-dependent. The system is formulated using a first-order linear differential equation and solved analytically under appropriate boundary conditions. The optimal inventory cycle length is derived by minimizing the average total cost function. Numerical illustrations and sensitivity analysis are presented to demonstrate the applicability of the proposed model. The results indicate that advertisement plays a significant role in accelerating demand and reducing inventory cycle time, although excessive advertisement may increase the total cost. The model provides valuable managerial insights for designing effective inventory and advertisement policies.

Keywords: *Inventory system, Advertisement-dependent demand, Differential equation, Optimal cycle time, Cost minimization.*

1. Introduction

In today's competitive market environment, advertisement has become an essential tool for stimulating customer demand. Traditional inventory models generally assume demand to be constant or time-dependent; however, such assumptions may not adequately capture real market behavior. Advertisement activities significantly influence consumer purchasing decisions and consequently affect the rate at which inventory is depleted. Incorporating advertisement effects into inventory modeling enables decision-makers to balance marketing efforts with operational efficiency. Motivated by this observation, the present study develops an inventory model in which demand is influenced by both inventory level and advertisement frequency. The model is formulated using differential equations, solved analytically, and validated through numerical illustration and graphical analysis. Inventory modeling has been extensively studied over the past several decades under various demand assumptions. Early classical inventory models, such as the Economic Order Quantity (EOQ) model, assumed constant demand and ignored marketing factors. However, with increasing market competition, researchers have recognized that demand is influenced by several external factors, including price, display, stock visibility, and advertisement. One of the earliest contributions to inventory modeling with variable demand was made by Silver and Peterson [7], who emphasized the importance of incorporating realistic demand structures into inventory decisions. Later, Levin et al. highlighted that marketing activities significantly affect consumer purchasing behavior and should be integrated with operational models. The concept of inventory-dependent demand was introduced by researchers such as Baker and Urban [1], who observed that higher inventory levels increase customer confidence and lead to higher demand. Subsequently, several authors extended this idea by incorporating stock-dependent demand into continuous-time inventory systems using differential equations. Advertisement-dependent demand has received increasing attention in recent years. Kotler [3] emphasized that advertisement plays a vital role in shaping demand patterns and market penetration. Inspired by this observation, researchers began incorporating advertisement as a decision variable in inventory systems. Urban and Baker [9] developed inventory models in which demand is influenced by promotional efforts and shelf display. Further studies by [2] Goyal and Gunasekaran examined the interaction between marketing decisions and inventory policies, concluding that coordinated advertisement and inventory planning leads to improved system performance. Teng and Chang [8] proposed inventory models with promotional effort-dependent demand and demonstrated that optimal advertisement policies significantly affect cycle time and profit. Several researchers have used power functions and exponential forms to represent advertisement effects on demand. For example, Sarkar and Majumder [5] incorporated advertisement frequency into demand functions and showed that demand increases nonlinearly with marketing effort. Pal, Sana, and Chaudhuri [4] analyzed inventory models with advertisement-dependent demand under various cost structures and highlighted the trade-off between holding cost and promotional expenses. More recently, Zanoni, Jaber, and Lazzari integrated advertisement-induced demand with sustainability considerations, showing that excessive promotion may lead to higher operational costs. Shaikh, Khan, and Panda [6] studied optimal pricing and advertisement strategies in inventory systems using differential equation-based formulations. Despite these contributions, relatively fewer studies have explicitly modeled advertisement impact using a power function of cost and frequency, combined with inventory-dependent demand in a continuous-time framework. Moreover, analytical solutions with clear boundary conditions and numerical validation remain limited. Motivated by these research gaps, the present study develops an inventory model incorporating both inventory-dependent and advertisement-dependent demand. The model is analytically solved using differential equations, and numerical as well as sensitivity analyses are provided to demonstrate its practical applicability. Pandey and Pandey [10] have developed an Inventory Model for Deteriorating Items considering two level storage with uniform demand and shortage under Inflation and completely backlogged. Pandey, H. and Pandey, A. [11] have developed an optimum inventory policy for exponentially deteriorating items, considering multi variate Consumption Rate with Partial Backlogging. Pandey et al. [12] have studied an EOQ Model with Ramp Type of Demand. Kumar et al [13] developed an Integrated Model with Variable Production and Demand Rate under Inflation. Pandey et al. [14] studied a Study of Production Inventory Policy with Stock Dependent Demand Rate. Pandey, A. [16] studied an EPQ Inventory Model for Deteriorating Items Considering Stock Dependent Demand and Time Varying Holding Cost. Again, Pandey et al. [17] investigated an EOQ MODEL WITH QUANTITY INCENTIVE STRATEGY FOR DETERIORATING ITEMS AND PARTIAL BACKLOGGING. Existing inventory management literature has extensively examined demand patterns influenced by either inventory level or advertisement activities. Classical inventory-dependent demand models assume that higher stock visibility stimulates demand, while advertisement-based models focus on promotional efforts as the primary driver of market demand. However, these two streams of research have largely evolved independently.

