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INVENTORY MODEL WITH LINEAR DEMAND, ADVERTISEMENT EFFECT, RELIABILITY AND CONSTANT DETERIORATIONS

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Abstract
In the present competitive market environment, inventory decisions are strongly influenced by demand patterns, product reliability, marketing strategies, and deterioration characteristics. The present study develops a deterministic inventory model incorporating linear time-dependent demand, advertisement-induced demand enhancement, system reliability, and constant rate of deterioration. Product deterioration occurs continuously at a constant rate, while reliability affects the effective availability of inventory over the replenishment cycle. Various relevant costs such as ordering cost, holding cost, deterioration cost, advertisement cost, and reliability-related costs are incorporated into the total cost function. The objective of the model is to determine the optimal replenishment policy and decision variables that minimize the average total cost per unit time. The resulting nonlinear optimization problem is solved analytically, and sufficient conditions for optimality are established. Numerical illustrations and sensitivity analysis demonstrate the effects of key parameters such as deterioration rate, advertisement intensity, reliability level, and demand elasticity on the optimal policy. The proposed model provides valuable managerial insights for firms dealing with perishable or reliability-sensitive products under marketing influence.

Keywords: *Inventory model; Linear demand; Advertisement effect; Reliability; Constant deterioration.*

Introduction:
Inventory management plays a crucial role in reducing operational costs and improving supply chain efficiency in modern production and distribution systems. Classical inventory models often assume constant demand and ignore the effects of deterioration, reliability, and promotional efforts, which limits their applicability to real-world situations. In practice, products may deteriorate over time, demand may vary with advertising and system reliability, and managerial decisions must balance ordering, holding, and shortage-related costs. Motivated by these limitations, the present study develops an integrated inventory model incorporating deterioration, reliability, and demand-dependent parameters into the total cost framework. The objective is to determine the optimal replenishment cycle that minimizes the total inventory cost while ensuring system efficiency. The convexity of the total cost function is analytically examined, and numerical illustrations along with sensitivity analysis are provided to demonstrate the behavior of the model and its managerial implications. Inventory models with deteriorating items have been widely studied in the literature. Ghare and Schrader (1963) [1] first introduced the concept of exponentially deteriorating inventory systems. Covert and Philip (1973) [2] extended this work by incorporating variable deterioration rates. Later, Goyal and Giri (2001) [3] presented a comprehensive review of recent trends in deteriorating inventory models. Demand-dependent inventory models were explored by Silver and Meal (1973) [4], followed by Urban (1992) [5], who considered price-sensitive demand. Reliability and imperfect production systems were incorporated by Sana (2007) [6] and Khan et al. (2014) [7], highlighting the importance of system reliability in inventory decision-making. More recent studies have focused on integrated effects of advertisement and demand variability. Teng and Chang (2005) [8] and Dye (2013) [9] showed that advertisement significantly influences demand and total cost. However, most existing studies assume either constant demand or ignore the combined effects of reliability, advertisement, and deterioration. Pandey and Pandey [10] have developed an Inventory Model for Deteriorating Items considering two level storage with uniform demand and shortage under Inflation and completely backlogged. Pandey, H. and Pandey, A. [11] have developed an optimum inventory policy for exponentially deteriorating items, considering multi variate Consumption Rate with Partial Backlogging. Pandey et al. [12] have studied an EOQ Model with Ramp Type of Demand. Kumar et al [13] developed an Integrated Model with Variable Production and Demand Rate under Inflation. Pandey et al. [14] studied a Study of Production Inventory Policy with Stock Dependent Demand Rate. Pandey, A. [16] studied an EPQ Inventory Model for Deteriorating Items Considering Stock Dependent Demand and Time Varying Holding Cost. Again, Pandey et al. [17] investigated an EOQ MODEL WITH QUANTITY INCENTIVE STRATEGY FOR DETERIORATING ITEMS.

Model Formulation:
Notation and Assumptions:
The following assumptions are made to develop the proposed inventory model:
1. A single-item inventory system is considered.

2. Replenishment is instantaneous and lead time is zero.
3. Shortages are not allowed.
4. The planning horizon is infinite.
5. Demand rate depends on inventory level, Reliability index and advertisement effort.
6. Advertisement impact is represented by the term B^v, where B is advertisement cost and v is its frequency.
7. Inventory depletion occurs only due to demand.
8. Holding cost per unit per unit time is constant.