Most advertisement-dependent inventory models assume a linear or exponential relationship between advertisement effort and demand, where advertisement is treated as a continuous control variable without explicitly accounting for the frequency of advertisement exposure. In practical marketing environments, repeated advertisement campaigns often exhibit nonlinear and cumulative effects, which are not adequately captured by existing demand formulations.

Furthermore, the majority of integrated inventory–marketing models consider advertisement expenditure as a one-dimensional decision variable, neglecting the interaction between advertisement cost and its repetition frequency. As a result, the dynamic influence of sustained advertisement campaigns on demand acceleration and inventory depletion remains insufficiently explored. Another limitation of the existing literature is the reliance on purely numerical or heuristic solution approaches. Several studies do not provide closed-form analytical solutions for inventory dynamics, making it difficult to derive clear managerial insights regarding optimal cycle length and cost behavior. Additionally, sensitivity analysis in many prior studies is restricted to cost parameters or holding costs, with limited emphasis on the systematic impact of advertisement frequency on demand, cycle time, and total cost. Motivated by these gaps, the present study develops an analytically tractable inventory model in which demand is jointly influenced by inventory level and advertisement frequency through a nonlinear power function of advertisement cost. By explicitly modeling advertisement frequency and deriving closed-form solutions, this study addresses an important theoretical and practical gap in the inventory–advertisement literature. In the present paper we develop an inventory model in which demand is influenced by advertisement activities. The advertisement effect is represented by a power function of advertisement cost and its frequency.

2. Assumptions and Notations:

The following assumptions are made to develop the proposed inventory model:

1. A single-item inventory system is considered.
2. Replenishment is instantaneous and lead time is zero.
3. Shortages are not allowed.
4. The planning horizon is infinite.
5. Demand rate depends on inventory level and advertisement effort.
6. Advertisement impact is represented by the term B^v, where B is advertisement cost and v is its frequency.
7. Inventory depletion occurs only due to demand.
8. Holding cost per unit per unit time is constant.

Notation:

Symbol	**Description**
$I(t)$	**Inventory level at time** t
a	**Basic demand rate**
k	**inventory-dependent demand parameter**
B	**Cost of one advertisement**
v	**Advertisement frequency**
B^v	**Advertisement-induced demand**
I_0	**Initial inventory level**
T	**Length of inventory cycle**
h	**Holding cost per unit per unit time**
HC	**Holding cost per cycle**

3. Model Formulation and Solution

The demand rate at any time t **is assumed to be:**

$$D = a + k\,I(t) + B^v$$

The term a represents basic demand, $k\,I(t)$ represents inventory-dependent demand and B^v captures the effect of advertisement on demand.

Since inventory decreases only due to demand, the governing equation is given by

$$\frac{dI(t)}{dt} = -(\mathrm{a} + \mathrm{k}\,\mathrm{I(t)} + \mathrm{B^v}) \tag{1}$$

With boundary conditions $I(0) = I_0$ and $I(T) = 0$

By applying boundary conditions, the final inventory level function becomes:

$$I(t) = \frac{a + B^v}{k}[e^{k(T-t)} - 1]$$

Determination of cycle time:

At $t = 0$ $\quad I_0 = \frac{a+B^v}{k}[e^{kT} - 1]$ (2)

Solving for T, the cycle time is obtained as:

$$T = \frac{1}{k}\ln\left(1 + \frac{kI_0}{a+B^v}\right) \tag{3}$$

Holding Cost:

The holding cost per cycle is calculated as

$$\mathrm{HC} = \mathrm{h}\int_0^{\mathrm{T}} \mathrm{I(t)dt}$$

$$\mathrm{HC} = \frac{\mathrm{h(a+B^v)}}{\mathrm{k^2}}\left(\mathrm{e^{kT}} - 1 - \mathrm{kT}\right) \tag{4}$$

Total Cost:

$$\mathrm{TC(T)} = \frac{\mathrm{HC} + \mathrm{B^v}}{\mathrm{T}}$$

$$\mathrm{TC(T)} = \frac{1}{\mathrm{T}}\left[\frac{\mathrm{h(a + B^v)}}{\mathrm{k^2}}\left(\mathrm{e^{kT}} - 1 - \mathrm{kT}\right) + \mathrm{B^v}\right]$$

Optiimization:

The optimal cycle time $\mathrm{T^*}$ **is obtained by solving**

$$\frac{\partial \mathrm{TC(T)}}{\partial \mathrm{T}} = 0$$

With condition $\frac{\partial^2 \mathrm{TC(T)}}{\partial \mathrm{T}^2} = 0$

Which ensure minimum total cost.

Numerical Analysis and Graphs.

For the model illustration we consider the following numerical Values

$\mathrm{a} = 50$ **,** $\mathrm{k} = 0.2, \mathrm{B} = 0, \mathrm{v} = 2, \mathrm{h} = 1.5$ per unit per unit time, $\mathrm{I_0} = 500$

Advertisement Impact $\mathrm{B^v} = 100$

Cycle length is obtained from equation (3)

$$\mathrm{T} = 2.554$$

Table 1: Numerical Illustration of the Model, Parameters and Results

Parameter	Value
Basic Demand(a)	**20**
Inventory Parameter (k)	**0.2**
Advertisement cost(B)	**10**
Advertisement frequency(v)	**2**
Advertisement effect (B^v)	**100**
Initial Inventory (I_0)	**500**
Cycle Time (T)	**2.554**
Holding Cost per cycle	**876.61**
Advertisement cost	**20**
Average Total Cost	**351.04**