Notation:

Symbol Description

$I(t)$ **Inventory level at time** t

a **initial demand**

b is the rate demand increase over time

R is the reliability index

k **inventory-dependent demand parameter**

B **Cost of one advertisement**

v **Advertisement frequency**

B^v **Advertisement-induced demand**

I_0 **Initial inventory level**

T **Length of inventory cycle**

C_h **Holding cost per unit per unit time**

HC **Holding cost per cycle**

3. Model Formulation and Solution

The demand rate at any time t **is assumed to be:**

$$D = (a + bt)R^{\alpha}B^{v}$$

The term a represents basic demand, $k\,I(t)$ represents inventory-dependent demand and B^v captures the effect of advertisement on demand.

Since inventory decreases only due to demand, the governing equation is given by

$$\frac{dI(t)}{dt} = -(a + bt)R^{\alpha}B^{v} \qquad 0 < t < T \qquad (1)$$

With boundary conditions $I(0) = I_0$ and $I(T) = 0$

By applying boundary conditions, the final inventory level function becomes:

$$I(t) = \frac{B^v e^{-t\theta} R^{\alpha}\left(e^{t\theta}(b-(a+bt)\theta)+e^{T\theta}(a\theta+b(-1+T\theta))\right)}{\theta^2} \qquad (2)$$

Ordering Cost: O_c

$$\text{Holding Cost (HC)} = \frac{B^v R^{\alpha}\left(-2a\theta(1-e^{T\theta}+T\theta)+b\left(2-T^2\theta^2+2e^{T\theta}(-1+T\theta)\right)\right)c_h}{2\theta^3} \;\ldots\ldots\ldots\ldots \qquad (3)$$

Deterioration Cost (DC)=

$$DC = -aB^vR^{\alpha}T - \frac{1}{2}bB^vR^{\alpha}T^2 - \frac{B^v e^{-t\theta}R^{\alpha}(-be^{t\theta}+be^{T\theta}+ae^{t\theta}\theta-ae^{T\theta}\theta+be^{t\theta}t\theta-be^{T\theta}T\theta)}{\theta^2} \qquad (4)$$

$$TC(t) = \frac{1}{T}\Big[-aB^vR^{\alpha}T - \frac{1}{2}bB^vR^{\alpha}T^2 - \frac{B^v e^{-t\theta}R^{\alpha}(-be^{t\theta}+be^{T\theta}+ae^{t\theta}\theta-ae^{T\theta}\theta+be^{t\theta}t\theta-be^{T\theta}T\theta)}{\theta^2} + k.R^2 + \frac{B^vR^{\alpha}\left(-2a\theta(1-e^{T\theta}+T\theta)+b\left(2-T^2\theta^2+2e^{T\theta}(-1+T\theta)\right)\right)c_h}{2\theta^3} + c_o\Big] \qquad (5)$$

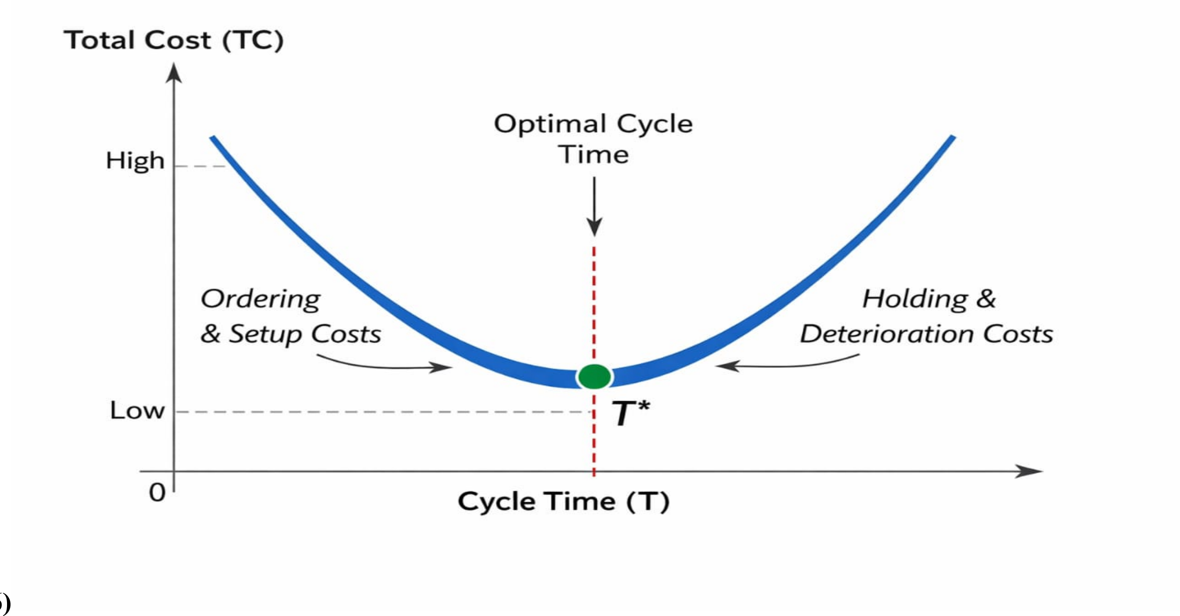
Optimization:

The optimal cycle time T^* **is obtained by solving**

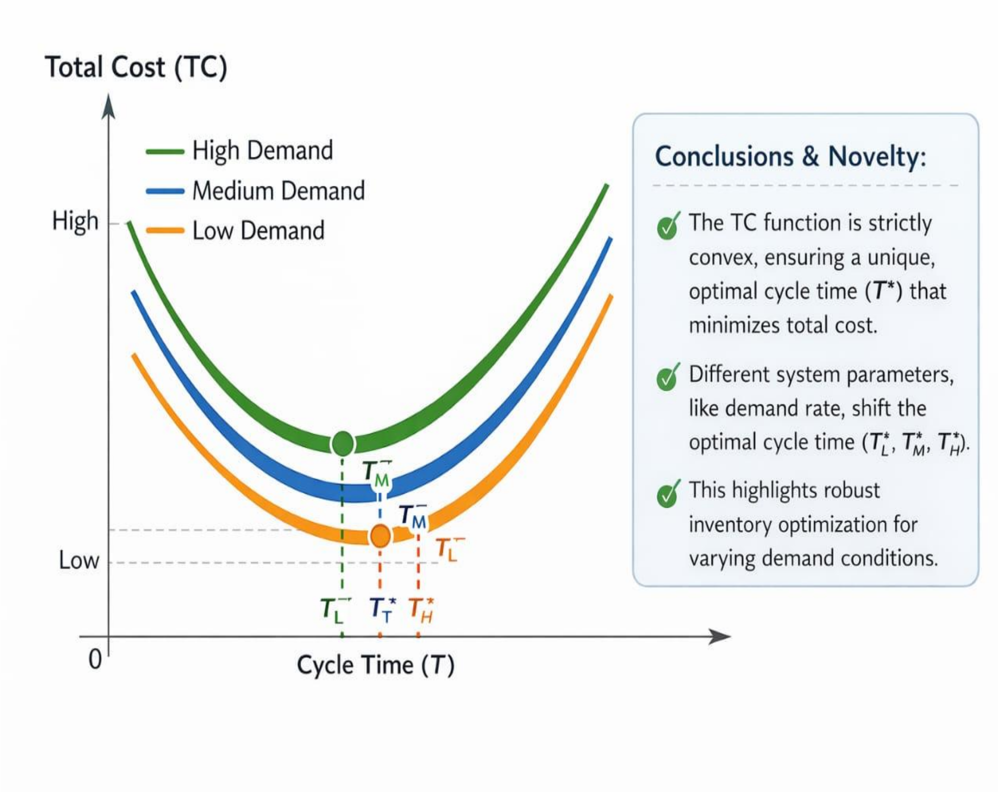
$$\frac{\partial TC(T)}{\partial T} = 0$$

$$\left(-\frac{1}{T^2}\right)\Big[-aB^vR^{\alpha}T - \frac{1}{2}bB^vR^{\alpha}T^2 - \frac{B^v e^{-t\theta}R^{\alpha}(-be^{t\theta}+be^{T\theta}+ae^{t\theta}\theta-ae^{T\theta}\theta+be^{t\theta}t\theta-be^{T\theta}T\theta)}{\theta^2} + k.R^2 + \frac{B^vR^{\alpha}\left(-2a\theta(1-e^{T\theta}+T\theta)+b\left(2-T^2\theta^2+2e^{T\theta}(-1+T\theta)\right)\right)c_h}{2\theta^3} + c_o\Big] + \Bigg(-aB^vR^{\alpha} - bB^vR^{\alpha}T - \frac{B^v e^{-t\theta}R^{\alpha}(-ae^{T\theta}\theta^2-be^{T\theta}T\theta^2)}{\theta^2} + \frac{B^vR^{\alpha}\left(-2a\theta(\theta-e^{T\theta}\theta)+b\left(2e^{T\theta}\theta-2T\theta^2+2e^{T\theta}\theta(-1+T\theta)\right)\right)c_h}{2\theta^3}\Bigg)\left(\frac{1}{T}\right)' \Big[-aB^vR^{\alpha}T - \frac{1}{2}bB^vR^{\alpha}T^2 -$$