Figure 1:Effect of Advertisement frequency on average total cost

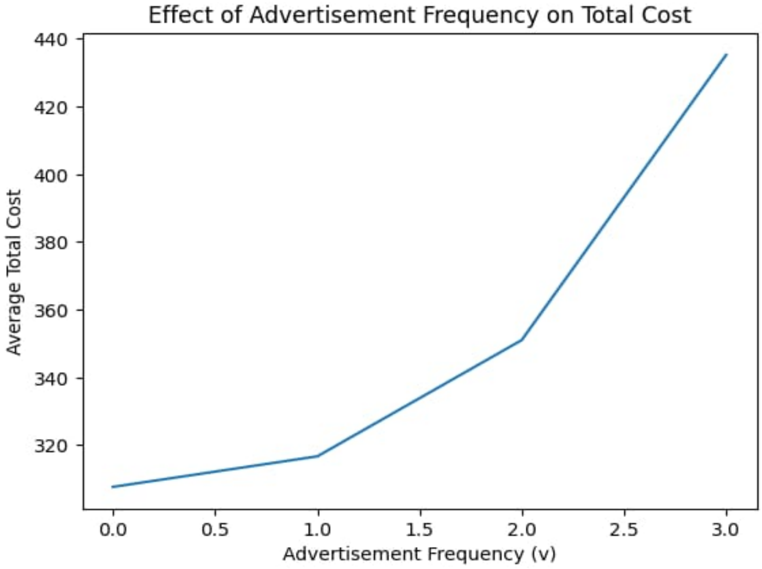
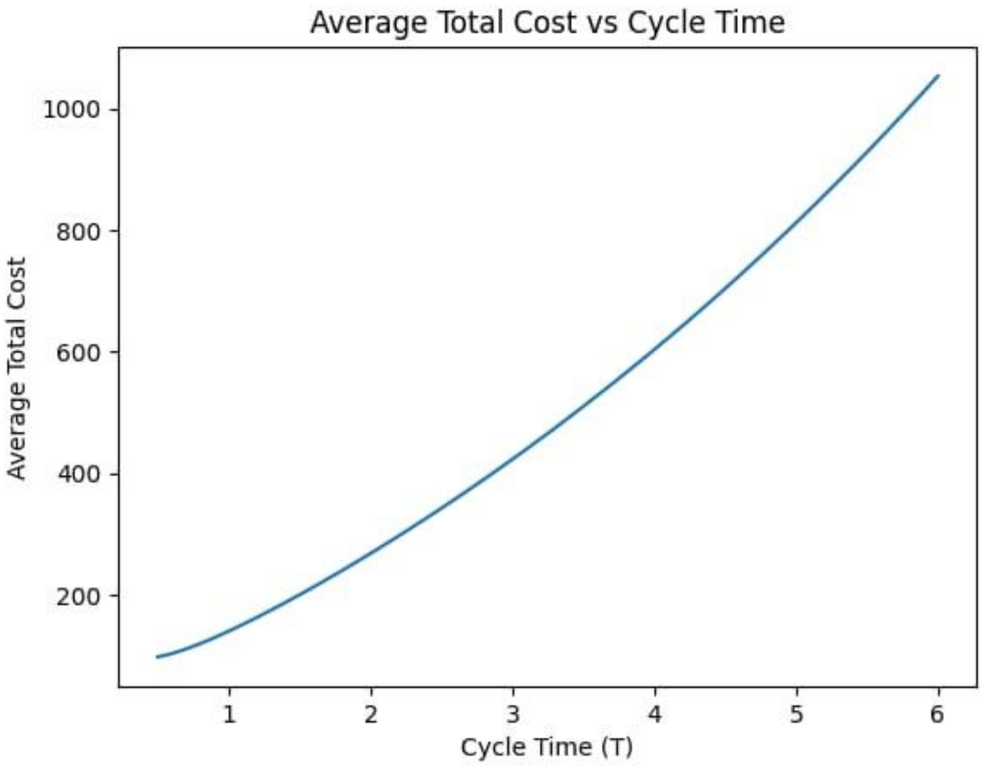
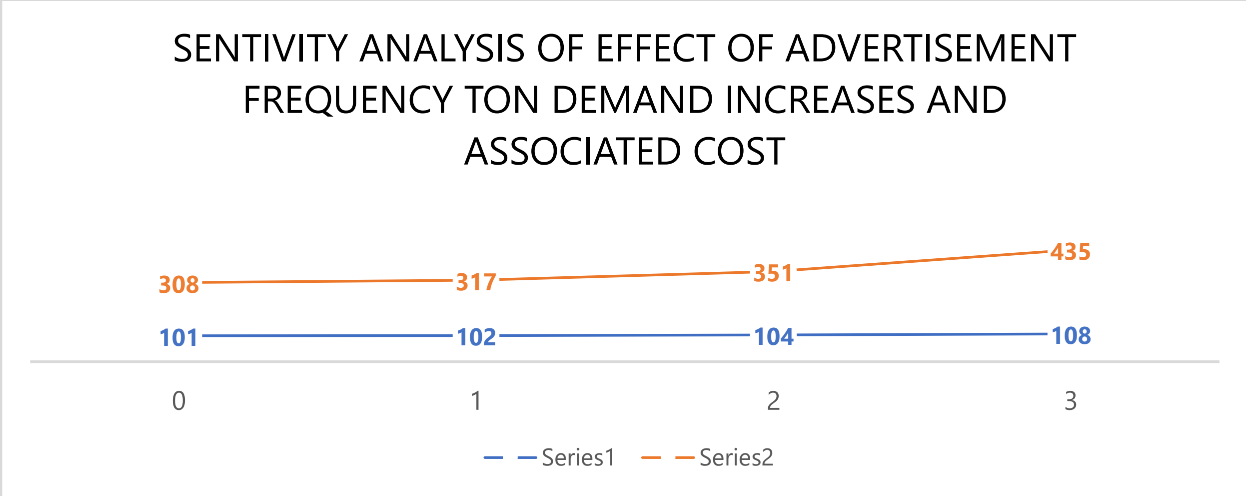


Figure: 2: Relationship between average total cost and inventory cycle time



Sensitivity analysis

Advertisement frequency (v)	Demand	Average total Cost
0	101	308
1	102	317
2	104	351
3	108	435



Conclusion

This paper presents an inventory model with advertisement-dependent demand. The system is formulated using a first-order differential equation and solved analytically under appropriate boundary conditions. Numerical illustration and sensitivity analysis demonstrate the significant role of advertisement in influencing demand and inventory behavior. The proposed model offers useful insights for managers in designing optimal inventory and advertisement strategies. Future research may extend the model by incorporating shortages, deterioration, or time-dependent advertisement effects.

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