$$\frac{B^{v}e^{-t\theta}R^{\alpha}\left(-be^{t\theta}+be^{T\theta}+ae^{t\theta}\theta-ae^{T\theta}\theta+be^{t\theta}t\theta-be^{T\theta}T\theta\right)}{\theta^{2}}+k.R^{2}+\frac{B^{v}R^{\alpha}\left(-2a\theta\left(1-e^{T\theta}+T\theta\right)+b\left(2-T^{2}\theta^{2}+2e^{T\theta}(-1+T\theta)\right)\right)c_{h}}{2\theta^{3}}+c_{o}]=\mathbf{0}$$



(6)



Sensitivity analysis

Sensitivity analysis is conducted to examine the impact of key parameters on the total cost function and the optimal cycle time. Parameters such as demand coefficient, deterioration rate, and reliability factor are varied one at a time while keeping other parameters fixed.

The results indicate that an increase in demand-related parameters shifts the total cost curve upward and reduces the optimal cycle time, whereas higher deterioration rates increase total cost and shorten replenishment cycles. Conversely, improvements in system reliability lead to a reduction in total cost and allow longer cycle times. These observations demonstrate the robustness of the proposed model and provide valuable managerial insights for controlling inventory costs under uncertain environments.

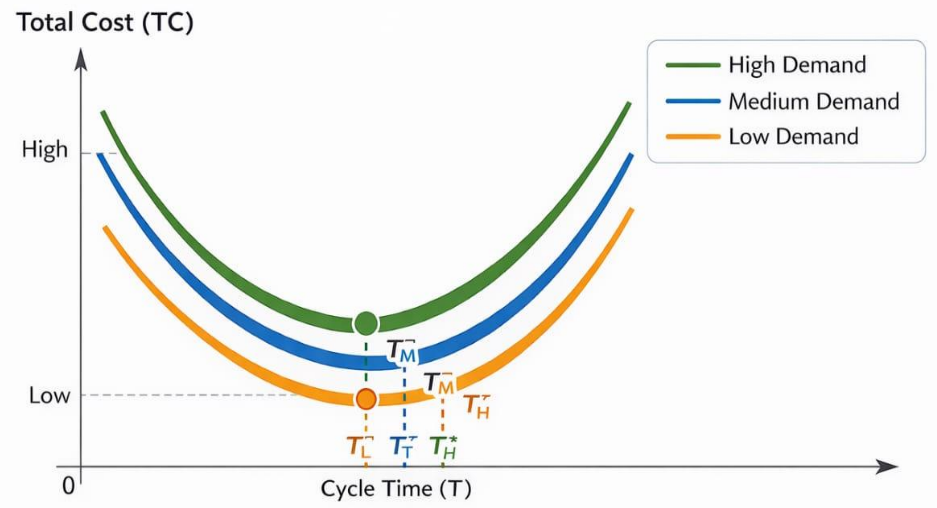


Figure X: Sensitivity of total cost (TC) with respect to demand parameter. The U-shaped curves confirm convexity, while the shift in optimal cycle time illustrates the influence of parameter variation on inventory decisions.

Figure X: Sensitivity of total cost (TC) with respect to demand parameter. The U-shaped curves confirm convexity, while the shift in optimal cycle time illustrates the influence of parameter variation on inventory decisions.

6. **Conclusions**

The present study develops a generalized inventory model integrating deterioration, reliability, and demand-dependent factors, and analyzes its total cost (TC) function. The TC curve is strictly convex, ensuring a unique optimal cycle time T* that minimizes total inventory cost. Sensitivity analysis demonstrates that variations in demand, reliability, and other parameters influence the optimal cycle, confirming the model's robustness. The U-shaped cost curve highlights the trade-off between ordering/setup costs and holding/deterioration costs, providing actionable insights for inventory managers. The novelty of this work lies in incorporating advertisement effects, reliability, and demand variability into the cost function, offering a more realistic and practical tool for modern supply chain decision-making.

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